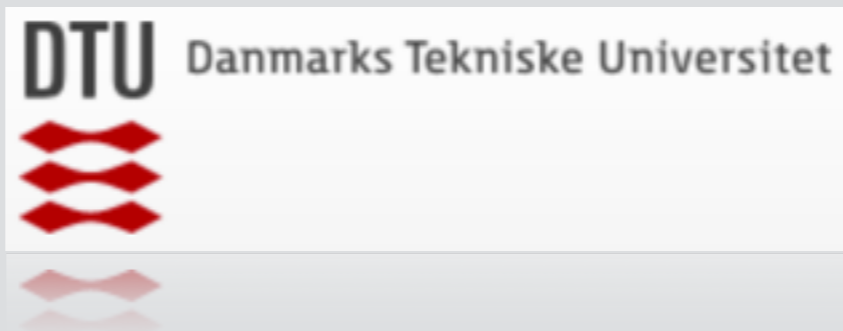


# Distributed MPC: a comprehensive overview and some recent advances

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May 31, 2016



# Model Predictive Control: motivations for its success

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## Fundamental features

- ☑ MPC is the most **successful** advanced control technique applied in the process industries
- ☑ MPC can ensure (for linear and nonlinear multivariable systems)
  - closed-loop **stability**
  - **constraint** satisfaction
  - **robustness** against modeling errors and disturbances
  - setpoint **tracking** (often **hierarchically**)
- ☑ MPC enhances the process **profitability**

# Large scale systems: some examples

Large industrial plants



Power generation networks



Systems of Systems

# MPC for large scale systems: different approaches

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## One or more MPCs?

### One **centralized MPC**

- Pros: global (**plant-wide**) optimality, stability
- Cons: limited **flexibility**, high **computational** cost

### Several **decentralized MPCs**

- Pros: **lower** computational cost, high **modularity**
- Cons: global **suboptimality**, stability issues

## Middle field...

### Distributed **MPC**

- Pros: high **modularity**, stability
- Pros/Cons: global optimality (high **computational** cost)

# Outline

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1. Comprehensive overview of (linear) distributed MPC algorithms
  - Models
  - Decentralized, Cooperative and Non-Cooperative MPC
  - Design and properties of Cooperative MPC
  - Cooperative MPC for tracking
2. Advances in cooperative MPC for tracking
  - Proposed approaches
  - Application results
3. Conclusions

# Part I

## A comprehensive overview of distributed MPC

# Models for distributed MPC - I

## Overall DLTI system

$$\begin{aligned}x^+ &= Ax + Bu \\ y &= Cx\end{aligned}$$

$x \in \mathbb{R}^n$ : current state  
 $x^+ \in \mathbb{R}^n$ : successor state  
 $u \in \mathbb{R}^m$ : manipulated input  
 $y \in \mathbb{R}^p$ : controlled output

## Local DLTI subsystems

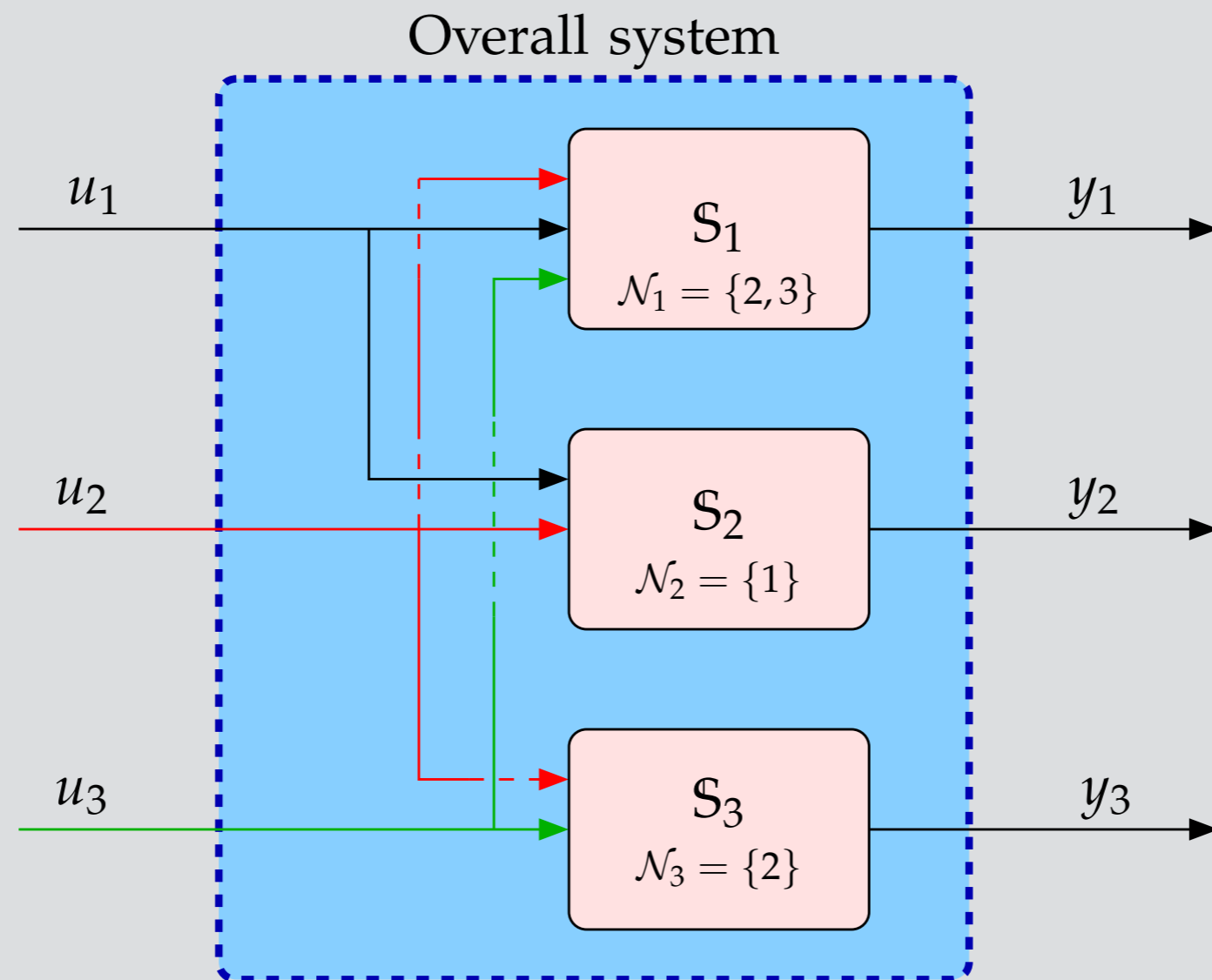
$$\begin{aligned}x_i^+ &= A_i x_i + B_i u_i + \sum_{j \in \mathcal{N}_i} B_{ij} u_j \\ y_i &= C_i x_i\end{aligned}$$

$M$ : number of subsystems  
 $\mathcal{N}_i$ : set of neighbors of subsystem  $i$

$x_i, x_i^+ \in \mathbb{R}^{n_i}$ : current/successor  $i$ -th state  
 $u_i \in \mathbb{R}^{m_i}$ : manipulated  $i$ -th input  
 $y_i \in \mathbb{R}^{p_i}$ : controlled  $i$ -th output

# Models for distributed MPC - II

## Interconnected systems and neighbors definition





# Models for distributed MPC - III

## Why not coupling through states?

- ✓ The state of each subsystem includes all modes from **local** and **neighboring inputs** to local **output**

- An example of 2 subsystems “**coupled by states**”:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^+ = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} B_{11} & 0 \\ 0 & B_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

- Equivalent** subsystems **coupled by inputs**:

$$\text{Subsystem 1: } x_1 \leftarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad A_1 \leftarrow \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}, \quad B_1 \leftarrow \begin{bmatrix} B_{11} \\ 0 \end{bmatrix}, \quad B_{12} \leftarrow \begin{bmatrix} 0 \\ B_{22} \end{bmatrix}, \quad C_1 \leftarrow [C_{11} \quad 0]$$

$$\text{Subsystem 2: } x_2 \leftarrow [x_2], \quad A_2 \leftarrow [A_{22}], \quad B_2 \leftarrow [B_{22}], \quad B_{21} \leftarrow [], \quad C_2 \leftarrow [C_{22}]$$

- ✓ Models obtained from **ID** naturally have **input couplings**

# Taxonomy

## Based on dynamics and objective of local controllers

- ✓ **Centralized MPC:** a single MPC computes **all inputs** to optimize a **global objective**
- ✓ **Decentralized MPC:** each MPC computes its **local input**, disregarding interacting dynamics, to optimize a **local objective**
- ✓ **Non-cooperative MPC:** each MPC computes its local input, considering interacting dynamics, to optimize a **local objective**
- ✓ **Cooperative MPC:** each MPC computes its local input, considering interacting dynamics, to optimize a **global objective**

# Decentralized MPC

Model (**decentralized**)

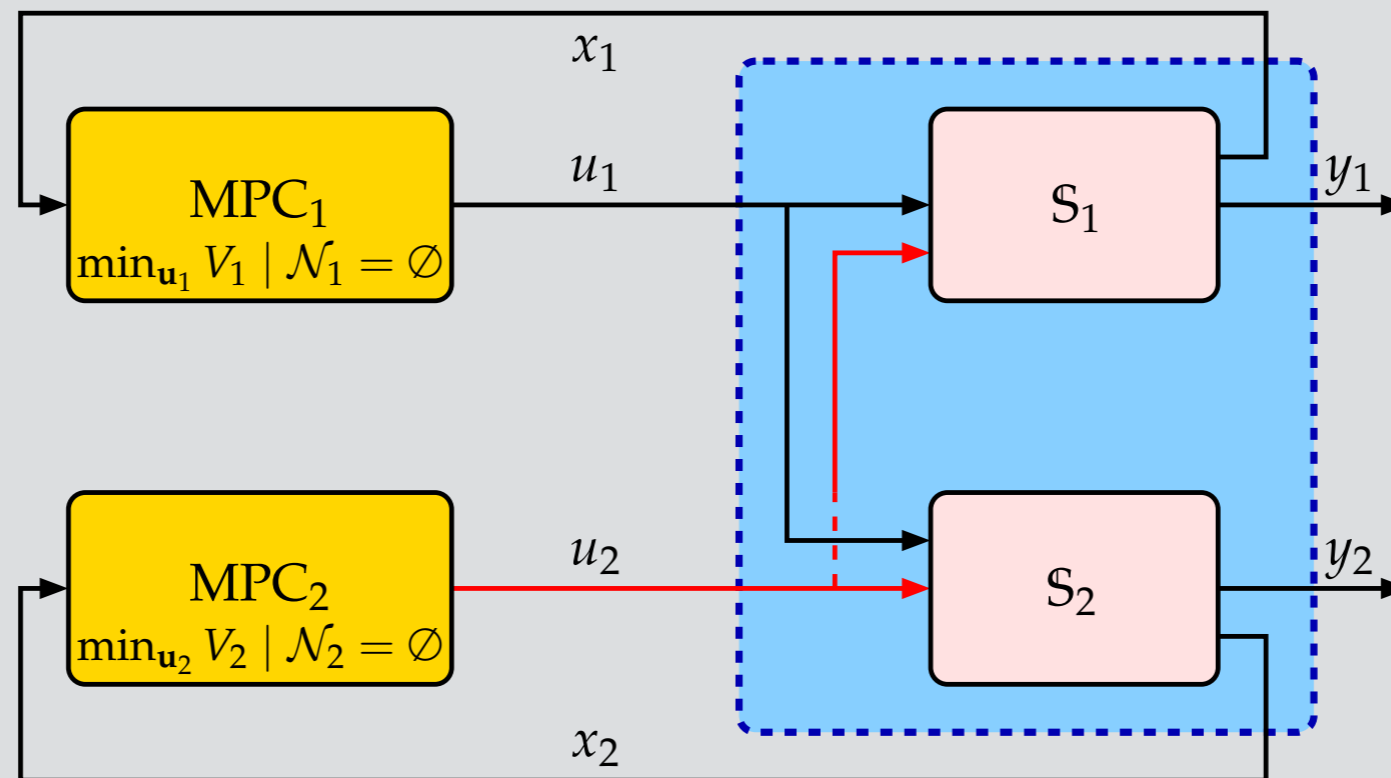
$$x_i^+ = A_i x_i + B_i u_i + \sum_{j \in \mathcal{N}_i} B_{ij} u_j$$

$$y_i = C_i x_i$$

Cost (**local**)

$$V_i = \sum_{k=0}^{N-1} (x_i(k)^T Q_i x_i(k) + u_i(k)^T R_i u_i(k)) + x_i(N)^T P_i x_i(N)$$

Reference Scheme



# Non-cooperative Distributed MPC

Model (**interacting**)

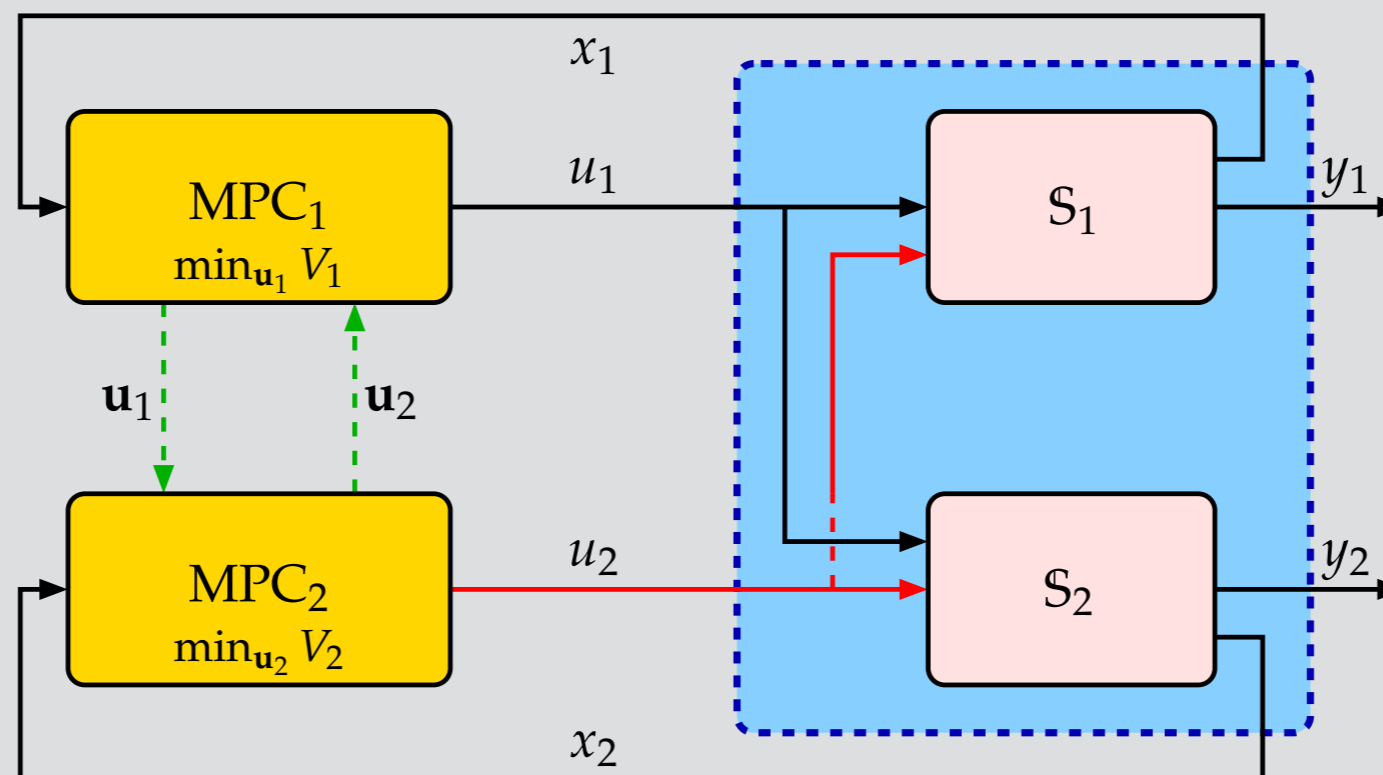
$$x_i^+ = A_i x_i + B_i u_i + \sum_{j \in \mathcal{N}_i} B_{ij} u_j$$

$$y_i = C_i x_i$$

Cost (**local**)

$$V_i = \sum_{k=0}^{N-1} (x_i(k)^T Q_i x_i(k) + u_i(k)^T R_i u_i(k)) + x_i(N)^T P_i x_i(N)$$

Reference Scheme



# Cooperative Distributed MPC

Model (**interacting**)

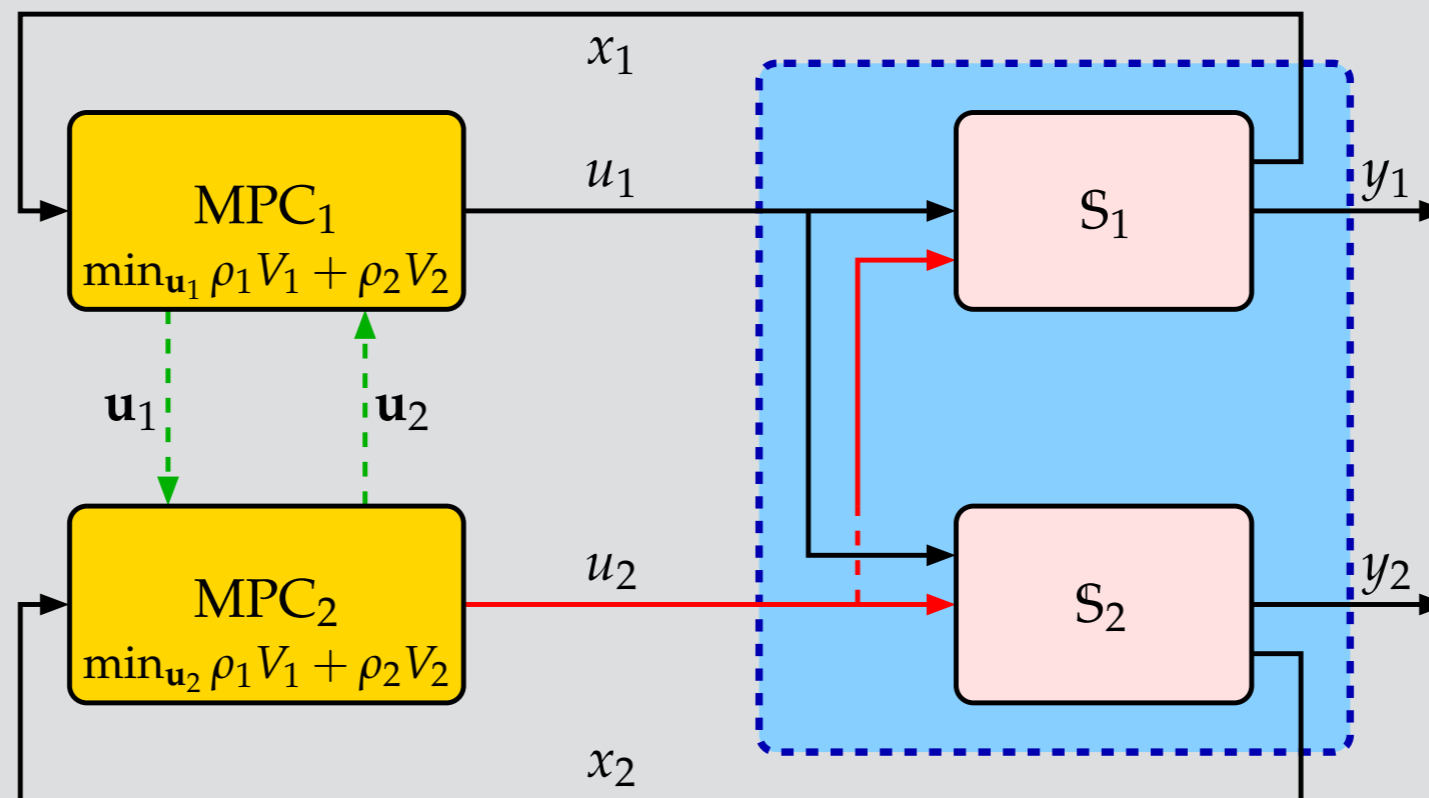
$$x_i^+ = A_i x_i + B_i u_i + \sum_{j \in \mathcal{N}_i} B_{ij} u_j$$

$$y_i = C_i x_i$$

Cost (**global**)

$$V = \sum_{j=1}^M \rho_j V_j = \sum_{k=0}^{N-1} (x(k)^T Q x(k) + u(k)^T R u(k)) + x(N)^T P x(N)$$

Reference Scheme



# Properties of different architectures

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## ☑ Decentralized MPC

- Due to neglected dynamics, **stability** (as well feasibility, tracking, etc.) are **not guaranteed**
- Possible remedies are based on **robust** (tube) **MPC** paradigms [Alessio et al., 2011; Riverso et al., 2013]

## ☑ Noncooperative distributed MPC

- Due to local objectives, **convergence** of iterations and **stability** is **not guaranteed**
- When convergence occurs, **Nash equilibrium** is reached. Still, stability may not hold [Rawlings and Mayne, 2009]

## ☑ Cooperative distributed MPC

- **Convergence** of iterations and **stability** is **guaranteed** [Rawlings and Mayne, 2009]
- **Global optimality** can be guaranteed [Stewart et al., 2011]

# Cooperative distributed MPC - I

## FHOCP and cooperative iterations

- Each local MPC, knowing **candidate sequences** of all other MPCs, solves the FHOCP

$$\mathbb{P}_i \left( x, \{ \mathbf{u}_j \}_{j \neq i} \right) : \min_{\mathbf{u}_i} V(x, \mathbf{u}) \quad \text{s.t.}$$

$$\mathbf{u}_i \in \mathcal{U}_i \left( x, \{ \mathbf{u}_j \}_{j \neq i} \right), \quad x(N) \in \mathbb{X}_f$$

Keep track of overall state dynamics

- Input **feasibility** space

$$\mathcal{U}_i \left( x, \{ \mathbf{u}_j \}_{j \neq i} \right) = \{ \mathbf{u}_i \mid u_i(k) \in \mathbb{U}_i, x(k) \in \mathbb{X} \}$$

- Cooperative** iterations, given  $\mathbf{u}_i^0$  : solution to  $\mathbb{P}_i \left( x, \{ \mathbf{u}_j^{[q-1]} \}_{j \neq i} \right)$

$$\mathbf{u}_i^{[q]} = w_i \mathbf{u}_i^0 + (1 - w_i) \mathbf{u}_i^{[q-1]}$$

# Cooperative distributed MPC - II

## Main properties

☑ **Feasibility** of each iterate

$$\mathbf{u}_i^{[q-1]} \in \mathcal{U}_i^N \Rightarrow \mathbf{u}_i^{[q]} \in \mathcal{U}_i^N, \text{ for all } i = 1, \dots, M \text{ and } q \in \mathbb{I}_{\geq 0}$$

☑ Cost **decrease** at each iteration

$$V(x(0), \mathbf{u}^{[q]}) \leq V(x(0), \mathbf{u}^{[q-1]}) \text{ for all } q \in \mathbb{I}_{\geq 0}$$

☑ Cost **convergence** to centralized optimum

$$\lim_{q \rightarrow \infty} V(x(0), \mathbf{u}^{[q]}) = \min_{\mathbf{u} \in \mathcal{U}^N} V(x(0), \mathbf{u})$$

☑ **Stability**, for any finite  $q$ , is proved via **suboptimal MPC** arguments

Can reduce  
computational  
requirements



# Tracking in centralized MPC

## Equilibrium target

✓ Any **equilibrium** solves: 
$$\begin{bmatrix} A - I & B & 0 \\ C & 0 & -I \end{bmatrix} \begin{bmatrix} x_s \\ u_s \\ y_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

✓ Given **desired setpoint**  $y_t$ , the optimal equilibrium is:

$$\min_{x_s, u_s, y_s} V_{ss}(y_s, y_t) \quad \text{s.t. above constraint and } x_s \in \mathbb{X}, \quad u_s \in \mathbb{U}$$

## Tracking MPC problem

✓ **Deviation** variables:  $\tilde{x} = x - x_s^0, \quad \tilde{u} = u - u_s^0$

$$\mathbb{P}(\tilde{x}) : \quad V^0(\tilde{x}) = \min_{\tilde{u}} \{ V(\tilde{x}(0), \tilde{u}) \mid \tilde{u} \in \tilde{\mathcal{U}}_N(\tilde{x}) \}$$

✓ **Single step** approaches are also possible [Limon et al., 2008]

# Distributed cooperative MPC for tracking - I

## Single step approach [Ferramosca et al., 2013]

☑ **Centralized** cost function, including centralized **target**

$$V_t(x, \mathbf{u}, x_s, u_s, y_s) = \sum_{k=0}^{N-1} \ell(x(k) - x_s, u(k) - u_s) + V_f(x(N) - x_s) \\ + V_{ss}(y_s, y_t)$$

$$\text{s.t. } x(0) = x$$

$$x(k+1) = Ax(k) + Bu(k)$$

$$\begin{bmatrix} A - I & B & 0 \\ C & 0 & -I \end{bmatrix} \begin{bmatrix} x_s \\ u_s \\ y_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

# Distributed cooperative MPC for tracking - II

## FHOCP and cooperative iterations

- Each local MPC, knowing **candidate** sequences of all other MPCs, solves the FHOCP

$$\mathbb{P}_i \left( x, \{ \mathbf{u}_j \}_{j \neq i} \right) : \min_{\mathbf{u}_i, x_s, u_s, y_s} V_t (x, \mathbf{u}, x_s, u_s, y_s) \quad \text{s.t.}$$

$$\mathbf{u}_i \in \mathcal{U}_i \left( x, \{ \mathbf{u}_j \}_{j \neq i} \right), \quad (x(N), y_s) \in \Omega$$

- $\Omega$  is an **invariant set** for tracking [Ferramosca et al. 2013]

- Cooperative** iterations, given  $\mathbf{u}_i^0 : \text{set of } \{ \mathbf{u}_j^0 \}_{j \neq i}$

$$\mathbf{u}_i^{[q]} = w_i \mathbf{u}_i^0 + (1 - w_i) \mathbf{u}_i^{[q-1]}$$

Need to keep track of centralized state dynamics

# Part II

## Advances in cooperative MPC for tracking

# Some reminders of graph theory

## Useful definitions

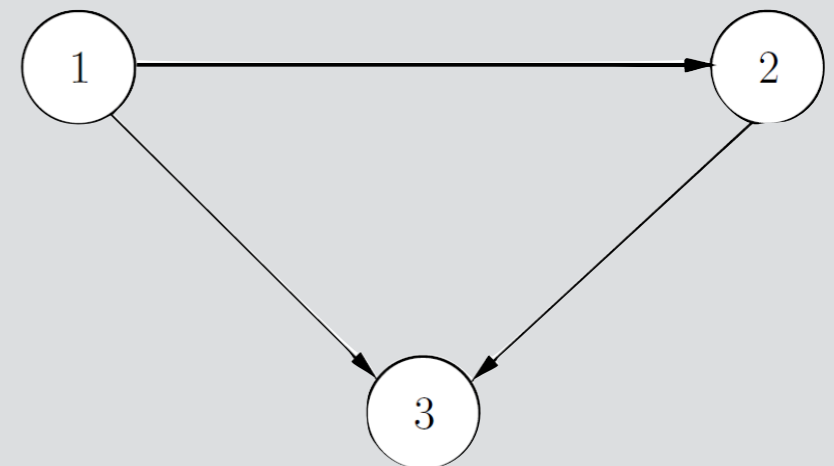
- A graph  $G = (\mathcal{V}, \mathcal{E})$  : set of **vertices**  $\mathcal{V}$  and **edges**  $\mathcal{E}$
- A **directed graph** is composed by oriented edges
- **Inlet** star:  $S_i^{IN} = \{v_j \in \mathcal{V} \mid (v_j, v_i) \in \mathcal{E}\}$
- **Outlet** star:  $S_i^{OUT} = \{v_j \in \mathcal{V} \mid (v_i, v_j) \in \mathcal{E}\}$

## An example

$$x_1^+ = A_1 x_1 + B_1 u_1$$

$$x_2^+ = A_2 x_2 + B_2 u_2 + B_{21} u_1$$

$$x_3^+ = A_3 x_3 + B_3 u_3 + B_{31} u_1 + B_{32} u_2$$



$$S_1^{IN} = \emptyset, S_1^{OUT} = \{2, 3\}, S_2^{IN} = \{1\}, S_2^{OUT} = \{3\}, S_3^{IN} = \{1, 2\}, S_3^{OUT} = \emptyset$$

# The augmented system - I

## Key observations

- ✓ The evolution of  $i$ -th subsystem is **influenced** by inputs of subsystems in its **inlet star**
- ✓ The input of  $i$ -th subsystem **influences** evolution of subsystems in its **outlet star**

$$x_i^+ = A_i x_i + B_i u_i + \sum_{k \in S_i^{IN}} B_{ik} u_k$$

$$x_j^+ = A_j x_j + B_{ji} u_i + \left( B_j u_j + \sum_{k \in S_j^{IN} \setminus \{i\}} B_{jk} u_k \right), \quad j \in S_i^{OUT}$$

- ✓ Other subsystems are **not affected** by the  $i$ -th subsystem input

$$x_j^+ = A_j x_j + \left( B_j u_j + \sum_{k \in S_j^{IN}} B_{jk} u_k \right), \quad j \notin S_i^{OUT}$$

# The augmented system - II

## Augmented i-th subsystem

- ✓ Augmented inlet star

$$\mathcal{S}_i^{IN} \leftarrow \mathcal{S}_i^{IN} \cup \mathcal{S}_i^{OUT} \cup \left( \bigcup_{j \in \mathcal{S}_i^{OUT}} \mathcal{S}_j^{IN} \setminus \{i\} \right)$$

- ✓ Augmented local variables and matrices

$$\bar{x}_i = \begin{bmatrix} x_i \\ [x_j]_{j \in \mathcal{S}_i^{OUT}} \end{bmatrix}, \quad \bar{u}_i = [u_k]_{k \in \mathcal{S}_i^{IN}}, \quad \bar{y}_i = \begin{bmatrix} y_i \\ [y_j]_{j \in \mathcal{S}_i^{OUT}} \end{bmatrix}$$

$$\bar{A}_i = \text{diag} \left\{ A_i, \{A_j\}_{j \in \mathcal{S}_i^{OUT}} \right\}, \quad \bar{B}_i = \begin{bmatrix} B_i \\ [B_{ji}]_{j \in \mathcal{S}_i^{OUT}} \end{bmatrix}, \quad \bar{B}_i^{IN} = [\text{hor}\{B_{ik}\}_{k \in \mathcal{S}_i^{IN}}, \text{hor}\{B_{jk}\}_{j \in \mathcal{S}_i^{OUT}, k \in \mathcal{S}_i^{IN}}]$$

- ✓ Final augmented dynamics

$$\bar{x}_i^+ = \bar{A}_i \bar{x}_i + \bar{B}_i u_i + \bar{B}_i^{IN} \bar{u}_i$$

$$\bar{y}_i = \bar{C}_i \bar{x}_i$$

# Augmented system based cooperative MPC for tracking - I

## Local (augmented system) cost function

☑ The global cost function can be **reduced** to a local (augmented) cost

$$V_{ti}(\cdot) = \sum_{k=0}^{N-1} \bar{\ell}_i(\bar{x}_i(k) - \bar{x}_{si}, u_i(k) - u_{si}) + \bar{V}_{fi}(\bar{x}_i(k) - \bar{x}_{si}) + \|y_s - y_t\|_T^2$$

s.t.

$$\bar{x}_i(0) = \bar{x}_i$$

$$\bar{x}_i(k+1) = \bar{A}_i \bar{x}_i(k) + \bar{B}_i u_i(k) + \bar{B}_i^{IN} \bar{u}_i(k)$$

Local (augmented) cost functions

$$\bar{\ell}_i(\bar{x}_i, u_i) = \frac{1}{2} (\bar{x}_i^T \bar{Q}_i \bar{x}_i + u_i^T R_i u_i) + \frac{1}{2} \bar{x}_i^T \bar{P}_i \bar{x}_i$$

The solution is the same as considering the global cost

☑ Discarded terms in the global function are **not affected** by  $u_i$



# Augmented system based cooperative MPC for tracking - II

## FHOCP and cooperative iterations

- Each local MPC, knowing **candidate** sequences of all MPCs in its **inlet star**, solves the FHOCP

$$\mathbb{P}_i \left( \bar{x}_i, \{\mathbf{u}_j\}_{j \in \mathcal{S}_i^N} \right) : \min_{\mathbf{u}_i, x_s, u_s, y_s} V_{ti}(\mathbf{u}_i, x_s, u_s, y_s) \text{ s.t.}$$

$$\mathbf{u}_i \in \bar{\mathcal{U}}_i \left( \bar{x}_i, \{\mathbf{u}_j\}_{j \in \mathcal{S}_i^N} \right), \quad (\bar{x}_i(N), \bar{y}_{si}) \in \bar{\Omega}_i$$

$$\begin{bmatrix} A - I & B & 0 \\ C & 0 & -I \end{bmatrix} \begin{bmatrix} x_s \\ u_s \\ y_s \end{bmatrix}$$

Need to keep track of augmented state dynamics only

- Input **feasibility** space

$$\bar{\mathcal{U}}_i \left( \bar{x}_i, \{\mathbf{u}_j\}_{j \in \mathcal{S}_i^N} \right) = \{ \mathbf{u}_i \mid u_i(k) \in \mathcal{U}_i, \bar{x}_i(k) \in \bar{\mathcal{X}}_i \}$$

- Cooperative iterations:

$$\mathbf{u}_i^{[q]} = w_i \mathbf{u}_i^0 + (1 - w_i) \mathbf{u}_i^{[q-1]}$$

# Overall algorithm

**Require:** Augmented subsystems,  $\mathbb{S}_i^N \forall i = 1 \dots M$ , tolerance  $\epsilon$ , maximum no. cooperative iterations  $q_{max}$ , convex combination weights  $w_i > 0$ , such that  $\sum_{i=1}^M w_i = 1$ .

- 1: Set  $q \leftarrow 0$  and  $e_i \leftarrow 2\epsilon$ .
- 2: **while**  $q < q_{max}$  and  $\exists i$  such that  $e_i > \epsilon$  **do**
- 3:      $q \leftarrow q + 1$
- 4:     **for**  $i = 1$  **to**  $M$  **do**
- 5:         Solve problem  $\mathbb{P}_i$  to obtain the optimal input sequence  $\mathbf{u}_i^0(x)$  and the centralized state-steady triple  $(x_s, u_s, y_s)$ .
- 6:         **if**  $q = 1$  **then**
- 7:              $\mathbf{u}_i^{[q-1]} = [u_{s_i}^T \quad \dots \quad u_{s_i}^T]^T$
- 8:         **end if**
- 9:         Define new iterate:  $\mathbf{u}_i^{[q]} = w_i \mathbf{u}_i^0 + (1 - w_i) \mathbf{u}_i^{[q-1]}$ .
- 10:         Compute convergence error:  $e_i = \frac{\|\mathbf{u}_i^{[q]} - \mathbf{u}_i^{[q-1]}\|}{1 + \|\mathbf{u}_i^{[q]}\|}$
- 11:     **end for**
- 12: **end while**
- 13: **return** Overall solution  $\mathbf{u} = (\mathbf{u}_1^{[q]}, \mathbf{u}_2^{[q]}, \dots, \mathbf{u}_M^{[q]})$ .

# A two-step variant

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## Step 1

- ☑ Solve the centralized target problem to obtain  $(x_s, u_s, y_s)$

## Step 2

- ☑ Each local MPC, knowing **candidate** sequences of all MPCs in its **inlet star**, solves the FHOCP with known  $(x_s, u_s, y_s)$
- ☑ Cooperative iterations: 
$$\mathbf{u}_i^{[q]} = w_i \mathbf{u}_i^0 + (1 - w_i) \mathbf{u}_i^{[q-1]}$$

# Complexity of different methods

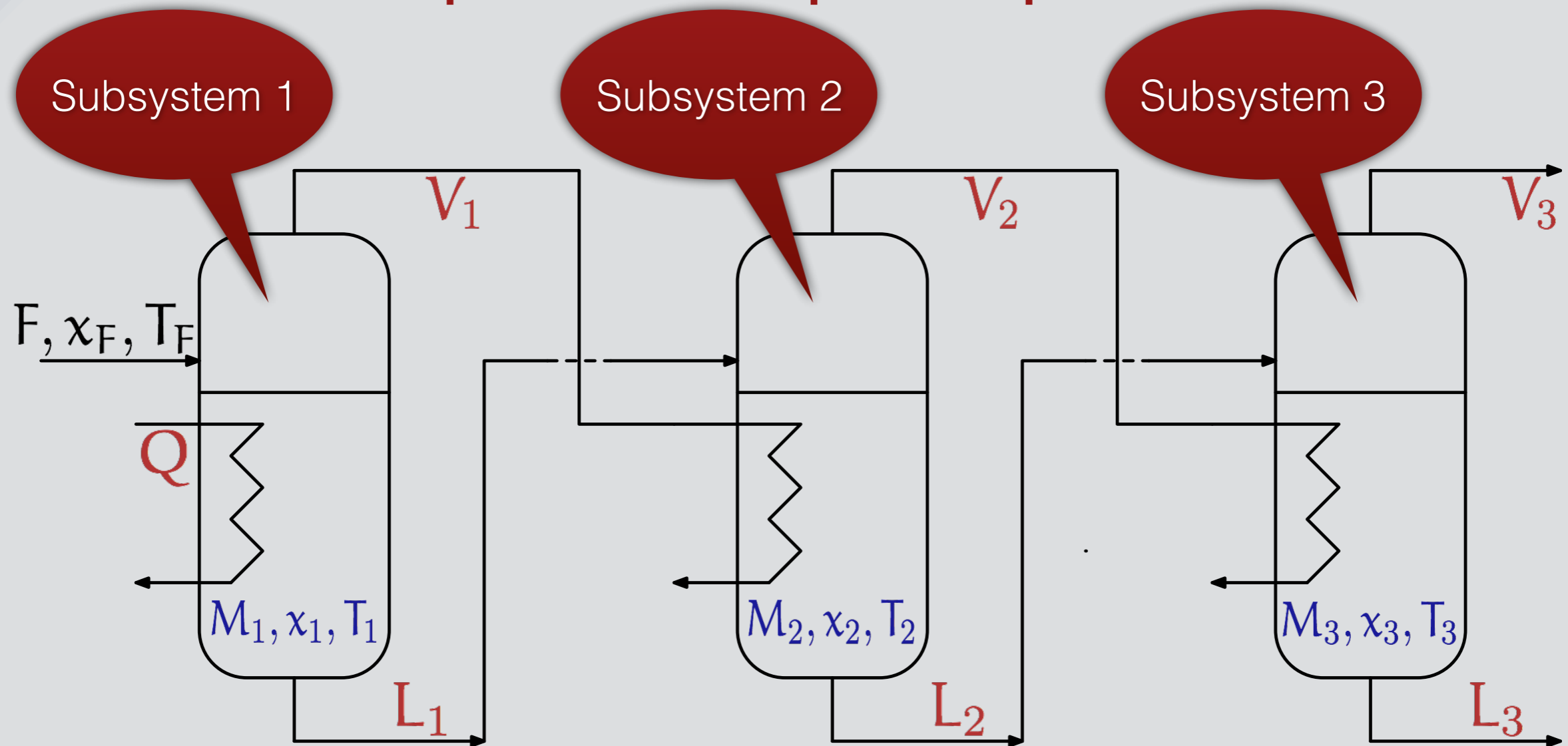
## Three alternatives

- DMPC0: available method [Ferramosca et al., 2013]
- DMPC1: proposed method (single step)
- DMPC2: proposed method (two steps)

	DMPC0	DMPC1	DMPC2
Prediction model	Centralized	Augmented	Augmented
Target calc. (TC)	Embedded	Embedded	Separate
TC decision var.	—	—	$(x_s, u_s, y_s)$
OCP decision var.	$(\mathbf{u}_i, \mathbf{x}, x_s, u_s, y_s)$	$(\mathbf{u}_i, \bar{\mathbf{x}}_i, x_s, u_s, y_s)$	$(\mathbf{u}_i, \bar{\mathbf{x}}_i)$

# Application example - I

## A triple effect evaporator process



# Application example - II

## Identified linear model

	$L_1$	$Q_1$	$V_1$	$L_2$	$V_2$	$L_3$	$V_3$
$M_1$	$-\frac{0.04797}{z-2.717}$	-	$-\frac{0.0339}{z-2.717}$	-	-	-	-
$T_1$	-	$\frac{0.564}{z-2.509}$	$-\frac{0.1745}{z-2.509}$	-	-	-	-
$\chi_1$	-	-	$\frac{0.009394}{z-2.549}$	-	-	-	-
$M_2$	$\frac{0.05726}{z-2.716}$	-	-	$-\frac{0.07207}{z-2.716}$	$-\frac{0.09465}{z-2.716}$	-	-
$T_2$	$\frac{0.008029z-0.01856}{z^2-4.913z+6.029}$	$\frac{0.089z-0.02579}{z^2-4.913z+6.029}$	$\frac{0.2431z-0.6396}{z^2-4.913z+6.029}$	-	$\frac{-0.6057z+1.451}{z^2-4.913z+6.029}$	-	-
$\chi_2$	$-\frac{0.01418}{z-2.604}$	-	$\frac{0.01038}{z-2.604}$	-	$\frac{0.02976}{z-2.604}$	-	-
$M_3$	-	-	-	$\frac{0.07503}{z-2.712}$	-	$-\frac{0.08504}{z-2.712}$	$-\frac{0.1255}{z-2.712}$
$T_3$	$\frac{0.001138z+0.03875}{z^2-4.898z+5.986}$	$\frac{-0.02526z+0.3423}{z^2-4.898z+5.986}$	$\frac{0.06671z-0.127}{z^2-4.898z+5.986}$	$\frac{0.09903z-0.2557}{z^2-4.898z+5.986}$	$\frac{2.472z-6.521}{z^2-4.898z+5.986}$	-	$\frac{-2.895z+7.385}{z^2-4.898z+5.986}$
$\chi_3$	$\frac{-0.01013z+0.01865}{z^2-5.241z+6.864}$	-	$\frac{0.004064z-0.005355}{z^2-5.241z+6.864}$	$\frac{-0.2224z+0.6029}{z^2-5.241z+6.864}$	$\frac{0.01244z-0.02893}{z^2-5.241z+6.864}$	-	$\frac{0.464z-1.249}{z^2-5.241z+6.864}$

## Neighbors and local (augmented) systems

☑ Neighbors sets:  $\mathcal{N}_1 = \emptyset$ ,  $\mathcal{N}_2 = \{1\}$ ,  $\mathcal{N}_3 = \{1, 2\}$

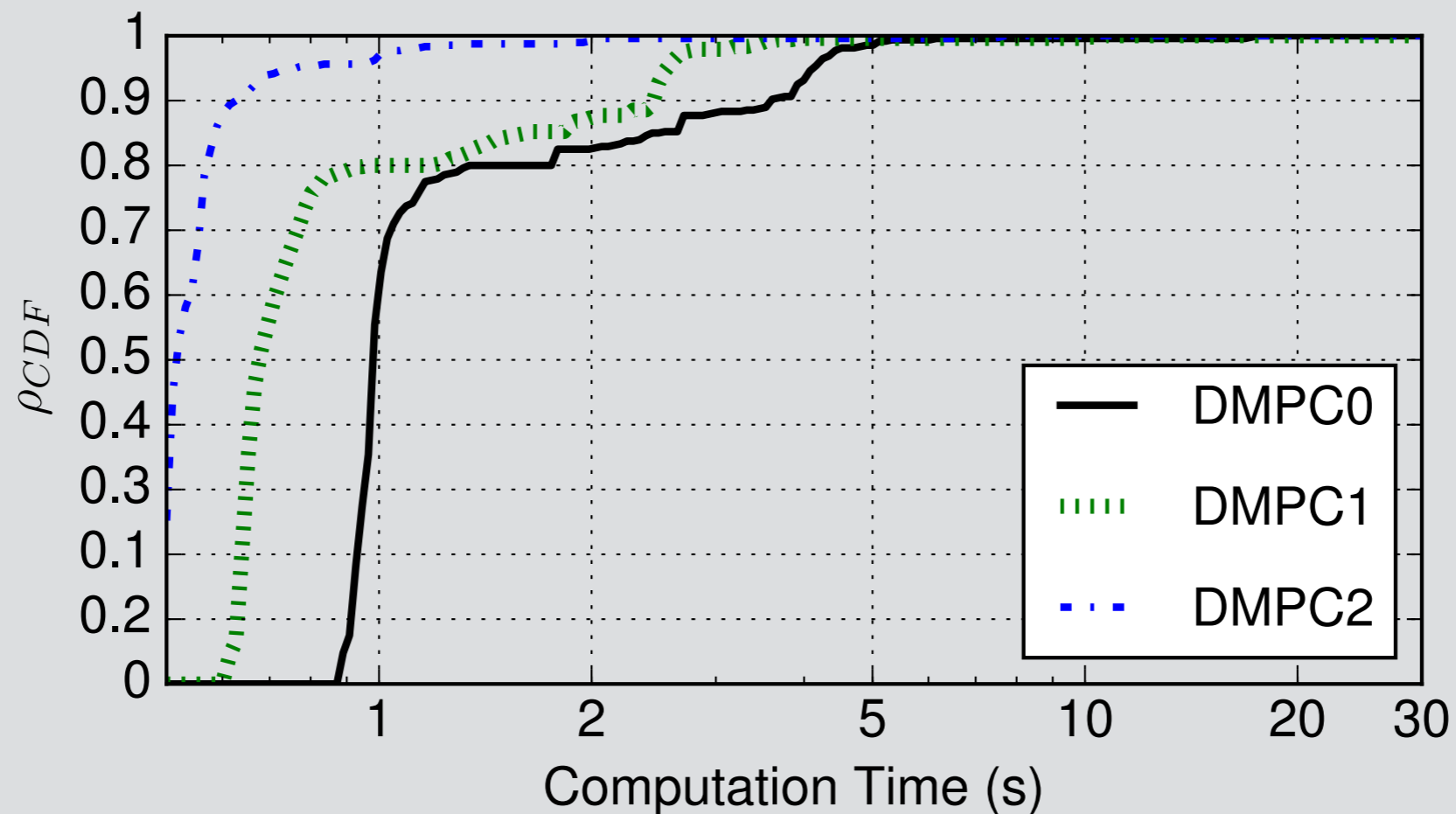
$$\bar{x}_1 = (x_1, x_2, x_3), \quad \bar{x}_2 = (x_2, x_3), \quad \bar{x}_3 = x_3$$

☑ Augmented systems:

$$\mathcal{S}_1^{IN} = \{2, 3\}, \quad \mathcal{S}_2^{IN} = \{1, 3\}, \quad \mathcal{S}_3^{IN} = \{1, 3\}$$

# Application example - III

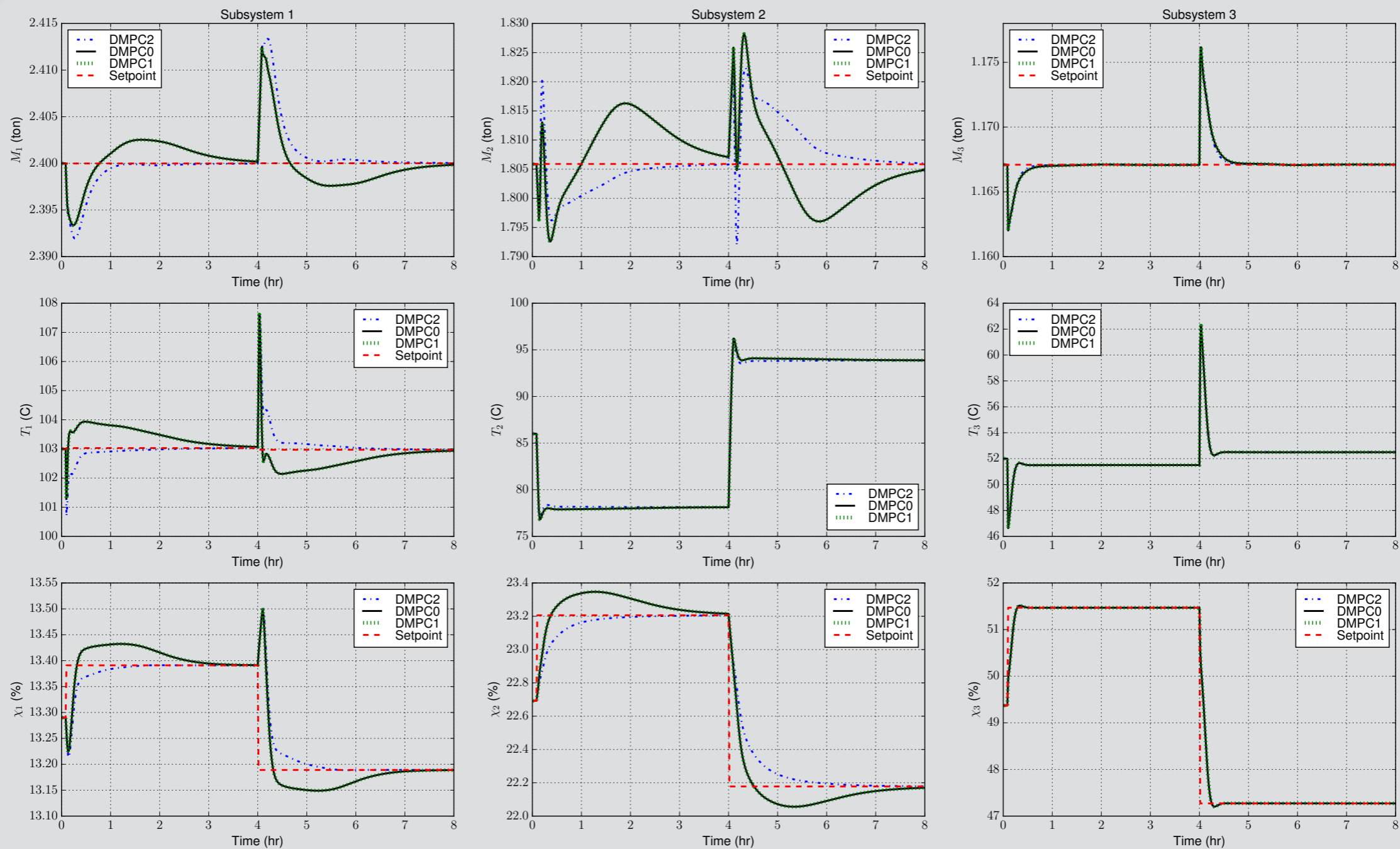
## Comparison of computation time



- Simulations performed in Matlab, MacBook Pro (3 GHz Intel Core i7, 16 GB RAM)
- Horizon  $N=100$ , QP solved (non condensed form) using quadprog (interior-point)
- Tested for 8 hours (480 samples) with two setpoint changes

# Application example - IV

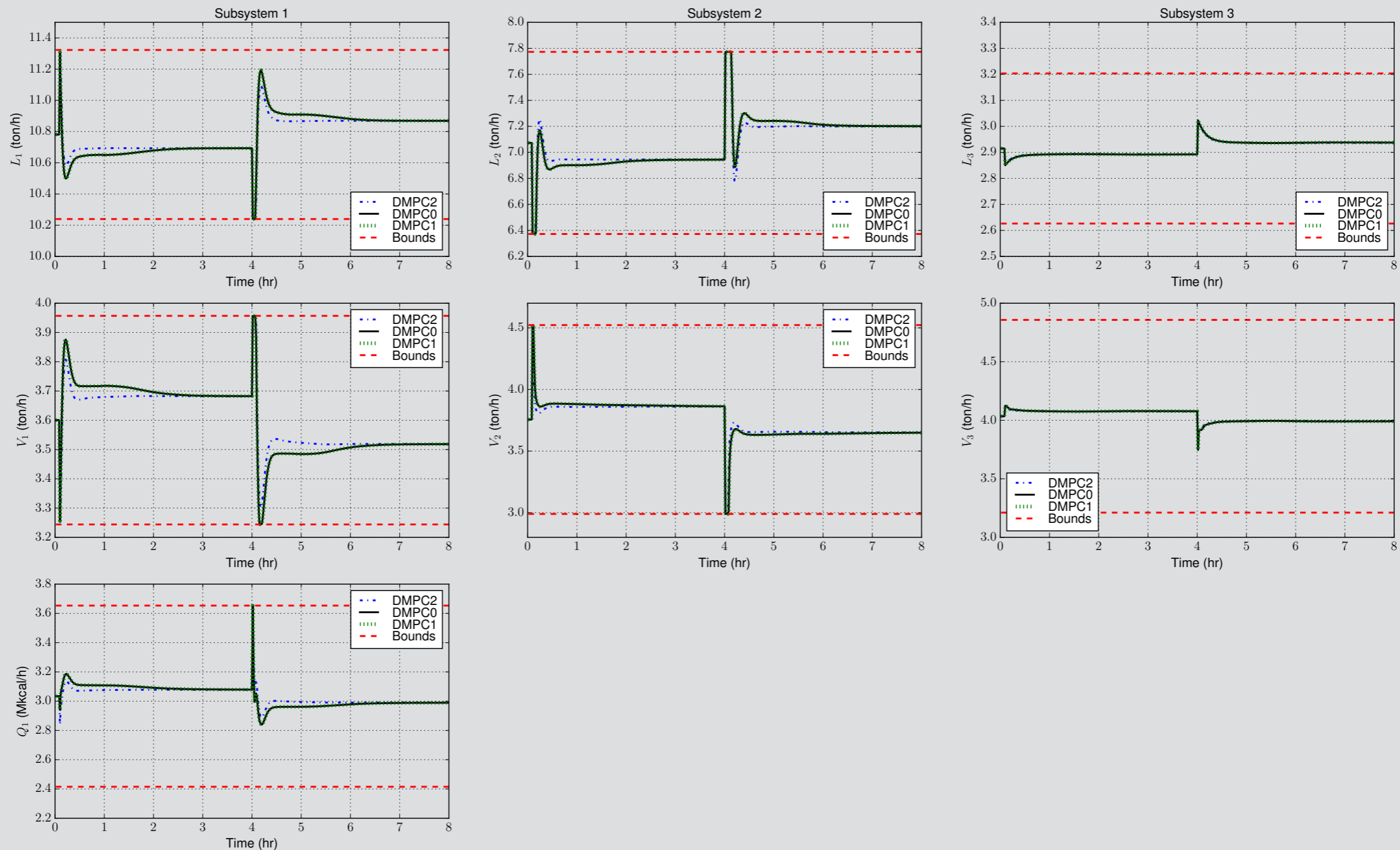
## Comparison of closed-loop outputs





# Application example - V

## Comparison of closed-loop inputs



# Part III

## Conclusions



# Conclusions - I

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## Summary

- ✓ Presented a **comprehensive overview** on linear distributed MPC
- ✓ Focus on **cooperative** algorithms, which share compelling properties with centralized architectures
- ✓ Discussed available **distributed** MPC for **tracking** algorithms
- ✓ Proposed **novel distributed** MPC for **tracking** approaches that rely “as local as possible” information instead of plant-wide state

# Conclusions - II

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## Take home messages

- ✓ Distributed MPC is a **solid** and reliable **alternative** to centralized MPC
- ✓ **Cooperative** architectures should be **preferred**
- ✓ Distributed MPC is preferable with respect to centralized MPC for **organizational** reasons, **not computational**

## Research directions

- ✓ Nonlinear distributed MPC
- ✓ Reconfigurability and reliability with respect to communication disruptions
- ✓ Distributed economic MPC

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