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Time Series Analysis
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Estimating the Trajectory of a Satellite[1]. Related to Chapter 10 of the book

1 Background

Gravity supplies the necessary centripetal force to hold a satellite in orbit about the earth. The circular orbit is a special case since orbits are generally ellipses, or hyperbolas in the case of objects which are merely deflected by the planet's gravity but not captured. Setting the gravity force from the universal law of gravity equal to the required centripetal force yields the description of the orbit. The orbit can be expressed in terms of the acceleration of gravity at the orbit.

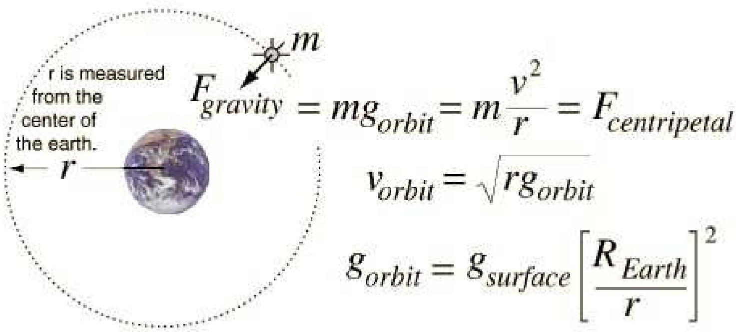

Figure 1: The orbit of a satellite

The force of gravity in keeping a satellite in circular motion is an example of centripetal force. In this assignment we assume a near circular motion of the satellite.

[1]The assignment is found at `www.imm.dtu.dk/~hm/time.series.analysis`

1.1 Purpose of the assignment

The purpose of this assignment is to apply the Kalman filter to do reconstruction and predictions for the trajectory of the orbiting satellite, based on the measurement $((r_1^m, \theta_1^m), \ldots, (r_n^m, \theta_n^m))$ available in polar coordinates.

1.2 The data

A measurement at time t, (r_t^m, θ_t^m) is obtained by utilizing a telescope, due to atmospheric disturbances imperfections in the telescope ect. the actual position are different from the true positions. We assume that the measurements are related to the actual positions $((r_1^p, \theta_1^p), \ldots, (r_n^p, \theta_n^p))$ through a set of equations

$$\begin{aligned} r_t^m &= r_t^p + \epsilon_{rt} \\ \theta_t^m &= \theta_t^p + \epsilon_{\theta t} \end{aligned}$$

where ϵ_{rt} and $\epsilon_{\theta t}$ are i.i.d. and specified by $\epsilon_{rt} \sim N(0, 2000^2)$ and $\epsilon_{\theta t} \sim N(0, .03^2)$. The 50 measurements are observed at equidistant time points during a period.

The measurements can be found in the file:

```
Satelliteorbit.csv
```

2 Task No. 1

Based on the assumption that the trajectory is well approximated by a circle we choose a state vector $\begin{pmatrix} r_t^p \\ \theta_t^p \\ v_{\theta,t}^p \end{pmatrix}$, where r_t^p, θ_t^p and $v_{\theta,t}$ represent respectively the true distance, the angle and the angle-velocity at time t describing the satellite. We introduce i.i.d. random perturbations

$$\begin{aligned} \epsilon_{rt}^p &\sim N(0, 500^2) \\ \epsilon_{\theta t}^p &\sim N(0, .005^2) \\ \epsilon_{v_\theta t}^p &\sim N(0, .005^2) \end{aligned}$$

to account for effects such as the orbit not being a perfect circle. We then specify a model by

$$\begin{aligned} r_t^p &= r_{t-1}^p + \epsilon_{rt}^p \\ \theta_t^p &= \theta_{t-1}^p + v_{\theta,t-1}^p + \epsilon_{\theta t}^p \\ v_{\theta,t}^p &= v_{\theta,t-1}^p + \epsilon_{v_\theta t}^p \end{aligned}$$

to describe the evolution in time of the system. From the supplied information formulate the system on state space form.

3 Part No. 2

The second part of this assignment is related to the implementation of the Kalman filter described in Theorem 10.2 in the Madsen (2007).

Remember that the Kalman filter is an algorithm consisting of a

- Initialization

followed by iterations

- Reconstruction.
- Prediction.

3.1 Task No. 2

Implement the Kalman filter for the state space model formulated in Task 2 in R, Splus, Matlab, Java, or Maple. You should provide a list of your code in the report.

3.2 Task No. 3

Use the implementation to reconstruct (estimate) the trajectory of the orbiting satellite.

3.3 Task No. 4

Use the implementation to predict the positions of the satellite for the 6 next time intervals, you should also predict the corresponding uncertainties.