Domain Modeling – A Primer

Dines Bjørner

The cover is my wife's rendition of the Austrian painter Hans Florey's 12 tone painting:


Photo shows my wife, Florey's painting and Gerhard Zeller

See: https://hans-florey.at/home/ausstellungen/nomos-2/ and https://hans-florey.at/

- This book manuscript is currently, July 17, 2026: 11:09 am, being translated:
  - **Chinese**: **领域科学与工程导论** [68]
    - Dr. Yang ShaoFa
    - Institute of Software
    - Chinese Academy of Sciences
    - Beijing
  - **Russian**: **Моделирование предметной области: основы** [59]
    - Dr. Mikhail Chupilko
    - Institute of Systems Programming
    - Russian Academy of Sciences
    - Moscow

Dines Bjørner[1]

Domain Modeling – A Primer[2]

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[1] In a possible Chinese publication of this primer Dr. Yang ShaoFa shall be listed as co-writer.
In a possible Russian publication of this primer Dr. Mikhail Chupilko shall be listed as co-writer.

[2] Primer: A small introductory book on a subject [https://www.merriam-webster.com/]

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Preface

The Triptych Dogma

In order to specify **software**,
we must understand its requirements.
In order to prescribe **requirements**,
we must understand the **domain**.
So we must **study**, **analyse** and **describe** domains.

Domains – What Are They ?

By a *domain* we shall understand a *rationally describable* segment of a *discrete dynamics* fragment of a *human assisted* reality, i.e., of the world. It includes its **endurants**, i.e., *solid and fluid entities* of **parts** and **living species**, and **perdurants** ▪

Endurants are either *natural* ["God-given"] or *artefactual* ["man-made"] and may be considered *atomic* or *compound* parts, or, as in this primer, further unanalysed *living species:* **plants** and **animals** – including *humans* ▪

Perdurants are here considered to be *actions*, *events* and *behaviours* ▪

Examples of domains are: *rail, road, sea and air transport; water, oil and gas pipelines; industrial manufacturing; the financial service industry: clients, banks, credit cards, stocks, etc.; consumer, retail and wholesale markets; health care;* et cetera.

Aim and Objectives

- The **aim** of this primer is to contribute to a methodology for analysing and describing domains.
- The **objectives** – in the sense of 'how is the aim achieved' – is reflected in the structure and contents and the didactic approach of this primer.
- The main elements of our approach – along one concept-axis – can be itemized:
  - There is the founding of our analysis & description approach in providing a base **philosophy**, cf. Chapter 2.
  - There is the application of ideas of **taxonomy** to understand the possibly hierarchical structuring of domain phenomena respectively the understanding of properties of phenomena and relations between them.
  - There are the notions **endurants** and **perdurants** – with *endurants* being the phenomena that can be observed, or conceived and described, as a "complete thing" at no matter which given snapshot of time [131, Vol. I, pg. 656], and *perdurants* being the phenomena for which only a fragment exists if we look at or touch them at any given snapshot in time [131, Vol. II, pg. 1552].
  - There is the introduction of base elements of **calculi** for analysing and describing domains.
  - There is the application of ideas of **ontology** to understand the possibly hierarchical structuring of these calculi.
  - And finally there is the notion of **transcendental deduction**, cf. Sect. 2.1.2, for "morphing" certain kinds of endurants into certain kinds of perdurants, Chapter 6.
- Along another conceptual-axis the below are further elements of our approach:
  - We consider domain descriptions, requirements prescriptions and software design specifications to be **mathematical** quantities.

- And we consider them basically in the sense of *recursive function theory*[3] [154, Hartley Rogers, 1952] and *type theory* [141, Benjamin Pierce, 1997].

Methodology

By a **method** we shall understand a set of **principles**[4] and **procedures**[5] for selecting and applying a set of **techniques**[6] and **tools**[7] to a problem in order to achieve an orderly construction of a **solution**, i.e., an **artefact**.

By **methodology** we shall understand the *study & application* of one or more methods.

By a **formal method** we shall understand a method

- whose *principles* include that of considering its artefacts as *mathematical* quantities, of *abstraction*, etc.;
- whose decisive *procedures* include that of
  - the sequential analysis & description of first endurants, then perdurants, and,
  - within the analysis & description of endurants, the sequential analysis & description of first their external qualities and then their internal qualities,
  - etc.;
- whose *techniques* include those of specific ways of specifying properties; and
- whose *tools* include those of one or more **formal languages**.

By a **language** we shall here understand a set of strings of characters, i.e., sentences, sentences which are structured according to some **syntax**, i.e., **grammar**, are given meaning by some **semantics**, and are used according to some **pragmatics**.

By a **formal language** we shall here understand a language whose *syntax* and *semantics* can both be expressed **mathematically** and for whose sentences one can **rationally reason** (*argue, prove*) **properties**.

We refer to Chapter 1 of [45] for an 8 page, approximately 50 entries set of concept definitions such as the above.

We refer to the Method index, Sect. D.4 on page 233.

•••

In this primer we shall use the formal specification language, RSL, the RAISE [8] Specification Language, [98] – and we shall notably rely on RSL's adaptation of **CSP**, Tony Hoare's *Communicating Sequential Processes* [114]; and we shall propagate a definitive method for the study and description of domains.

An Emphasis

When we say *domain analysis & description* we mean that the result of such a domain analysis & description is to be a model that describes a usually infinite set of domain instances. Domains exhibit endurants

[3] **A note of warning**: There is no recursion in domains ! [Cf. Item (ii) Sect. 4.7 on page 54.] Domain behaviours are, however, defined as tail-recursive functions.

[4] By a **principle** we mean: *a principle is a proposition or value that is a guide for behavior or evaluation* [Wikipedia], *i.e., code of conduct*

[5] By a **procedure** we mean: *instructions or recipes, a set of commands that show how to achieve some result, such as to prepare or make something* [Wikipedia], *i.e., an established way of doing something*

[6] By a **technique** we mean: *a technique, or skill, is the learned ability to perform an action with determined results with good execution often within a given amount of time, energy, or both* [Wikipedia], *i.e., a way of carrying out a particular task*

[7] By a **tool** we mean: *a tool is an object that can extend an individual's ability to modify features of the surrounding environment* [Wikipedia]

[8] **RAISE**: **R**igorous **A**pproach to **I**ndustrial **S**oftware **E**ngineering, [99]

and perdurants. A domain model is therefore something that defines the *nouns* (roughly speaking the endurants) and *verbs* (roughly speaking the perdurants) – and their combination – of a *language* spoken and used in writing by the practitioners of the domain. Not an instantiation of nouns, verbs and their combination, but all possible and sensible instantiations.

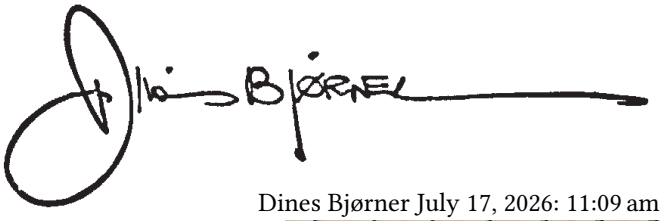
A Caveat

Experienced RSL [98] readers might observe our, perhaps cavalier (offhand), use of RSL. Perhaps, in some places, the syntax of RSL clauses is not quite right. Our non-use of RSL's module (Scheme, Class and Object) constructs force us to declare channels in the same way types, values and variables are introduced.

The DOMAIN Method

I have now worked on this approach to domain science engineering for some 15 years. It is time, I "humbly" suggest, to give a name to "my" approach. I unashamedly "baptise" it DOMAIN. The name DOMAIN is now meant to cover both the science and the engineering aspects of this approach.

In Chapters 4–6 we unveil, "bit–by–bit", the building blocks: procedures, techniques and tools – of a method. In Chapter 7 these are "assembled" into the DOMAIN method.



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Chapter 1
Introduction

Contents

We reiterate:

The Triptych Dogma

In order to specify $\mathcal{S}$**oftware**,
we must understand its requirements.
In order to prescribe $\mathcal{R}$**equirements**,
we must understand the $\mathcal{D}$**omain**.
So we must **study**, **analyse** and **describe** domains.

$\mathcal{D},\mathcal{S} \models \mathcal{R}$

In proofs of correctness ($\models$) of $\mathcal{S}$oftware with respect to $\mathcal{R}$equirements assumptions are often stated with respect to the $\mathcal{D}$omain. So this, therefore, alone justifies our focus on domains.

1.1 Why This Primer ?

This primer is intended as a **textbook**. The courses that we have in mind are, in the lectures, **to focus** on Chapters 3–6, i.e., Pages 23–122. The **serious students**, whether just readers or actual, physical course lecture attendants, are expected to study Chapters 1–2 as well as Chapter 8 and the Bibliography (Chapter 9) and the appendices on their own !

This primer is about how to *analyse & describe* man-made domains (including their possible interaction with nature). We emphasize the ampersand: '&'.[9] We justify competency in *Domain Science & Engineering* for two reasons. (i) For reasons of proper *engineering* software development – as indicated by the above **Triptych Dogma**. In possible proofs of software properties references are made, not only to the software code itself and the requirements, but also to the domain, the latter in the form of *assumptions about the domain*. In our mind no software development project ought be undertaken unless it more-or-less starts with a proper domain engineering phase. And (ii) for reasons of *scientifically* understanding our own everyday practical world: financial institutions, the transport industry (road, rail and air traffic, shipping), feeder systems (such as oil, gas, water and other such pipeline systems), etc.

1.2 Structure

The primer, beyond the present chapter, has, syntactically speaking, four elements:

1 **Chapter 2** covers the *philosophy* of Kai Sørlander [161–165].
Yes, a major contribution of [45] and this primer is to justify important domain concepts by their sheer inevitability in any world description.
2 **Chapters 3–6** presents *the methodology of domain engineering*. It is split into four chapters for practical and pragmatic reasons. Chapter 3 gives a "capsule introduction" into Chapters 4–6.
3 **Chapter 7** presents a rigorous, but not entirely formal, model of the method that we outline in Chapters 3–6. It is intended for self-study.
4 **Chapters 8–9** and **Appendices A–D** cover such things as 'closing remarks' (**8**), a 'bibliography' (**9**), a 'Road Transport' example (**A**), a 'Pipeline System' example (**B**), an 'RSL formal specification language' primer (**C**), and 'Indexes' to definitions, concepts, etc. (**D**).

1.3 Prerequisite Skills

The reader is expected to possess the following skills:

- To be reasonably versed in **discrete mathematics**: mathematical logic and set theory.
- To have had, even if only a fleeting, acquaintance with abstract specifications in the style of VDM [64, 65, 93], Z [175], CafeObj [95], Maude [79, 134], or the like – and thus to enjoy abstractions[10].
- To have reasonable experience with **functional programming** a la Standard ML or F [105, 109, 138] respectively [106] – or similar such language.
- To have reasonable experience with CSP [113–115, 155, 159].

The reader is further expected to possess the following mindset:

- To basically consider software as **mathematical objects**. That is: as quantities about which one can (and must) reason logically.
- To **think and "act" abstractly**. An essence of abstraction is expressed in the next section.
- To **act responsibly**[11], that is to make sure that You have indeed understood Your domain, that You have indeed reasoned about adequacy of your requirements, and You have indeed model-checked, proved and formally tested Your specifications.

[9] By not writing 'and', but '&', we shall emphasize that in *A*&*B* we are dealing with **one** concept which consists of both *A* and *B* "tightly interacting".

[10] Some say: *"Mathematics is the Science of Abstractions"* ! Others say that both *"Mathematics and Physics are Abstractions of Reality"*.

[11] It is, today, July 17, 2026: 11:09 am, very fashionable to propagate messages of 'ethics' to programmers – without even touching upon issues such as *"have You understood Your application domain thoroughly ?"*, or *"have You reasoned about adequacy of your requirements ?"*, or *"have You model-checked, proved and formally tested your specifications (descriptions and prescriptions) and Your code ?"*, etc.

1.4 Abstraction

Conception, my boy, fundamental brain-work,
is what makes the difference in all art.
D.G. Rossetti[12]: letter to T. H. Hall Caine[13]

Abstraction is a tool, used by the human mind, and to be applied in the process of describing (understanding) complex phenomena.

Abstraction is the most powerful such tool available to the human intellect.

Science proceeds by simplifying reality. The first step in simplification is abstraction. Abstraction (in the context of science) means leaving out of account all those empirical data which do not fit the particular, conceptual framework within which science at the moment happens to be working.

1.5 Software Engineering

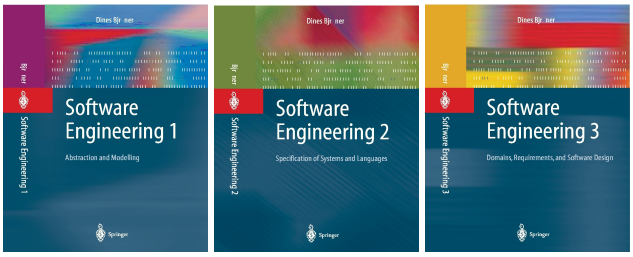
Software engineering to us, consists of domain and requirements engineering and software design and coding.

1.5.1 Domain Science & Engineering

This primer covers only the *application domain* of software development. There are two things to say about that. One is that facets of *requirements*, essential ones, is covered in [45, Chapter 8], general ones in [21, Software Engineering, III, Part V]; the other is that the pursuit of developing domain models is not just for the sake of software development, but also for the sake of just understanding the man-made world around us. Domain science and engineering can thus be pursued in-and-by itself.

1.5.2 Software Engineering

In 2006 these books were published: [19–21]:



1.5.2.1 Domain Engineering: 2016–2026

The first inklings of the specific approach [of this book] to domain science and engineering of [45] appeared in [29,33, 2010]. More-or-less "final" ideas were published, first in [40, 2017], then in [44, March

[12] Dante Gabrielli Rosetti, 1828–1882, English poet, illustrator, painter and translator.

[13] T. H. Hall Caine, 1853–1931, British novelist, dramatist, short story writer, poet and critic.

2019]. The book [45] with updates in this primer, then constitutes the most recent status of our work in domain science & engineering.

[21, Software Engineering, III, Part V] does not cover the *Domain Engineering* material covered in [45, Chapter 8: Domain Facets]. That latter was researched [28] and developed between the appearance of [21] and, obviously, [45].

Part V of [*Domain Engineering*] [21], except for Chapters 17–18, is still relevant. Chapters 17–18 of [21] are now to be replaced in any study by Chapters 4–7 of [45] **or** this primer !

1.5.2.2 Requirements Engineering

This primer does not show You how to proceed into software development according to the **Triptych Dogma**. This is strongly hinted at in [45, Chapter 9]. (That chapter is an adaptation of [25, May 2008].) Our approach to *requirements engineering* is rather different from that of both [129, A. van Lamswerde] and [122, M. A. Jackson] – to cite two relevant works. It is, we strongly think, commensurate with these works. We should like to see that someone could take up this line of research: making more precise, perhaps more formal, the ideas of *projection, intialisation, determination, extension* and *fitting*; and comparing, perhaps unifying our approach with that of Lamsweerde and Jackson.

1.5.2.3 Software Design

For the software design phase, after requirements engineering, we, of course, recommend [19,20, *Software Engineering* Vols. 1–2]

1.6 The Structuring of The Text

The reader will find that this text consists of "diverse" kinds of usually small paragraphs of texts: **definitions** – properly numbered and labeled; **examples** – properly numbered and labeled; **analysis predicate, function,** and **description prompt** "formalisations"; **method** principle, procedure, technique and tool paragraphs; – all of these delineated by closing ▪ s; – with short, usually one or two small paragraphs of introductory or otherwise explaining texts. All of these are "brought to You in living colours" ![14] So be prepared: Study such paragraphs: paragraph-by-paragraph. Each forms a separate "whole".

1.7 Self-Study

This primer is primarily intended to support actual, physical lectures. For self-study by B.Sc. and M.Sc. students and practicing novice software engineers we recommend to use this primer.

1.8 Two Examples

There are around 80 examples, scattered all over the first 120 pages. In addition we bring two larger examples:

- Road Transport, Appendix A, pages 161–171,
- Pipelines, Appendix B, pages 173–195.

[14] – as the NBC Television Network programmmes would "proudly" announce in the 1960s !

1.9 Relation to [45]

This primer is based on [45, Nov. 2021]. Chapter 2 is a complete rewrite of [45, Chapter 2]. Chapters 4–6 is a "condensation" of [45, Chapters 4–7]: [45, Chapter 6] has been shortened and appears in this primer as Sect. 2.1.2. From [45, Chapter 4] we have, in Chapter 4, omitted all material on – what is there referred to as *Conjoins*. And we have further sharpened the notion of *type names*. We have sharpened the focus on methods: principle, procedures, techniques and tools. You will find, in the *Indexes* section, Sect. D.4 on page 233, a summary of references to these. Work is still in progress on highlighting more of the method steps. Section 6.12 is new. Chapter 7 is new. It is based on [56][15].

1.10 The RAISE Specification Language, RSL, RSL$^+$ and RSL$^+$Text

The formal notation (to go with the informal text) of this primer is that of RSL [98], the **R**AISE **S**pecification **L**anguage, where RAISE stands for **R**igorous **A**pproach to **I**ndustrial **S**oftware **E**ngineering [99]. Other formal notations could be used instead. Replacement examples could, for example, be VDM [64,65,93], Z [175], or Alloy [120]. We are more using the RAISE specification language, RSL, than using the method. And we are using it in two ways:

- Informally, to present and explain the domain analysis & description methods of this primer, and
- formally, to present domain descriptions.

The informal RSL is an extended version, RSL$^+$.[16] RSL$^+$Text is the language of the RSL text generated in domain descriptions. The two ways are otherwise not related. One could use another specification language either for the informal or for the formal aspects.

1.11 Closing

The purpose of this introduction is to place the present primer in the context of my other books [19–21] on software development and possible lectures and self-study.

[15] https://www.imm.dtu.dk/~dibj/2026/metode/method-wx.pdf

[16] See Appendix Sect. C.13 on page 222.

Chapter 2
Kai Sørlander's Philosophy

Contents

Definition 1 **Philosophy**. Philosophy [17] is the study of general and fundamental questions, such as those about existence, reason, knowledge, values, mind, and language[18] ▪

2.1 Introduction

In philosophising questions are asked. One does not necessarily get answers to these questions. Questions are examined. Light is thrown on the questions and their derivative questions.

Philosophy is man's endeavour, our quest, for uncovering the necessary characteristics of our world and our situation as humans in that world.

We shall focus on the issues of metaphysics.

The treatment in this chapter is based very much on the works of the Danish philosopher Kai Sørlander (1944) [161–165, 1994–2022] both in contrast to and inspired by the German philosopher Immanuel Kant (1724–1804) [102]. In 2023, in collaboration with Kai Sørlander, [165] was translated into English [166].

The reason why we, as computer scientists, should be interested in philosophy, is that philosophers over more than 2500 years[19] have thought about existence: why is the world as it is – and computer scientists, like other scientists (notably physicists and economists), repeatedly model fragments of the

[17] From Greek: $\phi\iota\lambda o\sigma o\phi\iota\alpha$, philosophia, 'love of wisdom'

[18] Many of the 'definitions' in this primer are in the style used in philosophy. They are not in the 'precise' style commonly used in mathematics and computer science. You may wish to call them **characterisations**. In mathematics and computer science the definer usually has a formal base on which to build. In domain science & engineering we do not have a formal base, we have the "material" world of natural and man-made phenomena.

[19] – starting, one could claim, with:

- *Thales of Milet* 624–545 [everything originates from water] [139];
- *Anaximander* 610–546 ['apeiron' (the 'un-differentiated', 'the unlimited') is the origin] [82];
- *Anaximenes* 586–526 [air is the basis for everything] [136];
- *Heraklit of Efesos* 540–480 [fire is the basis and everything in nature is in never-ending "battle"] [5];
- *Empedokles* 490–430 [there are four base elements: fire, water, air and soil] [176];
- *Parminedes* 515–470 [everything that exists is eternal and immutable] [112];
- *Demokrit* 460–370 [all is built from atoms] [1];
- the Sophists: Protagoras, Gorgias (fifth and fourth centuries BC),
- *Socrates* (470–399) [2],
- *Plato* (424–347) [91],
- *Aristotle* (384–322) [6],
- etcetera.

After more than 1800 years came

- *René Descartes* (1596–1650) [86],
- *Baruch Spinoza* (1632–1677) [167],
- *John Locke* (1632–1704) [132],
- *George Berkeley* (1685–1753) [9],
- *David Hume* (1711–1776) [118],
- *Immanuel Kant* (1724–1804) [125],
- *Johan Gottlieb Fichte* (1762–1814) [123],
- *Georg Wilhelm Friedrich Hegel* (1770–1831) [110],
- *Friedrich Wilhelm Schelling* (1775–1864) [8],
- *Edmund Husserl* (1859–1938) [119],
- *Bertrand Russell* (1872–1970) [156–158, 171],
- *Ludwig Wittgenstein* (1889–1951) [173, 174],
- *Martin Heidegger* (1889–1976) [111],
- *Rudolf Karnap* (1891–1970) [74–76],
- *Karl Popper* (1902–1994) [144, 145],
- etcetera.

(This list is "pilfered" from [164, Pages 33–127].) [164] presents an analysis of the metaphysics of these philosophers. Except for those of Russell, Wittgenstein, Karnap and Popper, these references are just that.

world; and the reason why we focus on Kai Sørlander, is that his philosophy addresses issues that are crucial to our understanding of how we must proceed when modeling domains – and, we think, in a way that helps us model domains with a high assurance that our models are reasonable, can withstand close scrutiny. Kai Sørlander thinks and writes logically, rationally. The area of his philosophy that we are focusing on here is metaphysics.

2.1.1 Metaphysics

The branch of philosophy that we are focusing on is referred to as metaphysics. To explain that concept we quote from [Wikipedia]:

"Metaphysics is the branch of philosophy that studies the fundamental nature of reality, the first principles of being, identity and change, space and time, causality, necessity, and possibility.[20] *It includes questions about the nature of consciousness and the relationship between mind and matter, between substance and attribute, and between potentiality and actuality.*[21] *The word "metaphysics" comes from two Greek words that, together, literally mean "after or behind or among [the study of] the natural". It has been suggested that the term might have been coined by a first century editor who assembled various small selections of Aristotle's works into the treatise we now know by the name Metaphysics (μετα τα φυσικα, meta ta physika, literally 'after the Physics', another of Aristotle's works) [81].*

Metaphysics studies questions related to what it is for something to exist and what types of existence there are. Metaphysics seeks to answer, in an abstract and fully general manner, the questions:[22]

- *What is there ?*
- *What is it like ?*

Topics of metaphysical investigation include existence, objects and their properties, space and time, cause and effect, and possibility. Metaphysics is considered one of the four main branches of philosophy, along with epistemology, logic, and ethics" en.m.wikipedia.org/wiki/Metaphysics.

2.1.2 Transcendental Deductions

A crucial element in Kant's and Sørlander's philosophies is that of *transcendental deduction.*

It should be clear to the reader that in *domain analysis & description* we are reflecting on a number of philosophical issues; first and foremost on those of *ontology.* For this chapter we reflect on a subfield of epistemology, we reflect on issues of *transcendental* nature. Should you wish to follow-up on the concept of transcendentality, we refer to [102, Immanuel Kant], [117, Oxford Companion to Philosophy, pages 878–880], [4, The Cambridge Dictionary of Philosophy, pages 807–810], [73, The Blackwell Dictionary of Philosophy, pages 54–55 (1998)], and [164, Sørlander].

2.1.2.1 Some Definitions

Definition 2 **Transcendence.** By **transcendence** is meant something that can not be understood in principle ▪

Definition 3 **Transcendental.** By **transcendental** we shall understand the philosophical notion: **the a priori and intuitive basis of knowledge, independent of experience** ▪

[20] www.encyclopedia.com/philosophy-and-religion/philosophy/philosophy-terms-and-concepts/metaphysics

[21] Metaphysics. American Heritage Dictionary of the English Language (5th ed.). 2011.

[22] What is it (that is, whatever it is that there is) like? Hall, Ned (2012). "David Lewis's Metaphysics". In Edward N. Zalta (ed.). The Stanford Encyclopedia of Philosophy (Fall 2012 ed.). Center for the Study of Language and Information, Stanford University.

A priori knowledge or intuition is central: By *a priori* we mean that it not only precedes, but also determines rational thought.

Definition 4 **Transcendental Deduction.** By a **transcendental deduction** we shall understand the philosophical notion: **a** transcendental **"conversion" of one kind of knowledge into a seemingly different kind of knowledge** ▪

There are deductions and then there are deductions. Some are logical others are transcendental. With Krzysztof Brzeziński[23] we could rename the two into **analytic**, respectively **synthetic** deductions.

2.1.2.2 Some Informal Examples

Example 1 **Transcendental Deductions – Informal Examples.** We give some intuitive examples of transcendental deductions.

The first two examples

(i–ii) Time, henceforth referred to as 𝕋𝕀𝕄𝔼 and space, i.e., 𝕊ℙ𝔸ℂ𝔼, can be transcendentally (synthetically) deduced from the logical relations.

(i) 𝕊ℙ𝔸ℂ𝔼, cf. Sect. 2.4.9.1 on page 18, thus follows from the concepts of *distance* and *direction. Distance* is a relation that holds between any pair of two distinct entities. It is an *symmetric relation. Direction* is an *asymmetric relation.* Hence we conclude that 𝕊ℙ𝔸ℂ𝔼 is an unavoidable characteristics of every possible reality.

(ii) 𝕋𝕀𝕄𝔼, cf. Sect. 2.4.9.2 on page 18, can be transcendentally understood in the context of *states:* that an entity may accrue predicates (properties) that they do not actually accrue. These predicates, for any one particular entity, may be *incommensurable.* How is that possible ? Only if one and the same entity can exist in *different* **states**. It may exist in one state in which it accrues a certain predicate; and it may exist in another state in which it accrues another predicate. What can we say about more to come

(iii–v) Other examples of transcendental deduction are from the "domain" of programming languages.

There is the syntax of a programming language, and there are the programs that supposedly adhere to this syntax. Given that, the following are now transcendental deductions.

(iii) The software tool ***a syntax checker***, that takes a program and checks whether it satisfies the syntax, including the statically decidable context conditions, i.e., the *statics semantics* – such a tool is one of several forms of transcendental deductions.

(iv) The software tool, ***a model checker,*** for example SPIN [116], that takes a Promela statement, and proves, [checks], the program correct with respect to the statement.

(v-vi) A ***compiler*** and an ***interpreter*** for any programming language.

Yes, indeed, any ***abstract interpretation*** [84, 85] reflects a transcendental deduction: firstly, these examples show that there are many transcendental deductions; secondly, they show that there is no single-most preferred transcendental deduction ▪

A transcendental deduction, crudely speaking, is just any abstraction that can be "linked" to another, not by logical necessity, but by logical (and philosophical) possibility !

Definition 5 **Transcendentality.** By **transcendentality** we shall here mean the philosophical notion: "the state or condition of being transcendental" ▪

Example 2 **Transcendentality.** We[24] can speak of an automobile in at least three *senses:*

(i) The automobile as it is being "maintained, serviced, refueled";
(ii) the automobile as it "speeds" down its route; and
(iii) the automobile as it "appears" in an advertisement.

[23] Personal communication: The Technical University of Warsaw, Poland, 05 May 2026

The three *senses* are:

(i) as an **endurant** (here a *part*),
(ii) as a **perdurant** (as we shall see, a *behaviour*), and
(iii) as an **attribute** ▪

The above example, we claim, reflects *transcendentality* as follows:

(i) We have knowledge of an endurant (i.e., a part) being an endurant.
(ii) We are then to assume that the perdurant referred to in (ii) is an aspect of the endurant mentioned in (i) – where perdurants are to be assumed to represent a different kind of knowledge.
(iii) And, finally, we are to further assume that the attribute mentioned in (iii) is somehow related to both (i) and (ii) – where at least this attribute is to be assumed to represent yet a different kind of knowledge.

In other words: two (i–ii) kinds of different knowledge; that they relate *must indeed* be based on *a priori knowledge*. Someone claims that they relate ! The two statements (i–ii) are claimed to relate transcendentally.[25]

2.1.2.3 Bibliographical Note

The philosophical concept of *transcendental deduction* is a subtle one. Arguments of transcendental nature, across the literature of philosophy, do not follow set principles and techniques. We refer to [4, *The Cambridge Dictionary of Philosophy*, pages 807–810] and [73, *The Blackwell Companion to Philosophy*, Chapter 22: Kant (David Bell), pages 589–606, Bunnin and Tsui-James, eds.] for more on 'transcendence'.

2.2 The Philosophical Question

Sørlander focuses on the philosophical question of "**what is thus necessary that it could not, under any circumstances, be otherwise ?**".

To study and try answer that question Sørlander thinks rationally, that is, *reasons*, rather than express emotions. The German philosopher Immanuel Kant (1724–1804) suggests that our philosophising as to the philosophical question above must build on *"something which no person can consistently deny, and thus, something that every person can rationally justify, as a consequence of being able to think at all"*. Kant then goes on to build his philosophy [125] on *the possibility of self-awareness* – something of which we all are aware. Sørlander then, in for example [164], shows that this leads to solipsism[26], i.e., to nothing.

The next section elaborates on the above paragraph.

2.3 Three Axioms

2.3.1 The Possibility of Truth

Instead Sørlander suggests that **the possibility of truth** be the basis for the thinking of an answer to the highlighted question above. *The possibility of truth* is shared by all of us.

[24] We first came across this example when it was presented to us by Paul Lindgreen, an early Danish computer scientist (1936–2021) – and then as a problem of data modeling [130, 1983].

[25] – the attribute statement was "thrown" in "for good measure", i.e., to highlight the issue !

[26] Solipsism: the view or theory that the self is all that can be known to exist.

2.3.2 The Principle of Contradiction

Once we accept that *the possibility of truth* cannot be denied, we have also accepted **the principle of contradiction**, that is, that an assertion and its negation cannot both be true.

2.3.3 The Implicit Meaning Theory

We accept *the implicit meaning theory*.

Definition 6 **Implicit Meaning Theory**. The implicit meaning theory implies that there is a *mutual relationship* between the (α) *meaning of designations* and (β) *consistency relations between assertions*.

As an example of what "goes into" the *implicit meaning theory*, we bring, albeit from the world of computer science, that of the description of the **stack** data type (its endurant data types and perdurant operations).

Example 3 **An Implicit Meaning Theory**. **Narrative:**

α. **The Designations:**

1 Stacks, s:S, have elements, e:E;
2 the empty operation takes no arguments and yields a result stack;
3 the is_empty operation takes an argument stack and yields a Boolean value result.
4 the stack operation takes two arguments: an element and a stack and yields a result stack.
5 the unstack operation takes a non-empty argument stack and yields a stack result.
6 the top operation takes a non-empty argument stack and yields an element result.

β. **The Consistency Relations:**

7 an empty stack is_empty, and a stack with at least one element is not;
8 unstacking an argument stack, stack(e,s), results in the stack s; and
9 inquiring the top of a non-empty argument stack, stack(e,s), yields e.

Formalisation:

The designations:

type
1. E, S

value
2. empty: **Unit** → S
3. is_empty: S → **Bool**
4. stack: E × S → S
5. unstack: S $\xrightarrow{\sim}$ S[27]
6. top: S $\xrightarrow{\sim}$ E

The consistency relations:

axiom
7. is_empty(empty_S()) = **true**
7. is_empty(stack(e,s)) = **false**
8. unstack(stack(e,s)) = s
9. top(stack(e,s)) = e

2.3.4 A Domain Analysis & Description Core

The three concepts: (i) *the possibility of truth*, (ii) *the principle of contradiction* and (iii) *the implicit meaning theory* thus form the core – and imply that (a) *the indispensably necessary characteristics of any possible world, i.e., domain*, are *equivalent* with (b) *the similarly indispensably necessary conditions for any possible domain description.*

[27] $A \xrightarrow{\sim} B$ stands for partial functions, f, from A into B, i.e., there are some $a : A$ for which f is not defined.

2.4 The Deductions

2.4.1 Assertions

Definition 7 **Assertion**. An assertion is a declaration, an utterance, that something is the case.

Assertions may typically be either propositions or predicates.

2.4.2 The Logical Connectives

Any domain description must necessarily contain assertions. Assertions are expressed in terms of negation, $\sim$, conjunction, $\wedge$, disjunction, $\vee$, and implication, $\Rightarrow$.

2.4.2.1 $\sim$: Negation

Negation is defined by the principle of contradiction. If an assertion, a, holds, then its negation, $\sim a$, does not hold.

2.4.2.2 Simple Assertions

Simple assertions, i.e., propositions, are formed from assertions, for example a, b, by means of the logical connectives.

2.4.2.3 $\wedge$: Conjunction

The simple assertion $a \wedge b$ holds if both a and b hold.

2.4.2.4 $\vee$: Disjunction

The simple assertion $a \vee b$ holds if either of or both of a and b hold.

2.4.2.5 $\Rightarrow$: Implication

The simple assertion $a \Rightarrow b$ holds if a is *inconsistent* with the negation of b.

2.4.3 Modalities

2.4.3.1 Necessity

Definition 8 **Necessity**. An assertion is *necessarily true* if its truth ("true") follows from the definition of the designations by means of which it is expressed. Such an assertion holds under all circumstances.

Example 4 **Necessity**. *"It may rain someday"* is necessarily true.

2.4.3.2 Possibility

Definition 9 **Possibility**. An assertion is *possibly true* if its negation is not *necessarily true* ▪

Example 5 **Possibility**. *"It will rain tomorrow"* is possibly true.

2.4.4 Empirical Assertions

Definition 10 **Empirical Knowledge**. In philosophy, knowledge gained from experience – rather than from innate ideas or deductive reasoning – is empirical knowledge. In the sciences, knowledge gained from experiment and observation – rather than from theory – is empirical knowledge ▪

Example 6 **Expressing Empirical Knowledge**. There are innumerable ways of expressing empirical knowledge.

a. There are two automobiles in that garage.[28]
b. The two automobiles in that garage are distinct.[29]
c. The two automobiles in that garage are parked next to one another.[30]
d. That automobile, the one to the left, in that garage is [painted] red.[31]
e. The automobile to the right in that garage has just returned from a drive.[32]
f. The automobile, with Danish registration number AB 12345, is currently driving on the Copenhagen area city Holte road Fredsvej at position 'top of the hill'.[33]
g. The automobile on the roof of that garage is pink.

The pronoun 'that' shall be taken to mean that someone gestures at, points out, the garage in question. If there is no such garage then the assertion denotes the **chaos** value ! Statements (a.–g.) are assertions. The assertions contain *references* to quantities "outside the assertions" — 'outside' in the sense that they are not defined in the assertions. Assertion (g.) does not make sense, i.e., yields **chaos**. The term 'roof' has not been defined ▪

I: **The Object Language.**[34] *The language used in the above assertions is quite 'free-wheeling'. The language to be used in "our" domain descriptions is, i.e., will be, more rigid* ▪

Definition 11 **Empirical Assertion**. The domain description language of assertions, contain **references**, i.e., *designators*, and **operators**. All of these shall be properly defined in terms of names of *endurants* and their *unique identifiers*, *mereologies* and *attributes*; and in terms of their *perdurant "counterparts"* ▪

• • •

From Possible Predicates to Conceptual Logic Description Framework. The ability to deduce which type of predicates that a phenomenon of any domain can be ascribed is thus equivalent to deducing the conceptual logical conditions for every possible domain description.

[28] *The automobiles are solid* endurants, *and so is the garage, that is, they are both* parts.
[29] *Their distinctness gives rise to their respective, distinct, i.e.,* unique identifiers.
[30] *The topological ordering of the two automobiles is an example of their* mereology.
[31] *The red colour of the automobile is an* attribute *of that automobile.*
[32] *The fact that that automobile, to the right in the garage, has just returned from a drive, is a* possibly time-stamped attribute *of that automobile.*
[33] *The automobile in question is now a* perdurant *having a so-called* time-stamped progammable event attribute *of the Copenhagen area city of Holte, "top of the hill".*
[34] The prefix *I* indented paragraph designates an *I*nformal explication.

•••

By a so-called *transcendental deduction* we have shown that simple empirical assertions consist of a **subject** which **refers** to an independently existing entity and a **predicate** which ascribes a **property** to the referred entity [164, π 146 ℓ 1–5][35].

If the world, or as we shall put it, the domains, that we shall be concerned with, are *what can be described in simple assertions*, then any possible such world, i.e., domain must *primarily consist of such entities* [164, π 146 ℓ 5–7].

We shall therefore, in the following, explicate a system of concepts by means of which the entities, that may be referred to in simple assertions, can be described [164, π 146 ℓ 8–11].

$\mathcal{I}$: *These* concepts *are those of* entities, endurants, perdurants, unique identity, mereology *and* attributes. ▪

2.4.5 Identity and Difference

We can now assume that the world consists of an indefinite number of entities: Different empirical assertions may refer to distinct entities. Most immediately we can define two interconnected concepts: **identity** and **diversity**.

2.4.5.1 Identity

Definition 12 **Identical**. "An entity referred to by the name A is *identical* to an entity referred to by the name B if A cannot be ascribed a property which is incommensurable, that is logically inconsistent, with a property ascribed to B" [164, π 146 ℓ 14-23] ▪

2.4.5.2 Difference

Definition 13 **Different**. "A and B are *distinct*, differ from one another, if they can be ascribed incommensurable, that is logically inconsistent, properties." [164, π 146 ℓ 23-26] ▪

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"These definitions, by transcendental deduction, introduce the concepts of **identity** and **difference**. They can thus be assumed in any transcendental deduction of a domain description which, in principle, must be expressed in any possible language". [164, π 147 ℓ 1-5]

Definition 14 **Unique Identification**. By a *transcendental deduction* we introduce the concept of manifest, physical entities each being uniquely identified ▪

We make no assumptions about any representation of unique identifiers.

2.4.6 Relations

2.4.6.1 Identity and Difference

Definition 15 **Relation**. "Implicitly", from the two concepts of *identity* and *difference*, follows the concept of **relation**s. "A identical to B is a relational assertion. So is A different from B" [164, π 147 ℓ 6-10] ▪

[35] The reference [164, π 150 ℓ 1–5] refers to the [164] book by Kai Sørlander, πage 150, ℓines 1–5.

2.4.6.2 Symmetry

Definition 16 **Symmetry**. If A is identical to B then B must be identical to A. This expresses that the *identical to* relation is *symmetric*. And, if A is different from B then B must be different from A. This expresses that the *different from* relation is also *symmetric* ▪

2.4.6.3 Asymmetry

Definition 17 **Asymmetry**. A relation which holds between A and B but does not hold between B and A is *asymmetric* [164, π 147 ℓ 25–27] ▪

2.4.6.4 Transitivity

Definition 18 **Transitivity**. "If A is identical to B and if B is identical to C then A must be identical to C. So the relation *identical to* is *transitive*" [164, π 147-148 ℓ 28-30,1-4] ▪

The relation *different from* is not transitive.

2.4.6.5 Intransitivity

Definition 19 **Intransitivity**. If, on the other hand, we can logically deduce that a relation, $\mathcal{R}$ holds from A to B and the same relation, $\mathcal{R}$, holds from B to C but $\mathcal{R}$ does not hold from A to C then relation $\mathcal{R}$ is *intransitive* [164, π 148 ℓ 9–12] ▪

2.4.7 Sets, Quantifiers and Numbers

2.4.7.1 Sets

The possibility now exists that two or more entities may be prescribed the same property.

Definition 20 **Sets**. The "same properties" could, for example, be that two or more uniquely distinguished entities, $x, y, ..., z$, have [at least] one attribute kind (type) and value, (t, v), in common. This means that (t, v) distinguishes a set $s_{(s,v)}$ – by a *transcendental deduction*. A fact, just t likewise distinguishes a possibly other, most likely "larger", set s_t ▪

From the transcendentally deduced notion of set follows the relations: equality, $=$, inequality, $\neq$, proper subset, $\subset$, subset, $\subseteq$, set membership, $\in$, set intersection, $\cap$, set union, $\cup$, set subtraction, $\setminus$, set cardinality, **card**, etc. !

2.4.7.2 Quantifiers

By a further *transcendental deduction* we can place the *quantifiers* among the concepts that are necessary in order to describe domains.

Definition 21 **The Universal Quantifier**. The universal quantifier expresses that all members, x, of a set, s, possess a certain $\mathcal{P}$roperty: $\forall x : S \bullet \mathcal{P}(x)$ ▪

Definition 22 The Existential Quantifier. The existential quantifier expresses that at least one member, x, of a set, s, possesses a certain $\mathcal{P}$roperty: $\exists x : S \bullet \mathcal{P}(x)$ ▪

2.4.7.3 Numbers

Numbers can, again by *transcendental deduction*, be introduced, not as observable phenomena, but as a rational, logic consequence of sets.

Definition 23 Numbers. Numbers can be motivated, for example, as follows:

- Start with an empty set, say {}. It can be said to represent the number zero..
- Then insert the empty set, {}, to {} and You get {{}} – said to represent 1.
- Continue with adding {{}} to {{}} and You get {{},{{}}}, said to represent 2.
- And so forth – ad infinitum ▪

In this way one[36] can define the natural numbers. We could also do it by just postulating distinct entities which are then added, one by one to an initially empty set [164, π 150 ℓ 8-13].

We can then, still in the realm of philosophy, proceed with the introduction of the arithmetic operations designated by addition, +, subtraction, −, multiplication, ∗, division, ÷, equality, =, inequality, ≠, larger than, >, larger than or equal, ≥, smaller than, <, smaller than or equal, ≤, etcetera !

From explaining numbers on a purely philosophical basis one can now proceed mathematically into the realm of *number theory* [107].

2.4.8 Primary Entities

We now examine the concept of *primary objects*.

The next two definitions, in a sense, "fall outside" the line of the present philosophical inquiry. They will be "corrected" to then "fall inside" our inquiry.

Definition 24 Object. By an *object* we, in our context, mean something material that may be perceived by the senses[37] ▪

Definition 25 Primary Object. By a *primary object* we[38] mean an object that exists as its own *entity* independent[39] of other objects ▪

In the last definition we have used the term *entity*. That term, 'entity', will be used henceforth instead of the term 'object'.

We have deduced the relations *identity, difference, symmetry, asymmetry, transitivity* and *intransitivity* in Sects. 2.4.5–2.4.6. You may ask: *for what purpose ?* And our answer is: *to justify the next set of deductions.* First we reason that there is the possibility of there being many entities. We argue that that is possible due to there being the relation of asymmetry. If it holds between two entities then they must necessarily be ascribed different predicates, hence be distinct.

Similarly we can argue that two entities, B and C which both are asymmetric with respect to entity A may stand in a symmetric relation to one another. This opens for the *possibility* that every pair of distinct entities may stand in a pair of mutual relations: first the asymmetry relation that expresses their distinctness; secondly, the possibility of a symmetry relation which expresses the two entities individually with respect to one-another. *The above forms a transcendental basis for how two or more [primary] entities must necessarily be characterised by predicates.*

[36] https://en.wikipedia.org/wiki/Set-theoretic_definition_of_natural_numbers

[37] www.merriam-webster.com/dictionary/object

[38] help.hcltechsw.com/commerce/8.0.0/tutorials/tutorial/ttf_cmcdefineprimaryobject.html

[39] Yes, we know: we have not defined what is meant by 'as its own' and 'independent' !

2.4.9 Space and Time

The asymmetry and symmetry relations between entities cannot be *necessary* characteristics of every possible reality if they cannot also possess an *unavoidable rôle* in our own concrete reality. Next we examine two such *unavoidable rôles*.

2.4.9.1 Space

One pair of such rôles are *distance* and *direction*. *Distance* is a relation that holds between any pair of distinct entities. It is a symmetric relation. *Direction* is an asymmetric relation that also holds between any pair of distinct entities. Hence we conclude that **space** is an unavoidable characteristics of every possible reality. Hence we conclude that entities exist in space. They must "fill" some space, have *extension:*, *fill* some space, have *surface* and *form*. From this we can define the notions of spatial point, spatial straight line, spatial surface, etcetera. Thus we can philosophically motivate geometry.

2.4.9.2 Time

Primary empirical entities may accrue predicates that it is not logically necessary that they accrue. That is, it is logically possible that primary entities accrue predicates that they do not actually accrue. How is it possible that one and the same primary entity may accrue incommensurable predicates ?

That is only possible if one and the same primary entity can exist in **different states**. It may exist in one state in which it accrues a certain predicate. And it may exist in another state in which it accrues a therefrom incommensurable predicate.

What can we say about these states ? First that these states accrue different, incommensurable predicates. How can we assure that ! Only if the states stand in an asymmetric relation to one another. From this we can conclude that primary entities necessarily may exist in a number of states each of which stand in an asymmetric relation to its predecessor state.

This is a necessary characteristics of any possible world. So it is also a characteristics of our world. That relation is **time**. It possesses the *before*, *after*, *in-between*, and other [temporal] relations. We have thus deduced that every possible world must "occur in time" and that primary entities may exist in, before or after states.

From the above we can derive a whole algebra of temporal types and operations, for example:

- TIME and TIME INTERVAL types;
- addition of TIME and TIME INTERVAL to obtain TIME;
- addition of TIME INTERVALs to obtain TIME INTERVALs;
- subtraction of two TIMEs to become TIME INTERVALs; and
- subtraction of two TIME INTERVALs to obtain TIME INTERVAL.

2.4.10 The Causality Principle

But what is it that *causes* primary entities to undergo *state changes* ? Assertions about how a primary entity is at different times, such assertions must necessarily be logically independent. That follows from primary entities necessarily must accrue incommensurable predicates at different times. It is therefore logically impossible to conclude from how a primary entity is at one time to how it is at another time. How, therefore, can assertions about a primary entity at different times be about the same entity ?

We can therefore transcendentally deduce that there must be a *special implication-relationship* between assertions about how a primary entity is at different times. Such a *special implication-relationship* must depend on the *empirical circumstances* under which the primary entity exists. That is, we must deduce the conditions under which it is, at all, possible to consistently make statements about primary entities going

from one state in which it accrues a specific predicate to another state in which it accrues a therefrom incommensurable predicate. There must be something in the empirical circumstances which implicates the state transition. If the empirical circumstances are *stable* then there is nothing in these circumstances that imply entity changes. If the primary entity changes, then that assumes that there must have been a prior change in the circumstances – with those changes having that consequence. ...[40] We name such a change of the circumstances *a cause*. And we conclude that every change of a primary entity must have a cause. We also conclude that *equivalent causes* imply *equivalent effects*.

This form of implication is called the *causality principle*. It assumes logical implication. But it cannot be reduced to logical implication. It is logically necessary that every primary entity – and therefore every possible world – is subject to the *causality principle*. In this way Kai Sørlander transcendentally deduces the principle of causality. Every change has a cause. The same cause under the same circumstances leads to same effects.

2.4.11 Newton's Laws

Sørlander then shows how Newton's laws can be deduced. These laws, in summary, are:

- **Newton's First Law**: An entity at rest or moving at a constant speed in a straight line, will remain at rest or keep moving in a straight line at constant speed unless it is acted upon by a force.
- **Newton's Second Law**: When an entity is acted upon by a force, the time rate of change of its momentum equals the force.
- **Newton's Third Law**: To every action there is always opposed an equal reaction; or, the mutual actions of two bodies upon each other are always equal, and directed to contrary entities.

2.4.11.1 Kinematics

Above we have deduced that primary entities are in both space and time. They have *extent* in both space and time. That means that they may change with respect to their spatial properties: place and form. The change in place is the fundamental. A primary entity which changes place is said to be in *movement*. A primary entity in movement must follow a geometric route. It must move a certain length of route in a certain interval of time, i.e., have a *velocity:* speed and direction. A primary entity which changes velocity has an *acceleration*. That is, we have deduced the basics of *kinematics*. Kinematics imply that an entity changes if it goes from being at rest to moving, or if it goes from moving to being at rest. An entity also changes if it goes from moving at one velocity to moving at a different velocity. We introduce the notion of *momentum*. An entity has the same momentum at two times if it has the same velocity and acceleration.

2.4.11.2 Dynamics

When we, to the above, add that primary entities are in time, then they are subject to causality. That means that we are entering the doctrine of the influence of *forces* on primary entities. That is, *dynamics*.

2.4.11.3 Newton's First Law

When we combine kinematics with causality then we can deduce that if an entity changes momentum then there must be a cause in the circumstances which causally implies the change. We call that cause a *force*. The force must be proportional to the change in momentum. This implies that an entity which is

[40] We skip some of Sørlander's reasoning, [164, Page 162, lines 1–12]

not subject to an external force remains in the same momentum. This is **The Law of Inertia**, **Newton's First Law**.

2.4.11.4 Newton's Second Law

That a certain force is necessary in order to change an entity's momentum must imply that such an entity must provide a certain *resistance* against change of momentum. It must have a *mass*. From this it follows that the change of an entity's momentum not only must be proportional to the applied force but also inversely proportional to that entity's mass. This is **Newton's Second Law**.

2.4.11.5 Newton's Third Law

Where do the forces that influence the momentum of entities come from ? It must, it can only, be from primary entities. Primary entities must be the source of the forces that influence other entities. Here we shall argue one such reason. The next section, on universal gravitation, presents a second reason.

Primary entities may be in one another's way. Hence they may eventually collide. If a primary entity has a certain velocity it may collide with another primary entity crossing its way. In the mutual collision the two entities influence one another such that they change momentum. They influence each other with forces. Since neither of the two entities has any special rank, the forces by means of which they affect one another must be equal and oppositely directed. This is **Newton's Third Law**.

2.4.12 Universal Gravitation

But[41], really, how can primary entities be the source of forces that affect one another ? We must dig deeper ! How can primary entities have mass such that it requires force to change their momentum ? Our answer is that the reason they have mass must be due to mutual influence between the primary objects themselves. It must be an influence which is oppositely directed to that which they expose on one another when they collide. Because this, in principle, applies to all primary entities, these must be characterised by a mutual universal attraction. And that is what we call *universal gravitation*. That concept has profound implications.

• • •

We shall not go into details here but just, casually, as it was, mention that such concepts as speed limit, elementary particles and Einstein's theories are "more-or-less" transcendentally deduced !

2.4.13 Purpose, Life and Evolution

We shall briefly summarise Sørlander's analysis and deductions with respect to the concepts of *living species: plants* and *animals*, the latter including *humans*.

Up till now Sørlander's analyses and deductions have focused on the physical world, "culminating" in Newton's Laws and Einstein's theories.

If[42] there is to be language and meaning then, as a first condition, there must be the possibility that there are primary entities which are not locked-in "only" in that physical world deduced till now. This is only possible if such primary entities are additionally subject to a *purpose-causality*, one that is so

[41] This section is from [164, Pages 168–173]

[42] We now treat the material of [164, *Chapter 10, Pages 174–179*].

constructed as to *strive* to *maintain* its own *existence*. We shall refer to this kind of primary entities as *living species*.

2.4.13.1 Living Species

As living species they must be subject to all the physical conditions for existence and mutual influence. Additionally they must have a form which they are *causally determined to reach and maintain*. This development and maintenance must take place in a *substance exchange* with its surroundings. Living species need these substances in order to develop and maintain their form.

It must furthermore be possible to distinguish between two forms of living species: (i) one form which is characterised only by *development, form and substance exchange*; and (ii) another form which, additional to (i), is characterised by *being able to move*. The first form we call *plants*. The second form we call *animals*.

2.4.13.2 Animals

For animals to move they must (i) possess *sense organs*, (ii) *organs of movement* and (iii) *instincts, incentives,* or *feelings*. All these still subject to the physical laws and to satisfy motion.

This is only possible if animals are **not** built (like the elementary particles of physics) but by special physical units. These cells must satisfy the *purpose-causality* of animals. And we know, now, from the *biological sciences* that something like that is indeed the case. Indeed animals are built from cells all of which possess *genomes* for the whole animal and, for each such cell, a proper fraction of its genome controls whether it is part of a sensory organ, or a nerve, or a motion organ, or a more specific function. Thus it has transcendentally been deduced that such must be the case and biology has confirmed this.

2.4.13.2.1 Humans

We briefly summarise[43], in six steps, (i–vi), Sørlander's reasoning that leads from animals, in general, see above, to humans, in particular.

(i) First the concept of ***level of consciousness*** is introduced. On the basis of animals being able to *learn* from *experience* the concept of *consciousness level* is introduced. It is argued that *neurons* provide part of the basis for *learning* and the *consciousness level*.

(ii) Secondly the concept of ***social instincts*** is introduced. For animals to interact social instincts are required.

(iii) Thirdly the concept of ***sign language*** is introduced. In order for animals to interact some such animals, notably the humans, develop a sign language.

(iv) Fourthly the concept of ***language*** is introduced. The animals that we call *humans* finally develop their sign language into a language that can be spoken, heard and understood. Such a language, regardless of where it is developed, that is, regardless of which language it is, must assume, i.e., build on the same set of basic concepts as had been uncovered so far in our deductions of what must necessarily be in any description of any world.

We continue to summarise[44] Sørlander's reasoning that leads from generalities about humans to humans with knowledge and responsibility.

(v) Fifthly the concept of ***knowledge*** is introduced. An animal which is *conscious* must *sense* and must react to what it senses. To do so it must have *incentives* as causal conditions for its specific such actions. If the animal has, possesses, language, then it must be able to express that and what it senses and that it acts accordingly, and why it does so. It must be able to express that it can express this. That is, that what it expresses, is true. To express such assertions, with sufficient reasons for why they are true, is equivalent to *knowing* that they are true. Such animals, as possessing the above "skills", become persons, humans.

[43] [164, *Chapter 11, Pages 180–183*]

[44] [164, *Chapter 12, Pages 184–187*]

(vi) Sixthly the concept of ***responsibility*** is introduced. Humans conscious of their concrete situation, must also know that these situations change. They are conscious of earlier situations. Hence they have *memory*. So that they can formulate *experience* with respect to the *consequences* of their actions. Thus humans are (also) characterised by being able to understand the consequences of future actions. A person who considers how he ought act, can also be ascribed *responsibility* – and can be judged *morally*.

•••

This ends our exposé of Sørlander's metaphysics with respect to living species. That is, we shall cover neither non-human animals, nor plants.

2.5 Philosophy, Science and the Arts

We quote extensively from [162, Kai Sørlander, 1997].

- Page 178: *Philosophy, science and the arts are products of the human mind.*
- Page 179: *Philosophy, science and the arts each have their own goals.*

And:

- **Philosophers** seek to find the inescapable characteristics of any world.
- **Scientists** seek to determine how the world actually – and our situation in that world – is.
- **Artists** seek to create objects for experience.

We shall elaborate. [162, Page 180] "Simplifying, but not without an element of truth, we can relate the three concepts by the **modalities**:"

- ***philosophy*** is the **necessary**,
- ***science*** is the **real**, and
- ***art*** is the **possible**.

... Here we have, then, a distinction between philosophy and science. ... From [161] we can conclude the following about the results of philosophy and science. These results must be consistent [with one another]. This is a necessary condition for their being *correct*. ... ... The **real** must be a *concrete realisation* of the **necessary**.

2.6 A Word of Caution

The present chapter represents an attempt to give an English interpretation of Kai Sørlander's Philosophy. We otherwise refer to [166].

Chapter 3
Domains

Contents

This chapter is informal. Here we introduce You to important concepts of domains. Subsequent chapters will be more technical. They will define most of the domain concepts of this chapter properly.

3.1 Domain Definition

We repeat the definition of the concept of domains as first given on Page v.

Definition 26 Domain. By a *domain* we shall understand a *rationally describable* segment of a *discrete dynamics* fragment of a *human assisted* reality, i.e., of the world. It includes its **endurants**, i.e., *solid and fluid entities* of **parts** and **living species**, and **perdurants** ▪

Endurants are either *natural* ["God-given"] or *artefactual* ["man-made"] and may be considered *atomic* or *compound* parts, or, as in this primer, further unanalysed *living species:* **plants** and **animals** – including *humans* ▪

Perdurants are here considered to be *actions*, *events* and *behaviours* ▪

We exclude from our treatment of domains issues of biological and psychological matters.

Example 7 Domains. A few, more-or-less self-explanatory examples:

- **Rivers** – with their natural sources, deltas, tributaries, waterfalls, etc., and their man-made dams, harbours, locks, etc. – and their conveyage of materials (ships etc.) [49];
- **Road nets** – with street segments and intersections, traffic lights and automobiles – and the flow of these;
- **Pipelines** – with their wells, pipes, valves, pumps, forks, joins and wells and the flow of fluids [34]; and
- **Container terminals** – with their container vessels, containers, cranes, trucks, etc. – and the movement of all of these [42] ▪

The definition relies on the understanding of the terms *'rationally describable'*, *'discrete dynamics'*, *'human assisted'*, *'solid'* and *'fluid'*. The last two will be explained later. By ***rationally describable*** we mean that what is described can be understood, including reasoned about, in a rational, that is, logical manner – in other words ***logically tractable***. By ***discrete dynamics*** we imply that we shall basically rule out such domain phenomena which have properties that are continuous with respect to their time-wise, i.e., dynamic, behaviour. By ***human-assisted*** we mean that the domains – that we are interested in modeling – have, as an important property, that they possess man-made entities.

This primer presents a *method*, its *principles*, *procedures*, *techniques* and *tools*, for *analysing* &[45] *describing* domains.

3.2 Phenomena and Entities

Definition 27 Phenomena. By a *phenomenon* we shall understand a fact that is observed to exist or happen ▪

Some phenomena are rationally describable – to a large or full degree – others are not.

Definition 28 Entities. By an entity we shall understand a more-or-less rationally describable phenomenon ▪

Example 8 Phenomena and Entities. Some, but not necessarily all aspects of a river can be rationally described, hence can be still be considered entities. Similarly, many aspects of a road net can be rationally described, hence will be considered entities ▪

3.3 Endurants and Perdurants

The two concepts: endurants and perdurants, "go hand-in-hand". We choose to begin our analysis & description of domains with that of endurants.

[45] We use here the ampersand, '&', as in *A*&*B*, to emphasize that we are treating *A* and *B* as one concept.

3.3.1 Endurants

Definition 29 Endurants. Endurants are those quantities of domains that we can observe, in *space*, as "complete" entities at no matter which point in *time* – "material" entities that persist, endure ▪

To a "start" the domain analyser cum describers focus on endurants that they can see and touch. We can refer to these as *concrete endurants*. From these they may form *conceptually abstract endurants*.

Example 9 Endurants. Examples of "see and touch" endurants are: a street segment [link], a street intersection [hub], an automobile. Examples of "conceptually abstract" endurants could be: the Internet, the memoirs [of, and in a person's head], a document as yet unwritten[46] ▪

Domain endurants, when eventually modeled in software, typically become data. Hence the careful analysis of domain endurants is a prerequisite for subsequent careful conception and analyses of data structures for software, including data bases.

3.3.2 Perdurants

Definition 30 Perdurants. Perdurants are those quantities of domains for which only a fragment exists, in *space*, if we look at or touch them at any given snapshot in *time* ▪

Example 10 Perdurant. A moving automobile is an example of a perdurant ▪

Domain perdurants, when eventually modeled in software, typically become processes. Hence the careful analysis of domain perdurants is a prerequisite for subsequent careful conception and analyses of functions (procedures).

3.4 External and Internal Endurant Qualities

3.4.1 External Qualities

Definition 31 External Qualities. External qualities of endurants of a manifest domain are, in a simplifying sense, those we can see, touch and have spatial extent. They, so to speak, take form ▪

Example 11 External Qualities. An examples of external qualities of a domain are: the Cartesian product of sets of solid atomic street intersections, and of sets of solid atomic street segments, and of sets of solid automobiles of a road transport system where the Cartesian, sets, atomic, and solid reflect external qualities ▪

3.4.1.1 Discrete or Solid Endurants

Definition 32 Discrete or Solid Endurants. By a *solid* [or *discrete*] endurant we shall understand an endurant which is separate, individual or distinct in form or concept, or, rephrasing: have 'body' [or magnitude] of three-dimensions: length, breadth and depth [131, Vol. II, pg. 2046] ▪

[46] [41, https://www.imm.dtu.dk/ dibj/2025/documents/main.pdf]

Example 12 Solid Endurants. Examples of solid endurants are the wells, pipes, valves, pumps, forks, joins and sinks of pipelines. [These units may, however, and usually will, contain fluids, e.g., oil, gas or water] ▪

We shall mostly be analysing and describing solid endurants.

As we shall see, in the next chapter, we analyse and describe solid endurants as either parts or living species: animals and humans. We shall mostly be concerned with parts. That is, we shall just, as: "in passing", for the sake of completeness, mention living species !

3.4.1.2 Fluids

Definition 33 Fluid Endurants. By a *fluid endurant* we shall understand an endurant which is prolonged, without interruption, in an unbroken series or pattern; or, rephrasing: a substance (liquid, gas or plasma) having the property of flowing, consisting of particles that move among themselves [131, Vol. I, pg. 774] ▪

Example 13 Fluid Endurants. Examples of fluid endurants are: water, oil, gas, compressed air, smoke ▪

Fluids are otherwise liquid, or gaseous, or plasmatic, or granular[47], or plant products, i.e., chopped sugar cane, threshed, or otherwise[48], et cetera. Fluid endurants will be analysed and described in relation to solid endurants, viz. their "containers".

3.4.1.3 Parts

Definition 34 Parts. The non-living species solids are what we shall call parts ▪

Parts are the "work-horses" of man-made domains. That is, we shall mostly be concerned with the analysis and description of endurants into parts.

Example 14 Parts. The previous example of solids was also an example of parts ▪

We distinguish between atomic and compound parts.

3.4.1.3.1 Atomic Parts

Definition 35 Atomic Part, I. By an *atomic part* we shall understand a part which the domain analyser considers to be indivisible in the sense of not meaningfully divisible, for the purposes of the domain under consideration, that is, to not meaningfully consist of sub-parts ▪

Example 15 Atomic Parts. Examples of atomic parts are: a hub, i.e., a street intersection; a link, i.e., the stretch of road between two neighbouring hubs; and an automobile ▪

3.4.1.3.2 Compound Parts

We, pragmatically, distinguish between Cartesian-product-, and set- oriented parts. If Cartesian-oriented, to consist of two or more distinctly sort-named endurants (solids or fluids). If set-oriented, to consist of an indefinite number of zero, one or more parts.

[47] This is a purely pragmatic decision. "Of course" sand, gravel, soil, etc., are not fluids, but for our modeling purposes it is convenient to "compartmentalise" them as fluids !

[48] See footnote 47.

Definition 36 **Compound Part, I.** *Compound parts* are those which are either Cartesian- or are set- oriented parts ▪

Example 16 **Compound Parts.** An example of compound parts is: (i) a road net consisting of a set of hubs, i.e., street intersections or "end-of-streets", and (ii) a set of links, i.e., street segments (with no contained hubs), is a Cartesian compound. (iii) Each set of hubs and each set of links are part set compounds ▪

3.4.2 An Aside: An Upper Ontology

We have been reasonably careful to just introduce and state informal definitions of phenomena and some classes thereof. In the next chapter we shall, in a sense, "repeat" coverage of these phenomena. But then in a more analytic manner. Figure 3.1 is intended to indicate this.

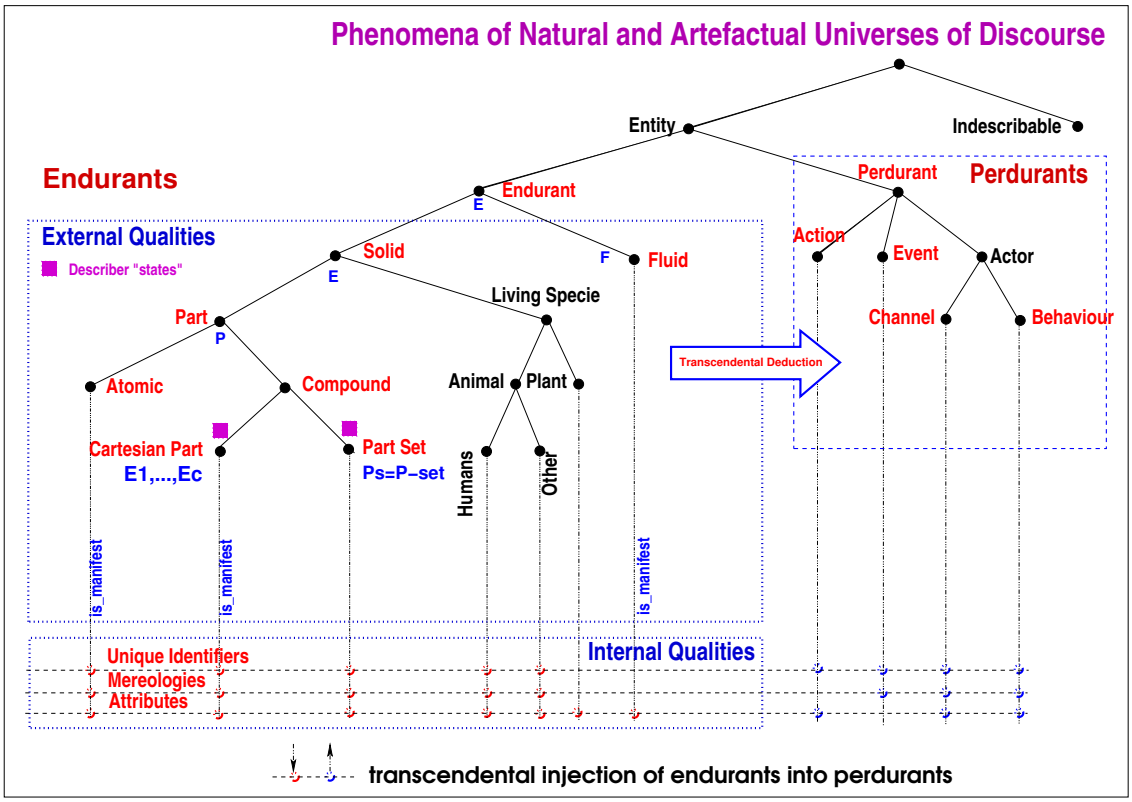

Fig. 3.1 An Upper Ontology: The Domain Analysis & Description Ontology

So far we have only touched upon the 'External Qualities' labeled, dotted-dashed box of the 'Endurants'-labeled **blue** dashed [- - - ... -] box of Fig. 3.1. In Chapter 4 we shall treat external qualities in more depth — more systematically: analytically and descriptionally.

• • •

In analysing and describing domains the domain analyser cum describers can be said to "traverse" both the domain and the graph of Fig. 3.1 on the preceding page.

By *traversal* we mean a repeated [a series of] analysis and description steps where each step – wrt. the graph of Fig. 3.1 on the previous page – "starts" at the root of Fig. 3.1 on the preceding page and [repeatedly] inquires the external qualities, of the domain elements being considered: are they describable; if so, are the endurants; if so, are they solid; if so, are they parts; if so, are they atomic; if not, are they Cartesian; etc. ! [We shall "formalise" this traversal, item 191, page 127 of Sect. 7.2.4.1.] ▪

3.4.3 Internal Qualities

Definition 37 Internal Qualities. Internal qualities are those properties [of endurants] that do not occupy *space* but can be measured or rationally spoken about ▪

Example 17 Internal qualities. Examples of internal qualities are the unique identity [cf. Sect. 3.4.3.1] of a part, the mereological relation [cf. Sect. 3.4.3.2] of a part to other parts, and the endurant attributes [cf. Sect. 3.4.3.3] such as temperature, length, colour ▪

3.4.3.1 Unique identity

Definition 38 Unique Identity. A unique identity is an immaterial property that distinguishes any two *spatially* distinct solids ▪

Example 18 Unique Identities. Each hub in a road net is uniquely identified, so is each link and each automobile ▪

3.4.3.2 Mereology

Definition 39 Mereology, I. Mereology is a theory of [endurant] part-hood relations: of the relations of an [endurant] part to a whole and the relations of [endurant] parts to [endurant] parts within that whole ▪

Example 19 Mereology. Examples of mereologies are that a link is topologically *connected* to exactly two specific hubs, that a hub is *connected* to zero, one or more specific links, and that links and hubs are *open* to specific subsets of automobiles ▪

3.4.3.3 Attribute

Definition 40 Attributes. Attributes are properties of endurants that are not *spatially* observable, but can be either physically (electronically, chemically, or otherwise) measured or can be objectively spoken about ▪

Example 20 Attributes. Examples of attributes are: links have lengths, and, that at any one time, zero, one or more automobiles are occupying each link[49] ▪

[49] Oh yes, it is, of course, spatially observable that a link has a length, but the measurement, say *123 meters* is not; and the number of cars on the link is also not spatially observable.

3.5 Prompts

3.5.1 Analysis Prompts

Definition 41 Analysis Prompt. An analysis prompt is a predicate or a function that may be posed by humans to segments of a domain. Observing the domain the analyser may then act upon the combination of the particular prompt (whether a predicate or a function, and then what particular one of these it is), thus "applying" it to a domain phenomenon, and yielding, in the minds of the humans, either a truth value or some other form of value ▪

3.5.1.1 Analysis Predicate

Definition 42 Analysis predicates. An analysis predicate is an analysis prompt which yields a truth value ▪

Example 21 Analysis Predicates. General examples of analysis predicates are: *"can an observable phenomenon be rationally described"*, i.e., an entity, *"is an entity a solid or a fluid"*, *"is a solid endurant a part or a living species"* ▪

3.5.1.2 Analysis Function

Definition 43 Analysis function. An analysis function is an analysis prompt which yields some RSL-Text ▪

Example 22 Analysis Functions. Two examples of analysis functions are: one yields the endurants of a Cartesian part and their respective sort names, another yields the set of parts of a part set and their common type ▪

3.5.2 Description Prompt

Definition 44 Description Prompt. A description prompt is a function that may be posed by humans who may then act upon it: [the human] "applying" it to a domain phenomenon, and [the human] "yielding", i.e., writing down, a narrative and formal RSL-Texts describing what is being observed [by that human] ▪

Example 23 Description Prompts. Description prompts result in RSL-Texts describing for example a (i) a Cartesian endurant, or (ii) its unique identifier, or (iii) its mereology, or (iv) its attributes, or (iv) other ▪

3.6 Perdurant Concepts

3.6.1 "Morphing" Parts into Behaviours

As already indicated we shall transcendentally deduce (perdurant) behaviours from those (endurant) parts which we, as domain analysers cum describers, have endowed with all three kinds of internal qualities: unique identifiers, mereologies and attributes. Chapter 6, will show how this deduction from endurants to perdurants can be accomplished.

3.6.2 State

Definition 45 **State, I.** A state is any set of the parts of a domain.

Example 24 **A Road System State.** The domain analyser cum describer may decide that a road system state consists of the road net aggregate (of hubs and links)[50], all the hubs, and all the links, and the automobile aggregate (of all the automobiles)[51], and all the individual automobiles.

3.6.3 Actors

Definition 46 **Actors.** An actor is anything that can initiate an action, an event or a behaviour.

3.6.3.1 Action

Definition 47 **Actions.** An action is a function that can purposefully change a state.

Example 25 **Road Net Actions.** These are some road net actions: The insertion of a new or removal of an existing hub; or the insertion of a new, or removal of an existing link;

3.6.3.2 Event

Definition 48 **Events.** An event is a function that surreptitiously changes a state.

Example 26 **Road Net Events.** These are some road net events: The blocking of a link due to a mud slide; the failing of a hub traffic signal due to power outage; the blocking of a link due to an automobile accident.

3.6.3.3 Behaviour

Definition 49 **Behaviours.** A behaviour is a set of sequences of actions, events and behaviours.

Example 27 **Road Net Traffic.** Road net traffic can be seen as a behaviour (i) of all the behaviours of automobiles, where each automobile behaviour is seen as sequence of start, stop, turn right, turn left, etc., actions; (ii) of all the behaviours of links where each link behaviour is seen as a set of sequences (i.e., behaviours) of "following" the link entering, link leaving, and movement of automobiles on the link; (iii) of all the behaviours of hubs (etc.); (iv) of the behaviour of the aggregate of roads, viz. *The Department of Roads*, and (v) of the behaviour of the aggregate of automobiles, viz, *The Department of Vehicles*.

[50] The road net aggregate, in its perdurant form, may "model" the *Department of Roads* of some country, province, or town.

[51] The automobile aggregate, in its perdurant form, may "model" the *Department of Vehicles* of some country, province, or town.

3.6.4 Channel

Definition 50 **Channel**. A channel is anything that allows synchronisation and communication of values between two behaviours ▪

We shall use Tony Hoare's CSP concept [114] to express synchronisation and communication of values between behaviours ***i*** and ***j***. Hence the behaviour ***i*** statement ***ch[j] ! value*** states that behaviour ***i*** offers, "outputs": !, ***value*** to the behaviour indicated by j. And behaviour j expresses ***ch[i] ?*** that it is willing to accept "input from & synchronise with" behaviour ***i***, ?, any ***value***.

3.7 Domain Analysis & Description

3.7.1 Domain Analysis

Definition 51 **Domain Analysis**. Domain analysis is the act of studying a domain as well as the result of that study in the form of **informal** statements ▪

3.7.2 Domain Description

Definition 52 **Domain Description**. Domain description is the act of describing a domain as well as the result of that act in the form of **narratives** and **formal** RSL-Text ▪

3.8 Closing

This chapter has introduced the main concepts of domains such as we shall treat (analyse and describe) domains.[52] The next three chapters shall now systematically treat the analysis and description of domains. That treatment takes concept by concept and provides proper definitions and introduces appropriate analysis and description prompts; one-by-one, in an almost pedantic, hence perhaps "slow" progression ! The reader may be excused if they, now-and-then, lose sight of "their way". Hence the present chapter. To show "the way": that, for example, when we treat external endurant qualities, there are still the internal endurant qualities, and that the whole thing leads of to perdurants: actors, actions, events and behaviours.

[52] We have omitted treatment of *living species: plants* and *animals* – the latter including *humans*. They will be treated in the next chapter !

Chapter 4
Endurants: External Domain Qualities

Contents

53

In this and the next chapter we shall analyse and describe *endurants*, that is, the *entities* that can be observed, or conceived and described, as a "complete thing" at no matter which given snapshot of time; alternatively an entity is endurant if it is capable of *enduring*, that is *persists*, *"holds out"* [131, Vol. I, pg. 656].

This modeling will focus on the **types** and **observers** of these endurants.

On one hand there are the domain phenomena of endurants. On the other hand there are means for analysing and describing these. The former are not formalised "before", or as, we analyse and describe them. The latter, 'the means', are assumed formalised.

Definition 53 Description, I. By a **description** of the external domain qualities we shall mean a pair of informal, narrative, and formal text – where these texts characterize the endurant entities of domains: their sorts, i.e., types, and the observers, i.e., informal, functions, that "dissects" the compound parts into endurants, usually sub-parts ▪

This chapter explains what is meant by *external qualities*, *endurants* and their *compound parts*.

Primary Modeling Tool, I

The tool with which we describe endurants will be the **type** and **function** concepts of, in this case, the formal specification language RSL [98].[54]

4.1 Universe of Discourse

The first analysis and description of a chosen domain is that of its universe of discourse.

Definition 54 Universe of Discourse, UoD. By a ***universe of discourse*** we shall understand the same as the ***domain of interest***, that is, the *domain* to be *analysed & described* ▪

Definition 55 Universe of Discourse Description. By a ***universe of discourse*** **description** we shall understand a pair

(i) a formal sort definition of the one line form:

type UoD

where UoD is a name, chosen by the domain analyser cum describers; and

(ii) a brief, a terse, informal, say at most a half page, text that mentions, in some form, what the domain analyser cum describers consider important characteristics of the domain ▪

We shall now consider these elements of description in some detail.

4.1.1 Identification

The **first task** of a domain analyser cum describer[55] is to settle upon the domain to be analysed and described. That domain has first to be given a *name*.

[53] Observe "insertion" of 'Domain Method' and 'Method' Domain' index range commands. This note is temporary: for editorial purposes.

[54] We could have chosen another formal specification language: VDM [64, 65, 93], Z [175], or Alloy [120], or other.

[55] Henceforth referred to as the domain modeler.

4.1.2 Sort and Types: Names and Meaning

We take an aside:

- Sorts are here sets of domain entities.
- Types are [more generally] sets of domain entities and mathematical quantities.
- Types have names T:𝕋.
- 𝕋 is the type of all type names.
- Sorts have names: the name of all sorts is 𝕊.
- The name of the sort of all names of sorts is Ξ [pronounced 'sigh'].
- If sort has name S:𝕊, then the name of that name is ηS:Ξ.

4.1.3 Naming

A **first decision** is to give a name to the overall domain sort, that is, the "kind" of the domain seen as an endurant, with that sort name being freely chosen by the domain modeler – with no such sort names having been chosen so far !

Mental Operation 1 The Naming Procedure: Throughout the analysis & description of domains the domain analyser cum describers must "create" names. These are names of the describable phenomena, i.e., endurants and perdurants, that are being analysed and described: of the external [solid (part) and fluid] as well as internal qualities [unique identifiers, mereologies and attributes]. And also (names of) functions over theses. Care must therefore be excerted in choosing appropriate names. A typical technique, especially in the context of domain engineering, is to choose names, or mnemonics[56] thereof ▪

4.1.4 Examples

Examples of UoDs: We refer to a number of Internet accessible experimental reports [48] of descriptions of the following domains:

- *railways* [15, 17, 62],
- *"The Market"* [16],
- *container shipping* [22],
- *Web systems* [30],
- *stock exchange* [31],
- *oil pipelines* [34],
- *credit card systems* [37],
- *weather information* [38],
- *swarms of drones* [39],
- *document systems* [41],
- *container terminals* [42],
- *retail systems* [46],
- *assembly plants* [43],
- *waterway systems* [49],
- *shipping* [50],
- *urban planning* [70].

4.1.5 Sketching

The **second task** of a domain modeler is to develop a *rough sketch narrative* of the domain. The rough-sketching of a domain is not a trivial matter. It is not done by a committee ! It usually requires repeated

[56] Mnemonic: a device such as a pattern of letters, ideas, or associations that assists in remembering something [Oxford Languages]

"trial sketches". To carry it out, i.e., the sketching, normally requires a combination of physical visits to domain examples, if possible; talking with domain professionals, at all levels; and reading relevant literature. It also includes searching the Internet for information. We shall show an example next.

Example 28 **Sketch of a Road Transport System UoD**. The road transport system that we have in mind consists of a road net and a set of automobiles (private, trucks, buses, etc.) such that the road net serves to convey automobiles. We consider the road net to consist of hubs, i.e., street intersections, and links, i.e., street segments between adjacent hubs[57] ▪

The sketching is necessarily informal. The domain analyser cum describers have yet to understand the domain. As they understand more-and-more of it, they can "go back" and improve upon the narrative. The whole purpose of domain modeling is to better understand the domain – and to formalise that understanding.

4.1.6 Universe of Discourse Description

The general universe of discourse, i.e., domain, description prompt can be expressed as follows:

Domain Description Prompt 1 describe_Universe_of_Discourse:
0. ***describe_Universe_of_Discourse*** **describer**

❝ **Naming**:
  type UoD
Rough Sketch:
  Text ❞

The above ❝ RSL-**Text** ❞ expresses that the *describe_Universe_of_Discourse()* domain describer generates RSL-**Text**. Here is another example rough sketch:

Example 29 **A Rough Sketch Domain Description**. The example is that of the production of rum, say of a **Rum Production** domain.

- the sowing, watering, and tending to of sugar cane plants;
- via the "burning" of these prior to harvest;
- the harvest;
- the collection of harvest from sugar cane fields to
- the chopping, crushing, (and sometimes repeated) boiling, cooling and centrifuging of sugar cane when making sugar and molasses (into A, B, and low grade batches);
- the fermentation, with water and yeast, producing a 'wash';
- the (pot still or column still) distilling of the wash into rum;
- the aging of rum in oak barrels;
- the charcoal filtration of rum;
- the blending of rum;
- the bottling of rum;
- the preparation of cases of rum for sales/export; and
- the transportation of these cases away from the rum distiller ▪

[57] This "rough" narrative fails to narrate what hubs, links, vehicles, automobiles are. In presenting it here we rely on your a priori understanding of these terms. But that is dangerous ! The danger, if we do not painstakingly narrate and formalise what we mean by all these terms, then readers (software designers, etc.) may make erroneous assumptions.

Some comments on Example 29: Each of the itemized items above is phrased in terms of perdurants. Behind each such perdurant lies some endurant. That is, in English, *"every noun can be verbed", and vice-versa.* It is in this way we anticipate the transcendental deduction, from endurants to perdurants.

We shall, in the next sections, not handle the issues related to the modeling of the above kind of examples. But will do so in Sects. 4.6 on page 52, 5.7 on page 96 and 6.10 on page 113.

• • •

Method Principle 1 ***From the "Overall" to The Details:*** Our first principle, as the first task in any new domain modeling project, is to "focus" on the "overall", that is, on the "entire", generic domain ▪

4.2 Entities

A core concept of domain modeling is that of an *entity*.

Definition 56 Entity. By an *entity* we shall understand a *phenomenon*, i.e., something that can be *observed*, i.e., be seen or touched by humans, *or* that can be *conceived* as an *abstraction* of an entity; alternatively, a phenomenon is an entity, *if it exists, it is "being", it is that which makes a "thing" what it is: essence, essential nature* [131, Vol. I, pg. 665]. If a phenomenon cannot be so **observed and described** then it is not en entity ▪

Mental Operation 2 **The is_E Observer:** The "mechanics" involved in order to analyse a phenomenon as to its domain sort is the *mental analysis operator* **is_**. It is combined with [prefixed to] the name for the sort of the phenomenon inquired. Applied to a value, e, of the sort E, i.e, **is_**E(e), it yields a truth value (**true** or **false**). By mental analysis operator we mean that it denotes an operation performed by the domain analyser cum describer: It is not computable. **is_**E are referred to as *observer functions* ▪

Analysis Predicate Prompt 1 **is_entity:** The domain analyser analyses "things" (ϕ, 'phenomena') into either entities or non-entities. The method provides the **domain analysis prompt**:

- **is_*entity*** – where **is_*entity*(ϕ)** holds if ϕ is an entity ▪ [58]

is_*entity* is said to be a *prerequisite prompt* for all other prompts.

Method 1: **is_*entity*** is a method **tool**.

On Analysis Prompts

The **is_*entity*** predicate function represents the first of a number of analysis prompts. They are "applied" by the domain analyser to phenomena of domains. They yield truth values, true or false, "left" in the mind of the domain analyser ▪

• • •

We have just shown how the **is_*entity*** predicate prompt can be applied to a universe of discourse. From now on we shall see prompts being applicable to increasingly more analysed entities. Figure 4.1 on the next page diagrams a **domain description ontology** of entities. That ontology indicates the sub-classes of endurants for which we shall introduce prompts, predicates and functions.

[58] ▪ marks the end of an analysis prompt definition.

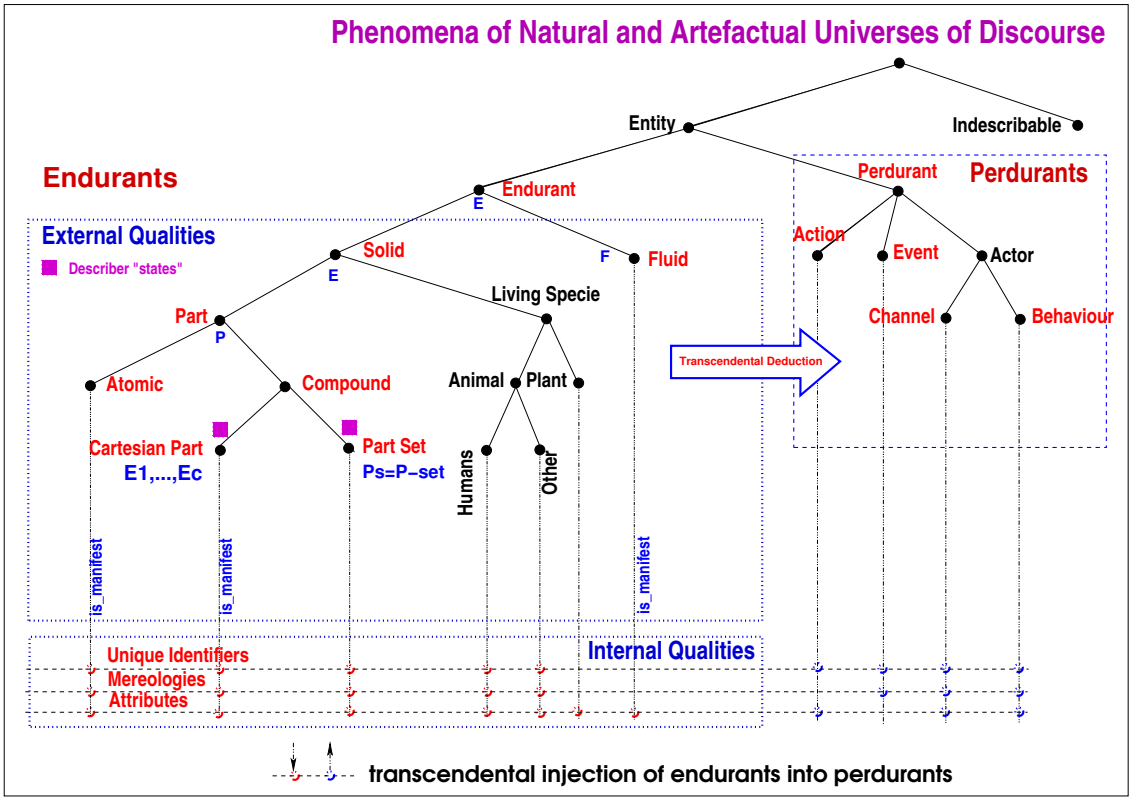

Fig. 4.1 The Upper Ontology – same as Fig. 3.1 on page 27

The present chapter shall focus only on the external qualities, that is, on the "contents" of the leftmost dash-dotted box.

• • •

Method Principle 2 ***Justifying Analysis along Philosophical Lines:*** The concept of *entities* as a main focal point is justified in *Kai Sørlander*'s philosophy [161–165, 1994–2022]. Entities are there referred to as *primary objects*. They are the ones about which we express predicates.

4.3 Endurants and Perdurants

Method Principle 3 ***Separation of Endurants and Perdurants:*** As we shall see in this primer, the domain analysis & description method calls for the separation of first considering the careful analysis & description of endurants, then considering perdurants. This principle is based on the transcendental deduction of the latter from the former.

4.3.1 Endurants

Definition 57 Endurant. By an *endurant*, to repeat, we shall understand an entity that can be observed, or conceived and described, as a "complete thing" at no matter which given snapshot of time; alternatively

an entity is an endurant if it is capable of *enduring*, that is, *persists*, *"holds out"* [131, Vol. I, pg. 656]. Were we to "freeze" time we would still be able to observe the entire endurant ▪

Example 30 **Natural and Artefactual Endurants.**
Geography Endurants: fields, meadows, lakes, rivers, forests, hills, mountains, et cetera.
Railway Track Endurants: a railway track, its net, its individual tracks, switch points, trains, their individual locomotives, signals, et cetera.
Road Transport System Endurants: the transport system, its road net aggregate and the aggregate of automobiles, the set of links (road segments) and hubs (road intersections) of the road net aggregate, these links and hubs, and the automobiles.

Analysis Predicate Prompt 2 **is_endurant:** The domain analyser analyses an entity, ϕ, into an endurant as prompted by the **domain analysis prompt**:

- **is_*endurant*** – ϕ is an endurant if **is_*endurant*(ϕ)** holds ▪

is_*entity* is a *prerequisite prompt* for **is_*endurant*.**

Method 2: **is_*endurant*** is a method **tool.**

4.3.2 Perdurants

Definition 58 **Perdurant.** By a *perdurant* we shall understand an entity for which only a fragment exists if we look at or touch them at any given snapshot in time. Were we to freeze time we would only see or touch a fragment of the perdurant [131, Vol. II, pg. 1552] ▪

Example 31 **Perdurants.** **Geography Perdurants:** the continuous changing of the weather (meteorology); the erosion of coastlines; the rising of some land area and the "sinking" of other land area; volcanic eruptions; earthquakes; et cetera. **Railway System Perdurants:** the ride of a train from one railway station to another; and the stop of a train at a railway station from some arrival time to some departure time ▪

Analysis Predicate Prompt 3 **is_perdurant:** The domain analyser analyses an entity *e* into a perdurant as prompted by the **domain analysis prompt**:

- **is_*perdurant*** – *e* is a perdurant if **is_*perdurant(e)*** holds.

is_*entity* is a *prerequisite prompt* for **is_*perdurant*** ▪

Method 3: **is_*perdurant*** is a method **tool.**

• • •

We repeat method principle 3 on the preceding page:

Method Principle 4 ***Separation of Endurants and Perdurants:*** First domain analyse & describe endurants; then domain analyse & describe perdurants ▪

4.4 Solids and Fluids

For *pragmatic* reasons we distinguish between solids and fluids.

Method Principle 5 ***Abstraction:*** The principle of abstraction is now brought into "full play": In analysing & describing entities the domain modeler is "free" to not consider all facets of entities, that is, to abstract. We refer to our characterisation of abstraction in Sect. 1.4 on page 3.

4.4.1 Solids

Definition 59 **Solid Endurant**. By a *solid endurant* we shall understand an endurant which is separate, individual or distinct in form or concept, or, rephrasing: a body or magnitude of three-dimensions, having length, breadth and thickness [131, Vol. II, pg. 2046] ▪

Analysis Predicate Prompt 4 **is_solid:** The domain analyser analyses endurants, *e*, into solid entities as prompted by the **domain analysis prompt**:

- **is_*solid*** – *e* is solid if **is_*solid(e)*** holds ▪

To simplify matters we shall allow separate elements of a solid endurant to be fluid! That is, a solid endurant, i.e., a part, may be conjoined with a fluid endurant, a fluid.

Method 4: **is_*solid*** is a method **tool**.

Example 32 **Artefactual Solid Endurants**. The individual endurants of the above example of **railway system** endurants, Example 30 on the previous page, were all solid. Here are examples of solid endurants of **pipeline systems**. A pipeline and its individual units: wells, pipes, valves, pumps, forks, joins, regulator, and sinks ▪

4.4.2 Fluids

Definition 60 **Fluid Endurant**. By a *fluid endurant* we shall understand an endurant which is prolonged, without interruption, in an unbroken series or pattern; or, rephrasing: a substance (liquid, gas or plasma) having the property of flowing, consisting of particles that move among themselves [131, Vol. I, pg. 774] ▪

Analysis Predicate Prompt 5 **is_fluid:** The domain analyser analyses endurants *e* into fluid entities as prompted by the **domain analysis prompt**:

- **is_*fluid*** – *e* is fluid if **is_*fluid(e)*** holds ▪

Method 5: **is_*fluid*** is a method **tool**.

Fluids are otherwise liquid, or gaseous, or plasmatic, or granular[59], or plant products[60], et cetera.

Example 33 **Fluids**. Specific examples of fluids are: water, oil, gas, compressed air, etc. A container, which we consider a solid endurant, may be *conjoined* with another, a fluid, like a gas pipeline unit may "contain" gas ▪

We shall, however, in this primer, extend the meaning of the word 'fluid' to also embody such entities as:

- granular substance: sand, salt, sugar, etc.;
- pieces of living species: chopped sugar cane, cut wooden logs, etc.;
- jams, marmalade, pieces of, f.ex., marinated herring, etc.;

This "extension" is required for our modeling so-called "soft pats", cf. Sects. 4.6 on page 52, 5.7 on page 96 and 6.10 on page 113.

4.5 Parts and Living Species

We analyse endurants into either of two kinds: *parts* and *living species*. The distinction between *parts* and *living species* is motivated in Kai Sørlander's Philosphy [161–165].

4.5.1 Parts

Definition 61 **Part**. By a *part* we shall understand a solid endurant existing in time and subject to laws of physics, including the *causality principle* and *gravitational pull*[61] ▪

Analysis Predicate Prompt 6 **is_part:** The domain analyser analyses "things" (e) into part. The method provides the **domain analysis prompt**:

- **is_part** – where **is_part**(e) holds if e is a part ▪

Method 6: **is_*part*** is a method **tool**.

Parts are either *natural* parts, or are *artefactual* parts, i.e. man-made. Natural and man-made parts are either *atomic* or *compound*.

[59] This is a purely pragmatic decision. "Of course" sand, gravel, soil, etc., are not fluids, but for our modeling purposes it is convenient to "compartmentalise" them as fluids !

[60] i.e., chopped sugar cane, threshed, or otherwise. See footnote 59.

[61] This characterisation is the result of our study of relations between philosophy and computing science, notably influenced by Kai Sørlander's Philosphy [161–165]

4.5.1.1 Atomic Parts

The term 'atomic' is, perhaps, misleading. It is not used in order to refer to nuclear physics. It is, however, chosen in relation to the notion of *atomism: a doctrine that the physical or physical and mental universe is composed of simple indivisible minute particles* [Merriam Webster].

Definition 62 Atomic Part. By an *atomic part* we shall understand a part which the domain analyser considers to be indivisible in the sense of not meaningfully divisible, for the purposes of the domain under consideration, that is, to not meaningfully consist of sub-parts ▪

Example 34 Some Atomic Parts. We refer to Example 32 on page 40: pipeline systems. The wells, pumps, valves, pipes, forks, joins and sinks can be considered atomic ▪

Analysis Predicate Prompt 7 is_atomic: The domain analyser analyses "things" (e) into atomic parts . The method provides the **domain analysis prompt**:

- is_atomic – where is_atomic(e) holds if e is an atomic part ▪

Method 7: is_*atomic* is a method **tool**.

4.5.1.2 Compound Parts

We, pragmatically, distinguish between Cartesian-product-, and set- oriented parts. That is, if Cartesian-product-oriented, to consist of two or more distinctly sort-named endurants (solids or fluids), or, if set-oriented, to consist of an indefinite number of zero, one or more identically sort-named parts.

Definition 63 Compound Part. *Compound part*s are those which are either Cartesian-product– or are set– oriented parts ▪

Analysis Predicate Prompt 8 is_compound: The domain analyser analyses "things" (e) into compound parts . The method provides the **domain analysis prompt**:

- is_compound – where is_compound(e) holds if e is a compound part ▪

Method 8: is_*compound* is a method **tool**.

Sub_Part Naming: The endurant elements of compounds, whether they be Cartesians or part set parts, are to be given sort names. For that purpose we introduce two mental observers: one for observing the names of the sorts of the elements of the Cartesian: **obs_Cartesian_sub_sorts**, and another for observing the name of the the sort of the part of the part sets: **obs_part_set_sub_part**. We shall introduce these observer operations below.

Mental Operation 3 **The obs_E Observer:** The "mechanics" involved in order to observe the components, Pi, of a compound, E, is the *description operator* **obs_**.[62] The mental operator is combined with [prefixed to] the name for the sort of phenomenon inquired and described. Applied to a value, e, of the sort E, i.e, **obs_**E(e), it yields the E component of [any] e:E. By a mental description operator we mean that it denotes an operation that appears in the RSL$^+$Text of a domain description and denotes an operation performed by the domain analyser cum describer: It is not computable. The **obs_**Ei are referred to as *observer functions* ▪

4.5.1.2.1 Cartesian Parts

Definition 64 **Cartesian Part**. A *Cartesian part* is a compound part which consists of an "indefinite number" of two or more parts of distinctly named sorts ▪

Definition 65 **Cartesian Part Sort**. The *Cartesian part sort* is the set of sorts of its parts ▪

Some clarification may be needed. (i) In mathematics, as in RSL [98], a value is a Cartesian value if it can be expressed, for example as $(a,b,...,c)$, where $a,b,...,c$ are mathematical (or, which is the same, RSL) values. Let the sort names of these be $A,B,...,C$ – with these being required to be distinct. We wrote "indefinite number": the meaning being that the number is fixed, finite, but not specific. (ii) The requirement: 'distinctly named' is pragmatic. If the domain modeler thinks that two or more of the components of a Cartesian part are [really] of the same sort, then that person is most likely confused and must come up with suitably distinct sort names for these "same sort" parts ! (iii) Why did we not write "definite number" ? Well, at the time of first analysing a Cartesian part, the domain modeler may not have thought of all the consequences of, i.e., analysed, the compound part. Additional sub-parts, of the Cartesian compound, may be "discovered", subsequently and can then, with the approach we are taking with respect to the modeling of these, be "freely" added subsequently !

Example 35 **Cartesian Automobiles**. We refer to Example 30 on page 39, the transport system sub-example. We there viewed (hubs, links and) automobiles as atomic parts. From another point of view we shall here understand automobiles as Cartesian parts: the engine train, the chassis, the car body, four doors (left front, left rear, right front, right rear), and the wheels. These may again be considered Cartesian parts.

Analysis Predicate Prompt 9 **is_Cartesian:** The domain analyser analyses "things" (e) into Cartesian parts . The method provides the **domain analysis prompt**:

- **is_Cartesian** – where **is_Cartesian(e)** holds if e is a Cartesian part ▪

Method 9: **is_*Cartesian*** is a method **tool**.

4.5.1.2.1.1 Determine Cartesian Part Sorts

The above analysis amounts to the analyser first "applying" the *domain analysis* prompt **is_*compound*(*e*)** to a solid endurant, *e*, where we now assume that the obtained truth value is **true**. Let us assume that

endurant $e{:}E$ consists of sub-endurants of sorts $\{E_1,E_2,...,E_m\}$. Since we cannot automatically guarantee that our domain descriptions secure that E and all these E_i $(1{\leq}i{\leq}m)$ denotes disjoint sets of entities *we must prove so !*

Mental Operation 4 **The obs_Cartesian_sub_sorts Observer:** This mental operator is invoked by the domain analyser cum describer when analysis has shown that a part is a Cartesian. As an operation, "applied" by the domain analyser cum describer, to a Cartesian compound, s:E, it yields a set of [mnemotechnically] "pleasing" and "suggestive" names for the sorts of the [perceived] Cartesian elements: E1, E2, ..., En ▪

On Determination Functions

Determination functions apply to compound parts and yield their sub-parts and the sorts of these. *That is, we observe the domain and our observation results in a focus on a subset of that domain and sort information about that subset.*

An RSL Extension

The **obs_Cartesian_sub_sorts_** and *** functions below involve the "generation" of sort names. Typically they involve RSL$^+$Text expressions of the form ηA,ηB,...,ηC.[63] That is, a "pattern" of sort names: A,B,...,C. These sort names are provided by the domain modeler. They are chosen as "full names", or as mnemonics, to capture an essence of the (to be) described sort. Repeated invocations, by the domain modeler, of these (...,sorts) analysis functions normally lead to new sort names distinct from previously chosen such names.

Observer Function Prompt 1 obs_Cartesian_sub_sorts: The domain analyser analyses a part into a Cartesian part. The method thus provides the **domain observer prompt**:

- *obs_Cartesian_su_sorts* — it directs the domain analyser to determine the definite number of endurants and corresponding distinct sorts of the part.

  value
  obs_Cartesian_sub_sorts_E → (E1×E2×...×En[64]) × (ηE1×ηE2×...×ηEn)[65]
  obs_Cartesian_sub_sorts_(e) **as** ((e1,...,en),(ηE1,...,ηEn))

  where by E, Ei we mean endurants, i.e., parts, and by ηEi we mean the names of the corresponding types ▪

Method 10: obs_Cartesian_sub_sorts is a method **tool**.

Calculation prompts apply to compound parts: Cartesians and sets, and yield an RSL-Text description.

[63] ηA,ηB,...,ηC are **names** of types. $\eta\theta$ is the type of all sort and type names !

[64] The identifiers E1, E2, ..., En could as well be written E_1, E_2, ..., E_n

[65] The ordering, ((e1,...,en),(ηE1,...,ηEn)), is pairwise arbitrary.

4.5.1.2.1.2 Describe Cartesian Part Sorts

Domain Description Prompt 2 describe_Cartesian_part_sorts: If **is_***Cartesian*(*e*) holds, then the analyser applies

the **domain description prompt** **obs_Cartesian_sub_sorts** resulting in the analyser writing down the endurant sorts and endurant sort observers.

The 1. **obs_Cartesian_sub_sorts Describer:**

let (_[66],(ηE$_1$,...,ηE$_m$)) = **obs_Cartesian_sub_sorts**(e) **in**
❝**Narration:**
[s] ... narrative text on sorts ...
[o] ... narrative text on sort observers ...
[p] ... narrative text on proof obligations ...
Formalisation:
type
[s] E$_1$, ❞...❝, E$_m$
value
[o] **obs_**E$_1$: E → E$_1$, ❞...❝, **obs_**E$_m$: E → E$_m$
proof obligation
[p] [Disjointness of endurant sorts]❞
end

Method 11: *describe_Cartesian_part_sorts* is a method **tool**.

Elaboration 1 Type, Values and Type Names: Note the use of quotes above. Please observe that when we write obs_E then obs_E is the name of a function. The E when juxtaposed to obs_ is now a name ▪

Example 36 A Road Transport System Domain: Cartesians. [67]

10 There is the *universe of discourse*, RTS.
It is composed from
11 a *road net*, RN, and
12 an *aggregate of automobiles*, AA.

type
10 RTS
11 RN
12 AA

value
11 obs_RN: RTS → RN
12 obs_AA: RTS → AA ▪

13 The road net consists of
a an aggregate, AH, of hubs and
b an aggregate, AL, of links.

[65] The use of the underscore, _ , shall inform the reader that there is no need, here, for naming a value.

type
13a AH
13b AL

value
13a obs_AH: RN → AH
13b obs_AL: RN → AL ▪

4.5.1.2.2 Part Sets

Definition 66 Part Sets. *Part sets* are those parts which, in a given context, are deemed to *meaningfully* consist of an indefinite number of of proper ["sibling"] *sub-parts* of the same sort ▪

Definition 67 Part Set Sorts. The *part set sort* is the common, i.e., the same, sort of its elements ▪

Analysis Predicate Prompt 10 is_part_set_sort: The domain analyser analyses a solid endurant, i.e., a part *p* into a set endurant:

- **is_*part_set_sort***: *p* is a composite endurant if **is_*part_set_sort(p)*** holds ▪

Method 12: is_*part_set_sort* is a method **tool**.

The **is_*part_set_sort*** predicate is informal. So are all the domain analysis predicates (and functions). That is, their values are "calculated" by a human, the domain analyser. That person observes fragments in the "real world". The determination of the predicate values, hence, are subjective.

4.5.1.2.2.1 Determine Part Set Sort

Mental Operation 5 The obs_Part_set_sub_sort Observer: This mental operator is invoked by the domain analyser cum describer when analysis has shown that a part is a set of parts. As an operation, "applied" by the domain analyser cum describer, to a part set compound, e:E, it yields a pair of [mnemotechnically] "pleasing" and "suggestive" names for the part set, e.g. PS, and the common sort, e.g. P, of the [perceived] part set elements ▪

[67] In example 36 on the preceding page' the **Narration** is not representative of what it should be. Here is a more reasonable narration:

- A road net is a set of hubs (road intersections) and links such that links are connected to adjacent hubs, and such that connected links and hubs form *roads* and where a road *is a thoroughfare, route, or way on land between two places that has been paved or otherwise improved to allow travel by foot or some form of conveyance, including a motor vehicle, cart, bicycle, or horse* [Wikipedia]

We bring this clarification here, once, and allow ourselves, with the reader's permission, to narrate only very stenographically.

Observer Function Prompt 2 determine_part_set_sort: The domain analyser observes parts into part set sorts. The method provides the **domain observer prompt**:

- determine_part_set_sort directs the domain analyser to determine the values and corresponding sorts of the part.

value
determine_part_set_sort: E → P-**set**×θP
determine_part_set_sort(e) **as** (ps,ηPn) ▪

Method 13: *determine_part_set_sort* is a method **tool**.

4.5.1.2.2.2 Describe Part Set Sort

Domain Description Prompt 3 describe_part_set_sort: If **is**_*part_set_sort*(*e*) holds, then the analyser "applies" the **domain description prompt**

- describe_part_set_sort(*e*)

resulting in the analyser writing down the *Part Set Sort and Sort Observers* domain description text according to the following schema:

2. describe_part_set_sort(e) **Describer**:

let (_,ηP) = determine___part_set_sort(e) **in**
❝**Narration**:
[s] ... narrative text on sort ...
[o] ... narrative text on sort observer ...
Formalisation:
type
[s] P
[s] Ps = P-**set**
value
[o] **obs**_Ps: E → Ps
end

Method 14: *describe_part_set_sort* is a method **tool**.

Elaboration 2 Type, Values and Type Names: Note the use of quotes above. Please observe that when we write obs_Ps then obs_Ps is the name of a function. The Ps, when juxtaposed to obs_ is now a name ▪

Example 37 Road Transport System: Sets of Hubs, Links and Automobiles. We refer to Example 36 on page 45.

14 The road net aggregate of road net hubs consists of a set of [atomic] hubs,
15 The road net aggregate of road net links consists of a set of [atomic] links,
16 The road net aggregate of automobiles consists of a set of [atomic] automobiles.

type
14. Hs = H-**set**, H
14. Ls − L-**set**, L
14. As = A-**set**, A

value
14. obs_Hs: AH → Hs
14. obs_Ls: AL → Ls
14. obs_As: AA → As ▪

Example 38 Alternative Rail Units.

17 The example is that of a railway system.
18 We focus on railway nets. They can be observed from the railway system.
19 The railway net embodies a set of [railway] net units.
20 A net unit is either a

a straight or curved **linear** unit, LU, or a
b switch, i.e., a **turnout** unit, TU, [68] or a
c cross-over unit, XU, or a
d single switched cross-over unit, SU, or a
e double switched cross-over slip unit, DU, or a
f **terminal** unit, LU.

We refer to Figure 4.2 on the facing page.

type
17. RS
18. RN

value
18. obs_RN: RS → RN

type
19. NUs = NU-**set**
20. NU = LU|PU|XU|SU|DU|TU

20a. LU :: LinU
20b. TU :: TurU
20c. XU :: COU
20d. SU :: SwiU
20e. DU :: DblU
20f. TU :: TerU

value
19. obs_NUs: RN → NUs ▪

The disjointness of the respective rail unit types is afforded by RSL's :: type definition construct.

• • •

Method Principle 6 ***Pedantic Steps of Development:*** This section, i.e., Sect. 4.5.1, has illustrated a principle of "small, pedantic" analysis & description steps. You could also call it a principle of separation of concerns ▪

4.5.1.3 Ontology and Endurant Taxonomy

We can speak of two kinds of ontologies[69]: the general ontologies of domain analysis & description, cf. Fig. 4.1 on page 38, and a specific domain's possible endurant ontologies. We shall here focus on a ["restricted"] concept of taxonomies.[70]

[68] https://en.wikipedia.org/wiki/Railroad_switch

[69] Ontology: a set of concepts and categories in a subject area or domain that shows their properties and the relations between them [Internet].

[70] Taxonomy: a scheme of classification, especially a hierarchical classification, in which things are organized into groups [Wikipedia].

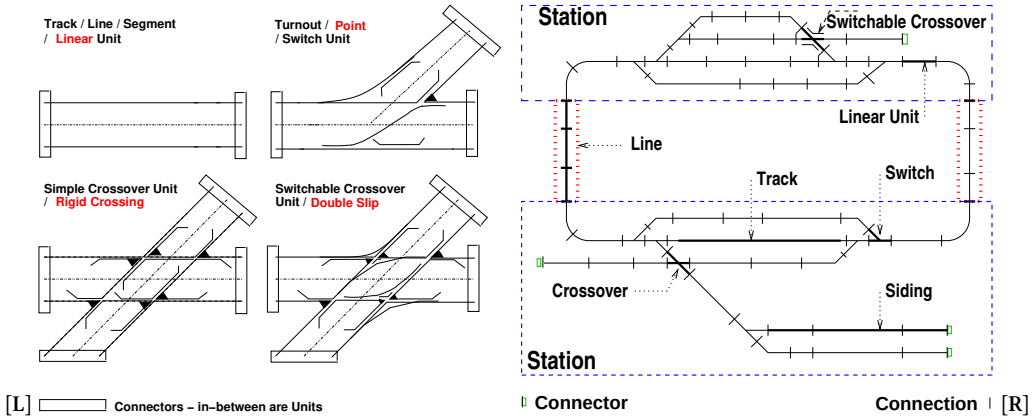

Fig. 4.2 **[L]**eft: Four of six net units (LU, TU, SU, DU); **[R]**ight: A railway net

Definition 68 **Endurant Taxonomy**. By an endurant taxonomy we shall understand a hierarchical structure, usually depicted as a(n "upside-down") tree, whose "root" designates a compound part and whose "siblings" (proper sub-trees) designate parts or fluids ▪

The 'restriction' amounts to considering only endurants. That is, not considering perdurants.

Method 15: *Endurant Taxonomy* is a method **technique**.

Example 39 **The Road Transport System Endurant Taxonomy**. Figure 4.3 shows a schematised taxonomy for the *Road Transport System* domain of Example 28 on page 36.

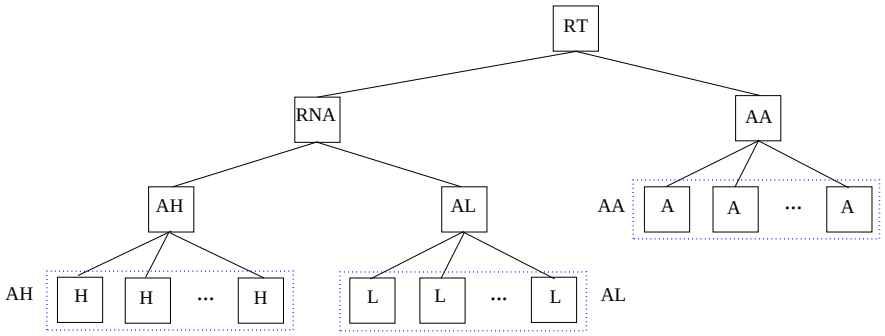

Fig. 4.3 A Road Transport System Taxonomy ▪

A visual domain model endurant taxonomy of the structure of parts do not add to the meaning of the domain model. It is merely of practical help in comprehending the possible complexity of the domain.

4.5.1.4 "Root" and "Sibling" Parts

For compound parts, cf. Sect. 4.5.1.2 on page 42, we introduce the specific domain taxonomy concepts of "root" and "sibling" parts. (We also refer to Fig. 4.3 on the previous page.)

When observing, as a human, a compound part, one may ask the question *"a tree — consisting of a specific domain taxonomy node labelled, e.g., X and the sub-trees labelled, e.g., $Y_1, Y_2, \ldots, Y_n$ — does that tree designate one "indivisible" part or does it designate $n+1$ parts?"* We shall, in general, consider the answer to be the latter: $n+1$!

We shall, in general, consider compound parts to consist of a "root" part and n "sibling parts and fluids". What the domain modeler observes appear as one part, "the whole", with n "embedded" sub-parts. What the domain modeler is asked to model is 1, the root part, and n, the sibling parts and fluids. The fact that the root part is separately modeled from the sibling parts, may seem to disappear in this separate modeling — but, as You shall see, in the next chapter, their relation: the siblings to "the whole", i.e., the root, will be modeled, specifically through their mereologies, as will be covered in Sect. 5.3, but also through their respective attributes, Sect. 5.4. We shall see this non-embbedness of root and sibling parts further accentuated in the modeling of their transcendentally deduced respective (perdurant) behaviours as distinct concurrent behaviours in Chapter 6.

4.5.2 Living Species

Living Species are either ***plants*** or ***animals***. Among animals we have the ***humans***.

Definition 69 **Living Species**. By a *living species* we shall understand a solid endurant, subject to laws of physics, and additionally subject to *causality of purpose*.

Living species must have some *form they can be developed to reach*; a form they must be *causally determined to maintain*. This *development and maintenance* must further engage in *exchanges of matter with an environment*. It must be possible that living species occur in two forms: **plants**, respectively **animals**, forms which are characterised by *development, form and exchange*, which, additionally, can be characterised by the *ability of purposeful movement* [161–165, Kai Sørlander] ▪

Analysis Predicate Prompt 11 **is**_living_species:

The domain analyser analyses "things" (ℓ) into living species. The method can thus be said to provide the **domain analysis prompt**:

- **is**_living_species – where **is**_living_species(ℓ) holds if ℓ is a living species ▪

Method 16: **is**_*living_species* is a method **tool**.

It is appropriate here to mention **Carl Linnaeus** (1707–1778). He was a Swedish botanist, zoologist, and physician who formalised, in the form of a binomial nomenclature, the modern system of naming organisms. He is known as the "father of modern taxonomy". We refer to his 'Species Plantarum' gutenberg.org/files/20771/20771-h/20771-h.htm.

4.5.2.1 Plants

Example 40 Plants. Although we have not yet come across domains for which the need to model the living species of plants were needed, we give some examples anyway: grass, tulip, rhododendron, oak tree.

Analysis Predicate Prompt 12 is_plant:

The domain analyser analyses "things" (ℓ) into a plant. The method can thus be said to provide the **domain analysis prompt**:

- is_plant – where is_plant(ℓ) holds if ℓ is a plant ▪

Method 17: **is_*plant*** is a method **tool**.

The predicate **is_*living_species*(ℓ)** is a prerequisite for **is_*plant*(ℓ)**.

4.5.2.2 Animals

Definition 70 Animal. We refer to the initial definition of *living species* above – while emphasizing the following traits: (i) a *form that animals can be developed to reach* and (ii) *causally determined to maintain* through (iii) *development and maintenance* in an *exchange of matter with an environment*, and (iv) *ability of purposeful movement* [161–165, Kai Sørlander] ▪

Analysis Predicate Prompt 13 is_animal:

The domain analyser analyses "things" (ℓ) into an animal. The method can thus be said to provide the **domain analysis prompt**:

- is_animal – where is_animal(ℓ) holds if ℓ is an animal ▪

Method 18: **is_*animal*** is a method **tool**.

The predicate **is_*living_species*(ℓ)** is a prerequisite for **is_*animal*(ℓ)**. We distinguish, motivated by [161–165, Kai Sørlander], between humans and other.

4.5.2.2.1 Humans

Definition 71 Human. A *human* (a *person*) is an *animal*, cf. Definition 70, with the additional properties of having *language*, being *conscious* of *having knowledge* (of its own situation), and *responsibility* [161–165, Kai Sørlander] ▪

Analysis Predicate Prompt 14 is_human:

The domain analyser analyses "things" (ℓ) into a human. The method can thus be said to provide the **domain analysis prompt**:

- **is**_human – where **is**_human(ℓ) holds if ℓ is a human ▪

Method 19: **is**_*human* is a method **tool**.

The predicate **is**_*animal*(ℓ) is a prerequisite for **is**_*human*(ℓ).

We have not, in our many experimental domain modeling efforts had occasion to model humans; or rather: we have modeled, for example, automobiles as possessing human qualities, i.e., "subsuming humans". We have found, in these experimental domain modeling efforts that we often confer anthropomorphic qualities on artefacts, that is, that these artefacts have human characteristics. You, the readers, are reminded that when some programmers try to explain their programs they do so using such phrases as *and here the program does ...* so-and-so !

4.5.2.2.2 Other

We shall skip any treatment of other than human animals !

•••

Method 20: *External Quality Analysis & Description* is a method **procedure**.

4.6 On Modeling "Soft" Parts, I

We introduce a notion of *soft parts*. This section, and its "companion" sections, Sects. 5.7 on page 96 and 6.10 on page 113, report on "work in progress". They are thus more speculative than definitive. Please bear that in mind.

Definition 72 Soft Parts. By a soft part we mean a part which is observed to consist of two endurants: an atomic part [considered to be a "soft" atomic part], i.e., a or the "container", and a fluid endurant, contained by the [soft] atomic part ▪

Example 41 Soft Parts. Examples of soft parts are:

- A bottle of milk: consists of the milk container and the contained milk. The milk container has its internal qualities [with, say, volume being of static attribute kind] and the contained milk has its internal qualities [with 'amount of milk' being of programmable attribute kind].
- A loaf of bread: consists of the bread (as a whole, and as a concept) and the bread "itself", for example in the form of its slices. The bread, as a whole, is of static attribute. The "itself" has a programmable attribute: number of slices, say 12 for a "full" bread, 0 for a bread for which there or no more slices.
- An automobile gas tank: consists of the [usually] metallic gas tank container and the contained gasoline or diesel oil. Et cetera ▪

In Example 29 on page 36 we – rather informally – used such terms as

- 'sugar plants',
- 'the burning of these',
- 'the harvest",
- 'sugar',
- 'molasses',
- 'wash',
- 'rum',
- 'barrels', '
- 'charcoal filtration of rum',
- 'blending of rum',
- 'bottling of rum',
- 'case of rum', and
- 'prepared cases'.

These all imply examples of soft parts.

Now: How are we to model "soft parts" ?

4.6.1 'Soft Part' Analysis Predicates

First we must be able to distinguish soft [atomic] parts from "other", i.e., "hard" [atomic] parts. To this end we introduce the part analysis predicates:

- **is_*soft***: The predicate **is**_*soft*a holds if a is atomic **and** is considered "soft".
- **is_*hard***: The predicate **is**_*hard*a holds if a is atomic **and** is **not** considered "soft", i.e., is hence considered "hard".

Then we must be able to "separately" observe the separate entities of an atomic soft part: the *container* and the *contained:*

- obs_container: Given a soft part, a, obs_container(a) yields the *hard* container entity of a.
- obs_contained: Given a soft part, a, obs_contained(a) yields the *soft* contained entity of a.

Figure 4.4 attempts to graphically illustrate some aspects of "softness".

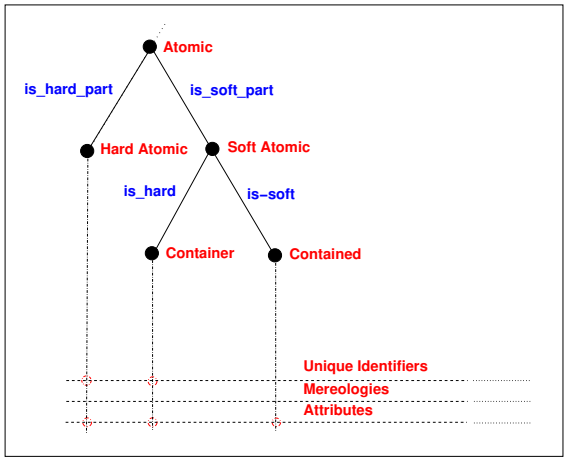

Fig. 4.4 Expanded segment of Fig. 4.1 on page 38

Of course, the the *hard* container entity and the *soft* contained entity of a soft part a are separate entities and could be considered "atomic" in-and-by-themselves – but there is no need to dwell on this !

Example 42 A Model of Soft Part "Atoms": A Grocery Store, I. We consider only a fragment of a grocery store.

21 A Grocery store is abstracted to consist just of a set, *Ss*, of Shelves.
22 A Shelf of a grocery store contains zero, one or more Wares of the same kind.
23 A Ware is a soft atom and is either a ware of kind *A*, or of kind *B*, ..., or of kind *C*.

24 Each *W*are is a soft atom: has a *k*ontainer part and a *c*ontained endurant which is a fluid.
25 *k*ontainer parts are hard and *c*ontained parts are soft.

Informally, in items 23-24:

type
21. G, Ss, S, Ws, W
21. Ss = S-**set**
21. Ws = W-**set**

value
22. obs_Ss: W → Ss
22. obs_Ws: S → Ws

type
23. W = W1 | W2 | ... | Wn
24. kW1, cW1, kW2, cW2, ..., kWn, cWn

value
24. obs_container: Wi → kWi
24. obs_contained: Wi → cWi

axiom
23. ∀ wi:Wi • **is**_soft_part(wi)
25. ∀ kwi:kWi • **is**_hard(kwi)
25. ∀ cwi:cWi • **is**_soft(cwi) ▪

This is all we need to say about the modeling of soft parts in this chapter. In Sect. 5.7 on page 96 we shall take up the topic of *modeling soft parts*.

4.7 Some Observations

Two observations must be made.

(i) The domain modeling procedures illustrated by the analysis functions determine_Cartesian_parts, determine_single_sort_part_set and determine_alternative_sorts_part_set yield names of endurant sorts. Some of these names may have already been encountered, i.e., discovered. That is, the domain modeler must carefully consider such possibilities.

(ii) Endurants are **not recursively definable**! This appears to come as a surprise to many computer scientists. Immediately many suggest that "tree-like" endurants like a river, or, indeed, a tree, should be defined recursively. But we posit that that is not the case. A river, for example, has a delta, its "root" so-to-speak, but the sub-trees of a recursively defined river endurant have no such "deltas"! Instead we define such "tree-like" endurants as graphs with appropriate mereologies – as introduced in the next chapter.

4.8 States

In our continued modeling we shall make good use of a concept of states.

Definition 73 State, II. By a *state* we shall understand any collection of one or more parts ▪

We omit consideration of "soft parts".

In Chapter 5 Sect. 5.4 we introduce the notion of *attributes*. Among attributes there are the *dynamic attributes*. They model that internal quality values may change dynamically. So we may wish, on occasion, to 'refine' our notion of state to be just those parts which have dynamic attributes.

4.8.1 State Calculation

Given any universe of discourse, uod:UoD, we can recursively calculate its "full" state, calc_parts({uod}).

26 Let e be any endurant. Let arg_parts be the parts to be calculated. Let res_parts be the parts calculated. Initialise the calculator with arg_parts={uod} and res_parts={}. Calculation stops with arg_parts empty and res_parts the result.
27 If **is_***Cartesian*(e)
28 then we obtain its immediate parts, determine_composite_part(e)
29 add them, as a set, to arg_parts, e removed from arg_parts and added to res_parts calculating the parts from that.
30 If **is_***single_sort_part_set*(e)
31 then the parts, ps, of the single sort set are determined,
32 added to arg_parts and e removed from arg_parts and added to res_parts calculating the parts from that.
33 If **is_***alternative_sorts_part_set*(e) then the parts, ((p1,_),(p2,_),...,(pn,_)), of the alternative sorts set are determined, added to arg_parts and e removed from arg_parts and added to res_parts calculating the parts from that.

```
value
26.  calc_parts: E-set → E-set → E-set
26.  calc_parts(arg_parts)(res_parts) ≡
26.    if arg_parts = {} then res_parts else
26.    let e • e ∈ arg_parts in
27.    is_Cartesian(e) →
28.      let ((e1,e2,...,en),_) = observe_Cartesian_part(e) in
29.      calc_parts(arg_parts\{e} ∪ {e1,e2,...,en})(res_parts ∪ {e}) end
30.    is_single_sort_part_set(e) →
31.      let ps = observe_single_sort_part_set(e) in
32.      calc_parts(arg_parts\{e}∪ ps)(res_parts ∪ {e}) end
33.    is_alternative_sort_part_set(e) →
33.      let ((p1,_),(p2,_),...,(pn,_)) = observe_alternative_sorts_part_set(e) in
33.      calc_parts(arg_parts\{e}∪{p1,p2,...,pn})(res_parts ∪ {e}) end
26.    end end
```

Method 21: *calc_parts* is a method **tool**.

Method Principle 7 ***Domain State:*** We have found, once all the state components, i.e., the endurant parts, have had their external qualities analysed, that it is then expedient to define the domain state. The domain state can then be the basis for several concepts of internal qualities.

Example 43 Constants and States.

34 Let there be given a universe of discourse, *rts* (Road Transport System). The set {*rts*} is an example of a state.

From that state we can calculate other states.

35 The set of all hubs, *hs*.
36 The set of all links, *ls*.
37 The set of all hubs and links, *hls*.
38 The set of all automobiles, *as*.

39 The set of all parts, *ps*.

```
value
34  rts:UoD
35  hs:H-set  ≡ obs_sH(obs_SH(obs_RN(rts)))
36  ls:L-set ≡ obs_sL(obs_SL(obs_RN(rts)))
37  hls:(H|L)-set ≡ hs ∪ ls
38  as:A-set ≡ obs_As(obs_AA(obs_RN(rts)))
39  ps:(UoD|H|L|A)-set ≡ rts ∪ hls ∪ as .
```

4.8.2 Updateable States

We shall, in Sect. 5.4, introduce the notion of parts, having dynamic attributes, that is, having internal qualities that may change. To cope with the modeling, in particular of so-called *monitor-able* attributes, we present the *state* as a global variable:

```
variable σ := calc_parts({uod})
```

4.9 An External Analysis and Description Procedure

We have covered the individual analysis and description steps of our approach to the external qualities modeling of domain endurants. We now suggest a 'formal' description of the process of linking all these analysis and description steps.

4.9.1 An Analysis & Description State

Common to all the discovery processes is an idea of a *notice board*. A notice board, at any time in the development of a domain description, is a repository of the analysis and description process. We suggest to model the notice board in terms of four global variables. The new variable holds the **parts** yet to be described; the ans variable holds the **sort names of parts** that have so far been described; the gen variable holds the **parts** that have so far been described; and the txt variable holds the RSL-Text so far generated. We model the txt variable as a map from endurant identifier names to RSL-Text.

Discovery Schema 0: The Notice Board

```
variable
    new := {uod} ,
    ans := { "UoD" } ,
    gen := {} ,
    txt:RSL-Text  := [ uid_UoD(uod) ↦ ⟨"type UoD"⟩ ]
```

4.9.2 A Domain Discovery Procedure, I

The discover_sorts pseudo program suggests a systematic way of proceeding through analysis, manifested by the **is_**⋯ predicates, to (→) description.

Some comments are in order. The e-set$_a$⊎e-set$_b$ expression yields a set of endurants that are either in e-set$_a$, or in e-set$_a$, or in both, but such that two endurants, e$_x$ and e$_y$ which are of the same endurant type, say E, and are in respective sets is only represented once in the result.

As this is the first time RSL-Text is put on the notice board we express this as:

- txt := txt ∪ [type_name(v) ↦ ⟨RSL-Text⟩]

Subsequent insertion of RSL-Text for internal quality descriptions and perdurants is then concatenated to the end of previously uploaded RSL-Text.

Discovery Schema 1: An External Qualities Domain Modeling

Process

```
value
discover_sorts: Unit → Unit
discover_sorts() ≡ while new ≠ {} do
    let v • v ∈ new in (new := new \ {v} ‖ gen := gen ∪ {v} ‖ ans := ans \ {type_of(v)}) ;
    is_atomic(v) → skip ,
    is_compound(v) →
      is_Cartesian(v) →
       let ((e1,...,en),(ηE1,...,ηEn))=determine_Cartesian_part_sorts(v) in
       (ans := ans ∪ {ηE1,...,ηEn} ‖ new := new ⊎ {e1,...,en}
          ‖ txt := txt ∪ [type_name(v) ↦ ⟨describe_Cartesian_part_sorts(v)⟩]) end,
      is_part_set(v) →
       let ({p1,...,pn},ηP)=determine_part_set_sort(v) in
       (ans := ans ∪ {ηP} ‖ new := new ⊎ {p1,...,pn} ‖
          txt := txt ∪ [type_name(v) ↦ describe_part_set_sort(v)]) end,
    end end
```

Method 22: *discover_sorts* is a method **procedure**.

4.10 Summary

We briefly summarise the main findings of this chapter. These are the main analysis predicates and functions, and the main description functions. These, to remind the reader, are (i) the *analysis*, the **is_**⋯,

predicates, (ii) the *analysis*, the determine_⋯, *functions*, (iii) the *state calculation* function, (iv) the *description* functions, and (v) the *domain discovery* procedure. They are summarised in this table:

External Qualities Predicates and Functions: Method Tools

- **Analysis Predicates:** These are the is_⋯ functions. The domain scientist cum engineer, i.e., the domain analyser cum describer, applies this to entities being observed in the domain. The answer is a truth value. Dependent on the truth value that person then goes on to apply, again informally, either a subsequent predicate, or some function.
- **Analysis Functions:** These are the determine_⋯ functions. They apply, respectively, to parts satisfying respective predicates.
- **State Calculation:** The state calculation function is given generally. The domain analyser cum describer must define this function for each domain studied.
- **Description Functions:** These calculation functions, in a sense, are the main "results" of this chapter.
- **Domain Discovery:** The procedure here being described, informally, guides the domain analyser cum describer to do the job!

#	Name	pg.
	Analysis Predicates	
1	is_entity	**37**
2	is_endurant	**39**
3	is_perdurant	**39**
4	is_solid	**40**
5	is_fluid	**40**
6	is_part	**41**
7	is_atomic	**42**
8	is_compound	**42**
9	is_Cartesian	**43**
10	is_part_set_sort	**46**
11	is_living_species	**50**
12	is_plant	**51**
13	is_animal	**51**
14	is_human	**52**
	Analysis Functions	
1	determ._Cart,_part_sorts	**44**
2	determ._part_set_sort	**47**
	State Calculation	
	calc_parts	**55**
	Description Functions	
1	descr._Universe_of_Discourse	**36**
2	descr._Cartesian_part_sorts	**45**
3	descr._part_set_sort	**47**
	Domain Discovery	
	discover_sorts	**57**

• • •

Please consider Fig. 4.1 [Page 38]. This chapter has covered the tree-like structure to the left in Fig. 4.1. The next chapter covers the items mentioned in the horisontal and vertical lines, also to the left in Fig. 4.1.

Chapter 5
Endurants: Internal and Universal Domain Qualities

Contents

Please consider Fig. 4.1 on page 38. The previous chapter covered the tree-like structure to the left in Fig. 4.1. This chapter covers the vertical and horizontal lines, also to the left, in Fig. 4.1.

• • •

In this chapter we introduce the concepts of internal qualities of endurants and some universal qualities of domains, and cover, first, the analysis and description of internal qualities: **unique identifiers** (Sect. 5.2 on page 62), **mereologies** (Sect. 5.3 on page 67) and **attributes** (Sect. 5.4 on page 71). There is, additionally, three **universal qualities**: **space**, **time** (Sect. 5.5 on page 87) and **intentionality** (Sect. 5.6 on page 90), where *intentionality* is "something" that expresses intention, design idea, purpose of artefacts – well, some would say, also of natural endurants.

As it turns out[71], to analyse and describe mereology we need to first analyse and describe unique identifiers; and to analyse and describe attributes we need to first analyse and describe mereologies. Hence:

[71] You, the first time reader cannot know this, i.e., the "turns out". Once we have developed and presented the material of this chapter, then you can see it; clearly !

Method Procedure 1 ***Sequential Analysis & Description of Internal Qualities:*** We advise that the domain modeler:

- **first** analyse & describe **unique identification** of all endurant sorts;
- **then** analyse & describe **mereologies** of all endurant sorts;
- **then** analyse & describe **attributes** of all endurant sorts; and,
- **finally**, to analyse & describe **intentionality**.

Definition 74 **Description, II.** By a **description** of the internal qualities of a domain we mean pairs of informal, narrative, and formal text which characterises the unique identifiers, mereologies, attributes, and possible intentional pulls of manifest parts (fluids and living species) ▪

This chapter explains what is meant by *unique identifier*, *mereology*, *attribute* and *intentional pull*.

5.1 Internal Qualities

We shall investigate the, as we shall call them, internal qualities of domains. That is, the properties of the entities to which we ascribe internal qualities. The outcome of this chapter is that the reader will be able to model the internal qualities of domains. Not just for a particular domain instance, but a possibly indefinite set of domain instances[72].

5.1.1 General Characterisation

External qualities of endurants of a manifest domain are, in a simplifying sense, those we can see and touch. They, so to speak, take form.

Internal qualities of endurants of a manifest domain are, in a less simplifying sense, those which we may not be able to see or "feel" when touching an endurant, but they can, as we now 'mandate' them, be reasoned about, as for **unique identifiers** and **mereologies**, or be measured by some **physical/chemical** means, or be "spoken of" by **intentional deduction**, and be reasoned about, as we do when we **attribute** properties to endurants.

5.1.2 Manifest Parts versus Structures

In [45] we covered a notion of *'structures'*. In this primer we shall treat the concept of 'structures' differently. We do so by distinguishing between manifest parts and structures.

5.1.2.1 Definitions

Definition 75 **Manifest Part.** By a manifest part we shall understand a part which 'manifests' itself either in a physical, visible manner, "occupying" an AREA or a VOLUME and a POSITION in SPACE, or in a conceptual manner forms an organisation in Your mind ! ▪

As we have already revealed, endurant parts can be transcendentally deduced into perdurant behaviours – with manifest parts indeed being so.

[72] By this we mean: You are not just analysing a specific domain, say the one manifested around the corner from where you are, but any instance, anywhere in the world, which satisfies what you have described.

Definition 76 Structure. By a structure we shall understand an endurant concept that allows the domain modeler to rationally decompose a domain analysis and/or its description into manageable, logically relevant sections, but where these abstract endurants are not further reflected upon in the domain analysis and description. Structures are therefore not transcendentally deduced into perdurant behaviourss ▪

We shall treat the notion of manifest further in Sect. 5.4.5.1.1 on page 83.

5.1.2.2 Analysis Predicates

Analysis Predicate Prompt 15 is_manifest: The method provides the **domain analysis prompt**:

- is_manifest – where is_manifest(p) holds if p is to be considered manifest ▪

Analysis Predicate Prompt 16 is_structure: The method provides the **domain analysis prompt**:

- is_structure – where is_structure(p) holds if p is to be considered a structure ▪

The obvious holds: is_manifest(p) ≡ ~ is_structure(p).

5.1.2.3 Examples

Example 44 Manifest Parts and Structures. We refer to Example 36 on page 45: the Road Transport System. We shall consider all atomic parts: hubs, links and automobiles as being manifest. (They are physical, visible and in 𝕊ℙ𝔸ℂ𝔼.) We shall consider road nets and aggregates of automobiles as being manifest. Road nets are physical, visible and in 𝕊ℙ𝔸ℂ𝔼. Aggregates of automobiles are here considered conceptual. The road net manifest part, apart from its aggregates of hubs and links, can be thought of as "representing" a *Department of Roads*[73]. The automobile aggregate apart from its automobiles, can be thought of as "representing" a *Department of Vehicles*[74]. We shall, at present, consider hub and link aggregates and hub and link sets as structures ▪

5.1.2.4 Modeling Consequence

If a part is considered manifest then we shall endow that part with all three kinds of internal qualities. If a part is considered a structure then we shall **not** endow that part with any of the three kinds of internal qualities.

5.2 Unique Identification

The concept of parts having unique identifiability, that is, that two parts, if they are the same, have the same unique identifier, and if they are not the same, then they have distinct identifiers, that concept is fundamental to our being able to analyse and describe internal qualities of endurants. So we are left with the issue of 'identity' ! (We refer to Sect. 2.4.5.1 on page 15.)

[73] – of some country, state, province, city or other.

[74] See above footnote.

Definition 77 **Uniqueness**. By uniqueness of parts we shall mean that any two spatially distinct parts are two unique parts – cannot be confused ▪

Definition 78 **Unique Identifier**. By a unique identifier we shall mean anything that can be used to distinguish any one part from any other spatially distinct parts ▪

5.2.1 On Uniqueness of Endurants

We therefore introduce the notion of unique identification of part endurants. We assume (i) that all part endurants, e, of any domain E, have *unique identifier*s, (ii) that *unique identifier*s (of part endurants e:E) are *abstract value*s (of the *unique identifier* sort UI of part endurants e:E), (iii) that distinct part endurant sorts, E_i and E_j, have distinctly named *unique identifier* sorts, say UI_i and UI_j[75], and (iv) that all ui_i:UI_i and ui_j:UI_j are distinct.

The names of unique identifier sorts, say UI, is entirely at the discretion of the *domain modeler*. If, for example, the sort name of a part is P, then it might be expedient to name the sort of the unique identifiers of its parts PI.

Representation of Unique Identifiers: Unique identifiers are abstractions. When we endow two endurants (say of the same sort) distinct unique identifiers then we are simply saying that these two endurants are distinct. We are not assuming anything about how these identifiers otherwise come about.
Identifiability of Endurants: From a philosophical point of view, and with basis in Kai Sørlander's Philosophy, cf. Paragraph **Identity, Difference and Relations** (Page 15), one can rationally argue that there are many endurants, and that they are unique, and hence uniquely identifiable. From an empirical point of view, and since one may eventually have a software development in mind, we may wonder how unique identifiablity can be accommodated.

Unique identifiability for solid endurants, even though they may be mobile, is straightforward: one can think of many ways of ascribing a unique identifier to any part. Hence one can think of many such unique identification schemas.

Unique identifiability for fluids may seem a bit more tricky. For this primer we shall not suggest to endow fluids with unique identification. We have simply not experimented with such part-fluids and fluid-parts domains – not enough – to suggest so.

5.2.2 Uniqueness Modeling Tools

The analysis method offers an observer function uid_E which when applied to a part endurant, e of sort E, yields the unique identifier, *ui*:EI, of e.

Domain Description Prompt 4 describe_unique_identifier: We can therefore apply the **domain description prompt**:

- describe_unique_identifier

to endurants e:E resulting in the analyser writing down the *Unique Identifier Type and Observer* domain description text according to the following schema:
3. describe_unique_identifier(e) **Observer**
❝ **Narration:**
[s] ... narrative text on unique identifier sort EI ...[76]
[u] ... narrative text on unique identifier observer uid_E ...

[75] This restriction is not necessary, but, for the time being, we can assume that it is.

[a] ... axiom on uniqueness of unique identifiers ...
Formalisation:
type
[s] UI
value
[u] uid_E: E → EI❞

is_part(e) is a prerequisite for describe_unique_identifier(e).

The unique identifier type name, EI above, chosen, of course, by the *domain modeler*, usually properly embodies the type name, E, of the endurant being analysed and mereology-described. Thus a part of type-name E might be given the mereology type name EI.

Generally we shall refer to these names by UI.

Observer Function Prompt 3 type_name, type_of, is_: Given *description schema* from Domain Description Prompt 4 we have, so-to-speak "in-reverse", that

∀ e:E • uid_E(e)=ui ⇒ type_of(ui)=ηUI ∧ type_name(ui)=UI ∧ is_UI(ui)

ηUI is a variable of type $\eta\mathbb{T}$. $\eta\mathbb{T}$ is the type of all domain endurant, unique identifier, mereology and attribute type names. By the subsequent UI we refer to the unique identifier type name value of ηUI.

Example 45 Unique Identifiers.

40 We assign unique identifiers to all parts.
41 By a road identifier we shall mean a link or a hub identifier.
42 Unique identifiers uniquely identify all parts.

a All hubs have distinct [unique] identifiers.
b All links have distinct identifiers.
c All automobiles have distinct identifiers.

type
40 H_UI, L_UI, A_UI
41 R_UI = H_UI | L_UI
value
42a uid_H: H → H_UI
42b uid_L: H → L_UI
42c uid_A: H → A_UI ▪

5.2.3 The Unique Identifier State

Given a universe of discourse we can calculate the set of the unique identifiers of all its parts.

```
value
    calculate_all_unique_identifiers: UoD → UI-set
    calculate_all_unique_identifiers(uod) ≡
        let parts = calc_parts({uod})({}) in { uid_E(e) | e:E • e ∈ parts } end
```

Example 46 Road Transport: Unique Identifier Auxiliary Functions.
Extract Parts from Their Unique Identifiers:

43 From the unique identifier of a part we can retrieve, $\wp$, the part having that identifier.

[76] The name, EI, of the unique identifier sort is determined, *"pulled out of a hat"*, by the domain modeler(s), i.e., the person(s) who "apply" the describe_unique_identifier(e) prompt.

```
type
43 P = H | L | A
value
43 ℘: H_UI→H | L_UI→L | A_UI→A
43 ℘(ui) ≡ let p:(H|L|A)•p∈ps∧uid_P(p)=ui in p end
```

We can speak of a unique identifier state:

```
variable
    uid_σ := discover_uids(uod)
value
    discover_uids: UoD → UI-set
    discover_uids(uod) ≡ calculate_all_unique_identifiers(uod) ▪
```

Example 47 Unique Road Transport System Identifiers. We can calculate:

44 the set, $h_{ui}s$, of unique hub identifiers;
45 the set, $l_{ui}s$, of unique link identifiers;
46 the set, $r_{ui}s$, of all unique hub and link, i.e., road identifiers;
47 the map, $hl_{ui}m$, from unique hub identifiers to the set of unique link identifiers of the links connected to the zero, one or more identified hubs;
48 the map, $lh_{ui}m$, from unique link identifiersto the set of unique hub iidentifiers of the two hubs connected to the identified link; and
49 the set, $a_{ui}s$, of unique automobile identifiers.

```
value
44  h_uis:H_UI-set ≡ {uid_H(h)|h:H•h ∈ hs}
45  l_uis:L_UI-set ≡ {uid_L(l)|l:L•l ∈ ls}
46  r_uis:R_UI-set ≡ h_uis∪l_uis
47  hl_uim:(H_UI →m L_UI-set) ≡
47      [h_ui↦luis|h_ui:H_UI,luis:L_UI-set•h_ui∈h_uis∧(_,luis,_)=mereo_H(η(h_ui))]
48  lh_uim:(L+UI →m H_UI-set) ≡
48      [l_ui↦huis | h_ui:L_UI,huis:H_UI-set • l_ui∈l_uis ∧ (_,huis,_)=mereo_L(η(l_ui))]
49  a_uis:A_UI-set ≡ {uid_A(a)|a:A•a ∈ as} ▪
```

5.2.4 A Domain Law: Uniqueness of Endurant Identifiers

We postulate that the unique identifier observer functions are about the uniqueness of the postulated endurant identifiers. But how is that guaranteed? We know, as *"an indisputable law of domains"*, that they are distinct, but our formulas do not guarantee that! So we must formalise their uniqueness.

Part Unique Identifiers never change

A Domain Law 1 Part Unique Identifiers never change:

50 The unique identifier of any part never change,

50. **axiom** uid_P(p)=uid ⇒ □ uid_P(p)=uid ▪

Domain Parts have Unique Identifiers

A Domain Law 2 Domain Parts have Unique Identifiers:

51 All parts of a described domain, uod, have unique identifiers.

axiom
51 **card** calc_parts({uod}) = **card** all_unique_identifiers(uod)

Example 48 Uniqueness of Road Net Identifiers. We must express the following axioms:

52 All hub identifiers are distinct.
53 All link identifiers are distinct.
54 All automobile identifiers are distinct.
55 All part identifiers are distinct.

axiom
52 **card** hs = **card** $h_{ui}s$
53 **card** ls = **card** $l_{ui}s$
54 **card** as = **card** $a_{ui}s$
55 **card** $\{h_{ui}s \cup l_{ui}s \cup a_{ui}s\}$ = **card** $h_{ui}s$+**card** $l_{ui}s$+**card** $a_{ui}s$ ▪

We ascribe, in principle, unique identifiers to all endurants whether natural or artefactual. We find, from our many experiments, cf. the *Universes of Discourse* example, Page 35, that we really focus on those domain entities which are artefactual endurants and their behavioural "counterparts".

Example 49 Rail Net Unique Identifiers.

56 With every rail net unit we associate a unique identifier.
57 That is, no two rail net units have the same identifier.
58 No two distinct trains have the same identifier.
59 Train identifiers are distinct from rail net unit identifiers.

type
56. NUI
value
56. uid_NU: NU → NUI
axiom
57. ∀ ui_i,ui_j:NUI • ui_i = ui_j ≡ uid_NU(ui_i)=uid_NU(ui_j)

5.2.4.1 Part Retrieval

Given the unique identifier, pi, of a part p, but not the part itself, and given the universe-of-discourse (uod) state σ, we can *retrieve* part, p, as follows:

```
value
    pi:PI, uod:UoD, σ
    retr_part: PI → P
    retr_part(pi) ≡ let p:P • p ∈ σ ∧ uid_P(p)=pi in p end
        pre: ∃ p:P • p ∈ σ ∧ uid_P(p)=pi
```

5.2.4.2 Unique Identification of Compounds

For structures we do not model their unique identification. But their components, whether the structures are *"Cartesian"* or *"sets"*, may very well be non-structures, hence be uniquely identifiable.

5.3 Mereology

Definition 79 **Mereology, II.** Mereology is the study and knowledge of parts and part relations.

Mereology, as a logical/philosophical discipline, can perhaps best be attributed to the Polish mathematician/logician Stanisław Leśniewski (1886–1939) [35, 77].

5.3.1 Endurant Relations

Which are the relations that can be relevant for "endurant-hood"? There are basically two relations: (i) spatial ones, and (ii) conceptual ones.

(i) Spatially two or more endurants may be *topologically* either adjacent to one another, like rails of a line, or within an endurant, like links and hubs of a road net, or an atomic part is conjoined to one or more fluids, or a fluid is conjoined to one or more parts. The latter two could also be considered conceptual "adjacencies".

(ii) Conceptually some parts, like automobiles, "belong" to an embedding endurant, like to an automobile club, or are registered in the local department of vehicles, or are 'intended' to drive on roads.

5.3.2 Mereology Modeling Tools

When the domain analyser decides that some endurants are related in a specifically enunciated mereology, the analyser has to decide on suitable *mereology type*s and *mereology observer*s (i.e., endurant relations). In general we express the mereology of an endurant, p:P, as a type expression[77] over unique identifiers of the spatially and/or conceptually related endurants:

type

$$\mathrm{MT} = \mathcal{M}(\mathrm{UI}_i,\mathrm{UI}_j,...,\mathrm{UI}_k)$$

Domain Description Prompt 5 describe_mereology:If has_mereology(p) holds for parts p of type P, then the analyser can apply the *domain description prompt*:

- describe_mereology

to parts of that type and write down the *Mereology Types and Observer* domain description text according to the following schema:

4. describe_mereology(e) **Describer**:
❝ **Narration**:
   [t] ... narrative text on mereology type ...
   [m] ... narrative text on mereology observer ...
   [a] ... narrative text on mereology type constraints ...

[77] We refer to Appendix Sect. C.2.1 on page 200 for more on RSL types.

Formalisation:
type
[t] MT = $\mathcal{M}$(UI$_i$,UI$_j$,...,UI$_k$)
value
[m] mereo_P: P → MT
axiom [Well−formedness of Domain Mereologies]
[a] $\mathcal{A}$: $\mathcal{A}$(MT) ❞.

$\mathcal{A}$(MT) is a predicate over possibly all unique identifier types of the domain description. To write down the concrete type definition for MT requires a bit of analysis and thinking,

Example 50 Mereology of a Road Net.

60 The mereology of hubs is a pair: (i) a set of automobile identifiers[78], and (ii) the set of unique identifiers of the links that it is connected to.[79]
61 The mereology of links is a pair: (i) a set of automobile identifiers, and (ii) the set of exactly the two distinct hubs they are connected to.
62 The mereology of an automobile is the set of the unique identifiers of all links and hubs along which it might travel[80].

We presently omit treatment of road net and automobile aggregate mereologies.

type
60 H_Mer = V_UI-**set** × L_UI-**set**
61 L_Mer = V_UI-**set** × H_UI-**set**
62 A_Mer = R_UI-**set**
value
60 mereo_H: H → H_Mer
61 mereo_L: L → L_Mer
62 mereo_A: A → A_Mer ▪

5.3.2.1 Invariance of Mereologies

For mereologies one can usually express some invariants. Such invariants express *"law-like properties"*, facts which are indisputable. We refer to Sect. 5.3.4 on the next page.

Example 51 Invariance of Road Nets. The observed mereologies must express identifiers of the state of road nets:

axiom
60 ∀ (auis,luis):H_Mer • luis ⊆ $l_{ui}s$ ∧ auis ⊆ $a_{ui}s$
61 ∀ (auis,huis):L_Mer • auis ⊆ $a_{ui}s$ ∧ huis ⊆ $h_{ui}s$ ∧ **card** huis = 2
62 ∀ ruis:A_Mer • ruis ⊆ $r_{ui}s$

63 For all hubs, h, and links, l, in the same road net,
64 if the hub h connects to link l then link l connects to hub h.

[78] This is just another way of saying that the meaning of hub mereologies involves the unique identifiers of those vehicles that might pass through the hub.
[79] The link identifiers designate the links, zero, one or more, that a hub is connected to.
[80] — that the automobile might pass through

axiom
63 ∀ h:H,l:L • h ∈ *hs* ∧ l ∈ *ls* ⇒
63 **let** (_,luis)=mereo_H(h), (_,huis)=mereo_L(l)
64 **in** uid_L(l)∈luis ≡ uid_H(h)∈huis **end**

65 For all links, l, and hubs, h_a,h_b, in the same road net,
66 if the l connects to hubs h_a and h_b, then h_a and h_b both connects to link l.

axiom
65 ∀ h_a,h_b:H,l:L • {h_a,h_b} ⊆ *hs* ∧ l ∈ *ls* ⇒
65 **let** (_,luis)=mereo_H(h), (_,huis)=mereo_L(l)
66 **in** uid_L(l)∈luis ≡ uid_H(h)∈huis **end** ▪

5.3.2.2 Deductions made from Mereologies

Once we have settled basic properties of the mereologies of a domain we can, like for unique identifiers, cf. Example 45 on page 64, *"play around"* with that concept: 'the mereology of a domain'.

Example 52 Consequences of a Road Net Mereology.

67 are there [isolated] units from which one can not "reach" other units ?
68 does the net consist of two or more "disjoint" nets ?
69 et cetera ▪

We leave it to the reader to narrate and formalise the above properly. (We refer to Appendix B.3.3.1 on page 180 which exemplifies to modeling of routes in networks.)

5.3.3 Formulation of Mereologies

The observe_mereology domain descriptor, Page 68, may give the impression that the mereo type MT can be described "at the point of issue" of the observe_mereology prompt. Since the MT type expression may, in general, depend on any part sort the mereo type MT can, for some domains, "first" be described when all part sorts have had their unique identifiers defined.

5.3.4 Fixed and Varying Mereologies

The mereology of parts is not necessarily fixed.

Definition 80 Fixed Mereology. By a **fixed mereology** we shall understand a mereology of a part which remains fixed over time.

Definition 81 Varying Mereology. By a **varying mereology** we shall understand a mereology of a part which may vary over time.

Example 53 Fixed and Varying Mereology. Let us consider a road net[81]. If hubs and links never change "affiliation", that is: hubs are in fixed relation to zero, one or more links, and links are in a fixed relation

[81] cf. Examples 28 on page 36, 36 on page 45, 37 on page 48, ?? on page ??, 44 on page 62, 45 on page 64, 48 on page 66, 49 on page 66, 50 on the facing page and 51.

to exactly two hubs then the mereology of Example 50 on page 68 is a *fixed mereology*. If, on the other hand hubs may be inserted into or removed from the net, and/or links may be removed from or inserted between any two existing hubs, then the mereology of Example 50 on page 68 is a *varying mereology* ▪

5.3.5 No Fluids Mereology

We comment on our decision, for this primer, to not endow fluids with mereologies. A first reason is that we "restrict" the concept of mereology to part endurants, that is, to solid endurants – those with "more-or-less" *fixed extents*. Fluids can be said to normally not have fixed extents, that is, they can "morph" from small into spatially extended forms. For domains of part-fluid conjoins this is particularly true. The fluids in such domains flow through and between parts. Some parts, at some times, embodying large, at other times small amounts of fluid. Some proper, but partial amount of fluid flowing from one part to a next. Et cetera. It is for the same reason that we do not endow fluids with identity. So, for this primer we decide to not suggest the modeling of fluid mereologies.

5.3.6 Some Modeling Observations

It is, in principle, possible to find examples of mereologies of natural parts: rivers: their confluence, lakes, oceans, mountain ranges, flat lands, etc. But in our experimental case studies, cf. Example on Page 35, we have found no really interesting such cases. All our experimental case studies appear to focus on the mereology of artefacts. And, finally, in modeling humans, we find that their mereology encompasses all other humans and all artefacts ! Humans cannot be tamed to refrain from interacting with everyone and everything.

Some domain models may emphasize *physical mereologies* based on spatial relations, others may emphasize *conceptual mereologies* based on logical "connections".

Example 54 Rail Net Mereology. We refer to Example 38 on page 48.

70 A linear rail unit is connected to exactly two distinct other units of any given rail net.
71 A point unit is connected to exactly three distinct other rail net units of any given rail net.
72 A rigid crossing unit is connected to exactly four distinct other rail net units of any given rail net.
73 A single slip unit, as well as a double slip unit is connected to exactly four distinct other rail net units of any given rail net.
74 A terminal unit is connected to exactly one distinct other rail net unit of any given rail net.
75 So we model the mereology of a railway net unit as a pair of sets of rail net unit unique identifiers distinct from that of the rail net unit.

```
value
75.  mereo_NU: NU → (UI-set×UI-set)
axiom
75.  ∀ nu:NU •
75.    let (uis_i,uis_o)=mereo_NU(nu) in
75.    case (card uis_i,card usi_o) =
70.      (is_LU(nu) → (1,1),
71.       is_PU(nu) → (1,2) ∨ (2,1),
72.       is_RU(nu) → (2,2),
73.       is_SU(nu) → (2,2), is_DU(nu) → (2,2),
74.       is_TU(nu) → (1,0) ∨ (0,1),
75.       _ → chaos) end
75.   ∧ uis_i∩uis_o={}
```

75. ∧ uid_NU(nu) ∉ (uis_i ∪ uis_o)
75. **end**

Figure 5.1 illustrates the mereology of four rail units.

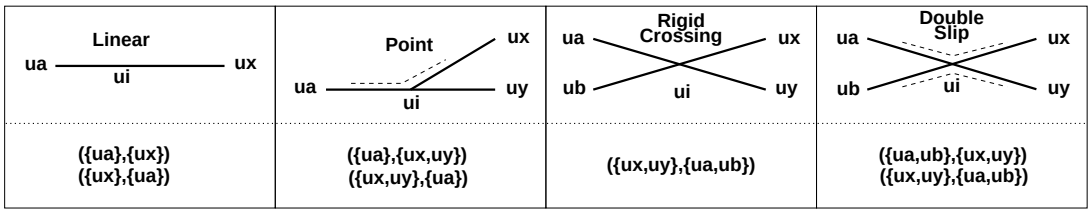

Fig. 5.1 Four Symmetric Rail Unit Mereologies ▪

5.4 Attributes

To recall: there are three sets of ***internal qualities***: unique identifiers, mereologies and attributes. Unique identifiers and mereologies are rather definite kinds of internal endurant qualities; attributes form more "free-wheeling" sets of *internal qualities*. Whereas, for this primer, we suggest to not endow fluids with unique identification and mereologies.

All endurants, i.e., including fluids, are endowed with attributes.

5.4.1 Inseparability of Attributes from Parts and Fluids

Parts and fluids are typically recognised because of their spatial form and are otherwise characterised by their intangible, but measurable attributes. That is, whereas endurants, whether solid (as are parts) or fluids, are physical, tangible, in the sense of being spatial [or being abstractions, i.e., concepts, of spatial endurants], attributes are intangible: cannot normally be touched[82], or seen[83], but can be objectively measured[84]. Thus, in our quest for describing domains where humans play an active rôle, we rule out subjective "attributes": feelings, sentiments, moods. Thus we shall abstain, in our domain science also from matters of aesthetics.

We equate all endurants — which have *the same type of unique identifiers, the same type of mereologies, and the same types of attributes* — with one sort. Thus removing an internal quality from an endurant makes no sense: the endurant of that type either becomes an endurant of another type or ceases to exist (i.e., becomes a non-entity) !

We can roughly distinguish between two kinds of attributes: those which can be motivated by **physical** (including chemical) **concerns**, and those, which, although they embody some form of 'physics measures', appear to reflect on **event histories**: *"if 'something', ϕ, has 'happened' to an endurant, e_a, then some 'commensurate thing', ψ, has 'happened' to another (one or more) endurants, e_b."* where the *'something'* and *'commensurate thing'* usually involve some 'interaction' between the two (or more) endurants. It can take

[82] One can see the red colour of a wall, but one touches the wall.

[83] One cannot see electric current, and one may touch an electric wire, but only if it conducts high voltage can one know that it is indeed an electric wire.

[84] That is, we restrict our domain analysis with respect to attributes to such quantities which are observable, say by mechanical, electrical or chemical instruments. Once objective measurements can be made of human feelings, beauty, and other, we may wish to include these "attributes" in our domain descriptions.

some reflection and analysis to properly identify endurants e_a and e_b and commensurate events ϕ and ψ. Example 73 shall illustrate the, as we shall call it, **intentional pull** of event histories.

5.4.2 Attribute Modeling Tools

5.4.2.1 Attribute Quality and Attribute Value

We distinguish between an **attribute** (as a logical proposition, of a name, i.e.) **type**, and an **attribute value**, as a value in some value space.

5.4.2.2 Concrete Attribute Types

By a *concrete type* we shall understand a sort (i.e., a type) which is defined in terms of some type expression: T = $\mathcal{T}$(...). This is indicated below by [=...].

5.4.2.3 Attribute Description

Let us recall that attributes cover qualities other than unique identifiers and mereology. Let us then consider that parts and fluids have one or more attributes. These attributes are qualities which help characterise "what it means" to be a part or a fluid. Note that we expect every part and fluid to have at least one attribute. The question is now, in general, how many and, particularly, which.

The describe_attributes description prompt is now defined.

Domain Description Prompt 6 describe_attributes:The domain analyser experiments, thinks and reflects about endurant, e, attributes. That process is initiated by the **domain description prompt**:

- describe_attributes(e).

The result of that **domain description prompt** is that the domain analyser cum describer writes down the *Attribute (Sorts or) Types and Observers* domain description text according to the following schema:

let {ηA$_1$, ..., ηA$_m$} = determine_attribute_type_names(e) **in**
❝**Narration:**
[t] ... narrative text on attribute sorts ...
some Ais may be concretely defined: [Ai=...]
[o] ... narrative text on attribute sort observers ...
[p] ... narrative text on attribute sort proof obligations ...
Formalisation:
type
[t] A$_1$[=...] , ..., A$_m$[=...]
value
[o] attr_A$_1$: E→A$_1$, ..., attr_A$_m$: E→A$_m$
proof obligation [Disjointness of Attribute Types]
[p] $\mathcal{PO}$: **let** P be any part sort **in** [the domain description]
[p] **let** a:(A$_1$|A$_2$|...|A$_m$) **in** is_A$_i$(a) ≠ is_A$_j$(a) [i≠i, i,j:[1..m]] **end end** ❞
end

Let A$_1$, ..., A$_n$ be the set of all conceivable attributes of endurant *e:E*. (Usually n is a rather large natural number, say in the order of a hundred conceivable such.) In any one domain model the domain analyser cum describer selects a modest subset, A$_1$, ..., A$_m$, i.e., $m < n$. Across many domain models for *"more-*

or-less the same" domain m varies and the attributes, A_1, ..., A_m, selected for one model may differ from those, A'_1, ..., $A'_{m'}$, chosen for another model.

The **type** definitions: A_1, ..., A_m, inform us that the domain analyser has decided to focus on the distinctly named A_1, ..., A_m attributes.[85] The **value** clauses $\text{attr\_A}_1\text{:P}\to\text{A}_1$, ..., $\text{attr\_A}_n\text{:P}\to\text{A}_n$ are then "automatically" given: if an endurant, e:E, has an attribute A_i then there is postulated, "by definition" [eureka] an attribute observer function $\text{attr\_A}_i\text{:E}\to\text{A}_i$ et cetera ▪

We cannot automatically, that is, syntactically, guarantee that our domain descriptions secure that the various attribute types for an endurant sort denote disjoint sets of values. Therefore we must prove it.

5.4.2.4 Attribute Categories

Michael A. Jackson [121] has suggested a hierarchy of attribute categories: from static to dynamic values – and within the dynamic value category: inert values, reactive values, active values – and within the dynamic active value category: autonomous values, biddable values and programmable values. We now review these attribute value types. The review is based on [121, M.A.Jackson].

Endurant attributes are either constant, i.e., **static**, or varying, i.e., **dynamic** attributes.

Attribute Category 1 By a ***static attribute***, a:A, is_static_attribute(a), we shall understand an attribute whose values are constants, i.e., cannot change ▪

Example 55 **Static Attributes**. Let us exemplify road net attributes in this and the next examples. And let us assume the following attributes: year of first link construction and link length at that time. We may consider both to be static attributes: The year first established, seems an obvious static attribute and the length is fixed at the time the road was first built.

Attribute Category 2 By a ***dynamic attribute***, a:A, is_dynamic_attribute(a), we shall understand an attribute whose values are variable, i.e., can change. Dynamic attributes are either *inert, reactive* or *active* attributes ▪

Attribute Category 3 By an ***inert attribute***, a:A, is_inert_attribute(a), we shall understand a dynamic attribute whose values only change as the result of external stimuli where these stimuli prescribe new values ▪

Example 56 **Inert Attribute**. And let us now further assume the following link attribute: link name. We may consider it to be an inert attribute: the name is not "assigned" to the link by the link itself, but probably by some road net authority which we are not modeling.

Attribute Category 4 By a ***reactive attribute***, a:A, is_reactive_attribute(a), we shall understand a dynamic attribute whose values, if they vary, change in response to external stimuli, where these stimuli either come from outside the domain of interest or from other endurants ▪

[85] The attribute type names are chosen by the domain analyser to reflect on domain phenomena.

Example 57 Reactive Attributes. Let us further assume the following two link attributes: "wear and tear", respectively "icy and slippery". We will consider those attributes to be reactive in that automobiles (another part) traveling the link, an external "force", typically causes the "wear and tear", respectively the weather (outside our domain) causes the "icy and slippery" property.

Attribute Category 5 By an ***active attribute***, a:A, is_active_attribute(a), we shall understand a dynamic attribute whose values change (also) by its own volition. Active attributes are either *autonomous*, or *biddable* or *programmable* attributes ▪

Attribute Category 6 By an , a:A, is_autonomous_attribute(a), we shall understand a dynamic active attribute whose values change only "on their own volition". The values of an autonomous attributes are a "law onto themselves and their surroundings" ▪

Example 58 Autonomous Attributes. We enlarge the scope of our examples of attribute categories to now also include automobiles (on the road net). In this example we assume that an automobile is driven by a human [behaviour]. These are some automobile attributes: velocity, acceleration, and moving straight, or turning left, or turning right. We shall assume that the notions of 'automobile' and driver' are synonymous We shall consider these three attributes to be autonomous. It is the driver, cum automobile, who decides whether the automobile should drive at constant velocity, including 0, or accelerate or decelerate, including stopping. And it is the driver who decides when to turn left or right, or not turn at all.

Attribute Category 7 By a ***biddable attribute***, a:A, is_biddable_attribute(a) we shall understand a dynamic active attribute whose values *are prescribed but may fail to be observed as retaining that value* ▪

Example 59 Biddable Attributes. In the context of automobiles these are some biddable attributes: turning the wheel, to drive right at a hub – with the automobile failing to turn right; pressing the accelerator, to obtain a higher speed – with the automobile failing to really gaining speed; pressing the brake, to stop – with the automobile failing to halt ▪

Attribute Category 8 By a ***programmable attribute***, a:A, is_programmable_attribute(a), we shall understand a dynamic active attribute whose values can be prescribed ▪

Example 60 Programmable Attribute. We continue with the automobile on the road net examples. In this example we assume that an automobile includes, as one inseparable entity, "the driver". These are some automobile attributes: position on a link, velocity, acceleration (incl. deceleration), and direction: straight, turning left, turning right. We shall now consider these three attributes to be programmable.

Figure 5.2 captures an attribute value ontology.
Figure 5.2 hints at three categories of dynamic attributes: **monitorable only**, **biddable** and **programmable** attributes.

Attribute Category 9 By a ***monitorable only attribute***, a:A, is_monitorable_only_attribute(a), we shall understand a dynamic active attribute which is either *inert* or *reactive* or *autonomous*.

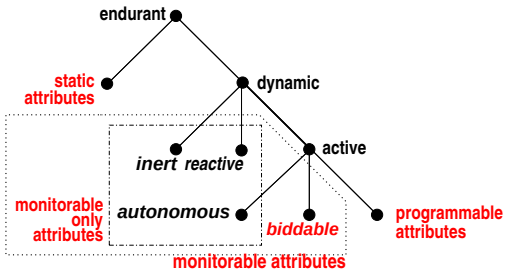

Fig. 5.2 Attribute Value Ontology

That is:

```
value
    is_monitorable_only: E → Bool
    is_monitorable_only(e) ≡ is_inert(e) ∨ is_reactive(e) ∨ is_autonomous(e)
```

We continue our road net examples – with a further four. We refer to Examples 61, 62, 63 on the next page and 68 on page 86..

Example 61 Road Net Attributes. We treat some attributes of the hubs of a road net.

76 There is a hub state. It is a set of pairs, (l_f,l_t), of link identifiers, where these link identifiers are in the mereology of the hub. The meaning of the hub state in which, e.g., (l_f,l_t) is an element, is that the hub is open, "green", for traffic *f*rom link l_f *t*o link l_t. If a hub state is empty then the hub is closed, i.e., "red" for traffic from any connected links to any other connected links.

77 There is a hub state space. It is a set of hub states. The current hub state must be in its state space. The meaning of the hub state space is that its states are all those the hub can attain.

78 Since we can think rationally about it, it can be described, hence we can model, as an attribute of hubs, a history of its traffic: the recording, per unique automobile identifier, of the time ordered presence in the hub of these vehicles. Hub history is an *event history*.

```
type
76  HΣ = (L_UI×L_UI)-set
77  HΩ = HΣ-set
78  H_Traffic = A_UI ~m~→ (𝕋𝕀𝕄𝔼 × VPos)*
axiom
76  ∀ h:H • obs_HΣ(h) ∈ obs_HΩ(h)
78  ∀ ht:H_Traffic,ui:A_UI •  ui ∈ dom ht ⇒ time_ordered(ht(ui))
value
76  attr_HΣ: H → HΣ
77  attr_HΩ: H → HΩ
78  attr_H_Traffic: H → H_Traffic
78   time_ordered: (𝕋𝕀𝕄𝔼 × VPos)* → Bool
78   time_ordered(tvpl) ≡ ...
```

In Item 78 we model the time-ordered sequence of traffic as a discrete sampling, i.e., $\xrightarrow[m]{}$, rather than as a continuous function, → ▪

Example 62 Invariance of Road Net Traffic States. We continue Example 61.

79 The link identifiers of hub states must be in the set, $l_{ui}s$, of the road net's link identifiers.

axiom
79 ∀ h:H • h ∈ *hs* ⇒
79 **let** hσ = attr_HΣ(h) **in**
79 ∀ (l$_{ui}$*i*,li$_{ui}$*i'*):(L_UI×L_UI) • (l$_{ui}$*i*,l$_{ui}$*i'*) ∈ hσ ⇒ {l$_{ui_i}$,l'$_{ui_i}$} ⊆ l$_{ui}$s **end** ▪

You may skip Example 63 in a first reading.

Example 63 Road Transport – Further Attributes.
Links:
We show just a few attributes.

80 There is a link state. It is a set of pairs, (h$_f$,h$_t$), of distinct hub identifiers, where these hub identifiers are in the mereology of the link. The meaning of a link state in which (h$_f$,h$_t$) is an element is that the link is open, "green", for traffic *f*rom hub h$_f$ *t*o hub h$_t$. Link states can have either 0, 1 or 2 elements.
81 There is a link state space. It is a set of link states. The meaning of the link state space is that its states are all those which the link can attain. The current link state must be in its state space. If a link state space is empty then the link is closed. If it has one element then it is a one-way link. If a one-way link, *l*, is imminent on a hub whose mereology designates that link, then the link is a "trap", i.e., a "blind cul-de-sac".
82 Since we can think rationally about it, it can be described, hence it can model, as an attribute of links, a history of its traffic: the recording, per unique automobile identifier, of the time ordered positions along the link (from one hub to the next) of these vehicles.
83 The hub identifiers of link states must be in the set, *h$_{ui}$s*, of the road net's hub identifiers.

type
80 LΣ = (H_UI×H_UI)-**set**
81 LΩ = LΣ-**set**
82 L_Traffic
82 L_Traffic = A_UI $\xrightarrow{m}$ (𝕋×(H_UI×Frac×H_UI))*
82 Frac = **Real**, **axiom** frac:Fract • 0<frac<1
value
80 attr_LΣ: L → LΣ
81 attr_LΩ: L → LΩ
82 attr_L_Traffic: L → L_Traffic
axiom
80 ∀ lσ:LΣ•**card** lσ≤2
80 ∀ l:L • obs_LΣ(l) ∈ obs_LΩ(l)
82 ∀ lt:L_Traffic,ui:A_UI•ui ∈ **dom** ht ⇒ time_ordered(ht(ui))
83 ∀ l:L•l ∈ *ls* ⇒
83 **let** lσ =attr_LΣ(l) **in** ∀ (h$_{ui}$*i*,h$_{ui}$*i'*):(H_UI×H_UI)•(h$_{ui}$*i*,h$_{ui}$*i'*)∈ lσ⇒{h$_{ui_i}$,h'$_{ui_i}$}⊆h$_{ui}$s **end**

Automobiles: We illustrate but a few attributes:

84 Automobiles have static number plate registration numbers.
85 Automobiles have dynamic positions on the road net:

a either *at a hub* identified by some h_ui,
b or *on a link*, some *fraction, frac:Fract* down an *identified link, l_ui*, from one of its *identified connecting hubs*, fh_ui, in the direction of the other *identified hub*, th_ui.
c Fraction is a real properly between 0 and 1.

type
84 RegNo
85 APos == atHub | onLink
85a atHub :: h_ui:H_UI

85b onLink :: fh_ui:H_UI × l_ui:L_UI × frac:Fract × th_ui:H_UI
85c Fract = **Real**
axiom
85c frac:Fract • 0<frac<1
value
84 attr_RegNo: A → RegNo
85 attr_APos: A → APos

Obvious attributes that are not illustrated are those of velocity and acceleration, forward or backward movement, turning right, left or going straight, etc. The *acceleration, deceleration, even velocity,* or *turning right, turning left, moving straight,* or *forward* or *backward* are seen as *command actions*. As such they denote actions by the automobile — such as pressing the accelerator, or lifting accelerator pressure or *braking*, or *turning the wheel* in one direction or another, etc. As actions they have a kind of counterpart in the velocity, the acceleration, etc. attributes. In Items 78 Pg. 75 and 82 Pg. 76, we illustrated an aspect of domain analysis & description that may seem, and at least some decades ago would have seemed, strange: namely that if we can think, hence speak, about it, then we can model it "as a fact" in the domain. The case in point is that we include among hub and link attributes their histories of the timed whereabouts of automobiles[86] ▪

5.4.2.5 Calculating Attribute Category Type Names

One can calculate sets of all attribute type names, of static, monitorable and programmable attribute types of parts and fluids with the following **domain analysis prompt**s:

- determine_attr_names,
- sta_attr_types,
- mon_attr_types, and
- pro_attr_types.

determine_attr_type_names applies to parts and yields a set of all attribute names of that part. sta_attr_types applies to parts and yields a set of attribute names of *static* attributes of that part.[87] mon_attr_types applies to parts and yields a set of attribute names of *monitorable* attributes of that part. pro_attr_types applies to parts and yields a set of attribute names of *programmable* attributes of that part.

Observer Function Prompt 4 determine_attr_type_names:
value
determine_attr_type_names: P → ηA-**set**
determine_attr_type_names(p) **as** {ηA1,ηA,...,ηAm }

Observer Function Prompt 5 sta_attr_type_names:
value
sta_attr_type_names: P → $\eta\mathbb{A}\times\eta\mathbb{A}\times...\times\eta\mathbb{A}$
sta_attr_type_names(p) **as** (ηA1,ηA2,...,ηAn)
where: {ηA1,ηA2,...,ηAn} ⊆ determine_attr_type_names(p)

[86] In this day and age of road cameras and satellite surveillance these traffic recordings may not appear so strange: We now know, at least in principle, of technologies that can record approximations to the hub and link traffic attributes.
[87] $\eta\mathbb{A}$ is the type of all attribute types.

```
    ∧ let anms = determine_attribute_type_names(p)
        ∀ anm:ηA • anm ∈ anms \ {ηA1,ηA2,...,ηAn}
            ⇒ ~ is_static_attribute{anm}
    ∧ ∀ anm:ηA • anm ∈ {ηA1,ηA2,...,ηAn}
            ⇒ is_static_attribute{anm} end
```

Observer Function Prompt 6 mon_attr_type_names:

```
value
    mon_attr_type_names: P → ηA×ηA×...×ηA
    mon_attr_type_names(p) as (ηA1,ηA2,...,ηAn)
        where: {ηA1,ηA2,...,ηAn} ⊆ determine_attr_type_names(p)
            ∧ let anms = determine_attribute_type_names(p)
                ∀ anm:ηA • anm ∈ anms \ {ηA1,ηA2,...,ηAn}
                    ⇒ ~ is_monitorable_attribute{anm}
            ∧ ∀ anm:ηA • anm ∈ {ηA1,ηA2,...,ηAn}
                    ⇒ is_monitorable_attribute{anm} end
```

Observer Function Prompt 7 pro_attr_type_names:

```
value
    pro_attr_type_names: P → ηA×ηA×...×ηA
    pro_attr_type_names(p) as (ηA1,ηA2,...,ηAn)
        where: {ηA1,ηA2,...,ηAn} ⊆ determine_attr_type_names(p)
            ∧ let anms = determine_attribute_type_names(p)
                ∀ anm:ηA • anm ∈ anms \ {ηA1,ηA2,...,ηAn}
                    ⇒ ~ is_monitorable_attribute{anm}
            ∧ ∀ anm:ηA • anm ∈ {ηA1,ηA2,...,ηAn}
                    ⇒ is_monitorable_attribute{anm} end
```

Some comments are in order. The analyse_attribute_type_names function is, as throughout, meta-linguistic, that is, informal, not-computable, but decidable by the domain modeler. Applying it to a part or fluid yields, at the discretion of the domain modeler, a set of attribute type names "freely" chosen by the domain modeler. The sta_attr_type_names, the mon_attr_type_names, and the pro_attr_type_names functions are likewise meta-linguistic; their definition here relies on the likewise meta-linguistic is_static, is_monitorable and is_programmable analysis predicates.

5.4.2.6 Calculating Attribute Values

Let (ηA1, ηA2, ... , ηAn) be a grouping of attribute types for part p (or fluid f). Then (attr_A1(p), attr_A2(p), ... ,attr_An(p)) (respectively f) yields (a1, a2, ... , an), the grouping of values for these attribute types.

We can "formalise" this conversion:

```
value
    types_to_values: ηA1 × ηA2 × ... × ηAn → A1 × A2 × ... × An
```

5.4.3 Operations on Monitorable Attributes of Parts

We remind the reader of the notions of states in general, Sect. 4.8 and of updateable states, Sect. 4.8.2 on page 56 in specific. For every domain description there is possibly an updateable state. There is such a state if there is at least one part with at least one monitorable attribute. Below, as in Sect. 4.8.2, we refer to the updateable states as σ.

Given a part, p, with attribute A, the simple operation attr_A(p) thus yields the value of attribute A for that part. But what if, if what we have is just the global state σ of the set of all monitorable parts of a given universe-of-discourse, uod, the unique identifier, uid_P(p), of a part of σ, and the name, ηA, of an attribute of p ? Then how do we ascertain the attribute value for A of p, and, for *biddable* attributes A, "update" p, in σ, to some A value ? Here is how we express these two issues.

5.4.3.1 Evaluation of Monitorable Attributes

86 Let pi:PI be the unique identifier of any part, p, with monitorable attributes, let A be a monitorable attribute of p, and let ηA be the name of attribute A.
87 Evaluation of the [current] attribute A value of p is defined by function read_A_from_P – retr_part(pi) is defined in Sect. 5.2.4.1 on page 66.

value
86. pi:PI, a:A, ηA:$\eta\mathbb{T}$

87. read_A_from_P: PI × $\mathbb{T}$ → **read** σ
87. read_A(pi,ηA) ≡ attr_A(retr_part(pi))

5.4.3.2 Update of Biddable Attributes

88 The update of a monitorable attribute A, with attribute name ηA of part p, identified by pi, to a new value, s, effects so by over-writing, the global part state σ:
89 Part p is retrieved from the global state.
90 A new part, p′ is formed such that p′ is like part p:

a same unique identifier,
b same mereology,
c same attributes values,
d except for A.

91 That new p' replaces p in σ.

value
86. σ, a:A, pi:PI, ηA:$\eta\mathbb{T}$

88. update_P_with_A: PI × A × $\eta\mathbb{T}$ → **write** σ
88. update_P_with_A(pi,a,ηA) ≡
89. **let** p = retr_part(pi) **in**
90. **let** p′:P •
90a. uid_P(p′)=pi
90b. ∧ mereo_P(p)=mereo_P(p′)
90c. ∧ ∀ ηA′ ∈ analyse_attribute_type_names(p) \ {ηA} ⇒ attr_A′(p)=attr_A′(p′)
90d. ∧ attr_A(p′)=a **in**
91. σ := σ \ {p} ∪ {p′}
88. **end end**

5.4.4 Physics Attributes

In this section we shall muse about the kind of attributes that are typical of natural parts, but which may also be relevant as attributes of artefacts.

Typically, when physicists write computer programs, intended for calculating physics behaviours, they lump all of these into the **type Real**, thereby hiding some important physics dimensions. In this section we shall review that which is missing !

The subject of physical dimensions in programming languages is rather decisively treated in David Kennedy's 1996 PhD Thesis [127] — so there really is no point in trying to cast new light on this subject other than to remind the reader of what these physical dimensions are all about.

5.4.4.1 SI: The International System of Quantities

In physics we operate on values of attributes of manifest, i.e., physical phenomena. The type of some of these attributes are recorded in well known tables, cf. Tables 5.1–5.3. Table 5.1 shows the base units of physics.

Base quantity	Name	Type
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

Table 5.1 Base SI Units

Table 5.2 shows the units of physics derived from the base units. Table 5.3 shows further units of physics

Name	Type	Derived Quantity	Derived Type
radian	rad	angle	m/m
steradian	sr	solid angle	$m^2 \times m^{-2}$
Hertz	Hz	frequency	s^{-1}
newton	N	force, weight	$kg \times m \times s^{-2}$
pascal	Pa	pressure, stress	N/m^2
joule	J	energy, work, heat	N×m
watt	W	power, radiant flux	J/s
coulomb	C	electric charge	s×A
volt	V	electromotive force	W/A ($kg \times m^2 \times s^{-3} \times A^{-1}$)
farad	F	capacitance	C/V ($kg^{-1} \times \cdot\ m^{-2} \times s^4 \times A^2$)
ohm	Ω	electrical resistance	V/A ($kg \times m^2 \times s^3 \times A^2$)
siemens	S	electrical conductance	A/V ($kg1 \times m^2 \times s^3 \times A^2$)
weber	Wb	magnetic flux	V×s ($kg \times m^2 \times s^{-2} \times A^{-1}$)
tesla	T	magnetic flux density	Wb/m^2 ($kg \times s^2 \times A^{-1}$)
henry	H	inductance	Wb/A ($kg \times m^2 \times s^{-2} \times A^2$)
degree Celsius	^{0}C	temp. rel. to 273.15 K	K
lumen	lm	luminous flux	cd×sr (cd)
lux	lx	illuminance	lm/m^2 ($m^2 \times cd$)

Table 5.2 Derived SI Units

derived from the base units. *velocity* is speed [with three dimensional direction] and is, for example, given as

- *velocity*, meter per second with direction: $m/s\ (r,r,r)$
- *acceleration*, meter per second squared, m/s^2
- *direction: (longitude,latitude,azimuth)* measured in radian: (r,r,r)

Name	Explanation	Derived Type
area	square meter	m^2
volume	cubic meter	m^3
speed	meter per second	m/s
wave number	reciprocal meter	m-1
mass density	kilogram per cubic meter	kg/m^3
specific volume	cubic meter per kilogram	m3/kg
current density	ampere per square meter	A/m^2
magnetic field strength	ampere per meter	A/m
substance concentration	mole per cubic meter	mol/m3
luminance	candela per square meter	cd/m^2
mass fraction	kilogram per kilogram	kg/kg = 1

Table 5.3 Further SI Units

Table 5.4 shows standard prefixes for SI units of measure and Tables 5.5 show fractions of SI units.

Prefix name		deca	hecto	kilo	mega	giga
Prefix symbol		da	h	k	M	G
Factor	10^0	10^1	10^2	10^3	10^6	10^9
Prefix name		tera	peta	exa	zetta	yotta
Prefix symbol		T	P	E	Z	Y
Factor		10^{12}	10^{15}	10^{18}	10^{21}	10^{24}

Table 5.4 Standard Prefixes for SI Units of Measure

Prefix name		deca	hecto	kilo	mega	giga
Prefix symbol		da	h	k	M	G
Factor	10^0	10^1	10^2	10^3	10^6	10^9
Prefix name		tera	peta	exa	zetta	yotta
Prefix symbol		T	P	E	Z	Y
Factor		10^{12}	10^{15}	10^{18}	10^{21}	10^{24}
Prefix name		deci	centi	milli	micro	nano
Prefix symbol		d	c	m	μ	n
Factor	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-6}	10^{-9}
Prefix name		pico	femto	atto	zepto	yocto
Prefix symbol		p	f	a	z	y
Factor		10^{-12}	10^{-15}	10^{-18}	10^{-21}	10^{-24}

Table 5.5 SI Units of Measure and Fractions

• • •

The point in including this material is that when modeling, i.e., describing domains we must be extremely careful in not falling into the trap of modeling physics types, etc., as we do in programming – by simple **Real**s. We claim, without evidence, that many trivial programming mistakes are due to confusions between especially derived SI units, fractions and prefixes.

5.4.4.2 Units are Indivisible

A volt, $kg \times m^2 \times s^{-3} \times A^{-1}$, see Table 5.2, is "indivisible". It is not a composite structure of mass, length, time, and electric current – in some intricate relationship.

• • •

Physical attributes may ascribe mass and volume to endurants. But they do not reveal the substance, i.e., the material from which the endurant is made. That is done by chemical attributes.

5.4.4.3 Chemical Elements

The chemical elements are, to us, what makes up MATTER. The *mole*, mol, substance is about chemical molecules. A mole contains exactly $6.02214076 \times 10^{23}$ (the Avogadro number) constituent particles, usually atoms, molecules, or ions – of the elements, cf. 'The Periodic Table', en.wikipedia.orgwiki/Periodic_table, cf. Fig. 5.3.

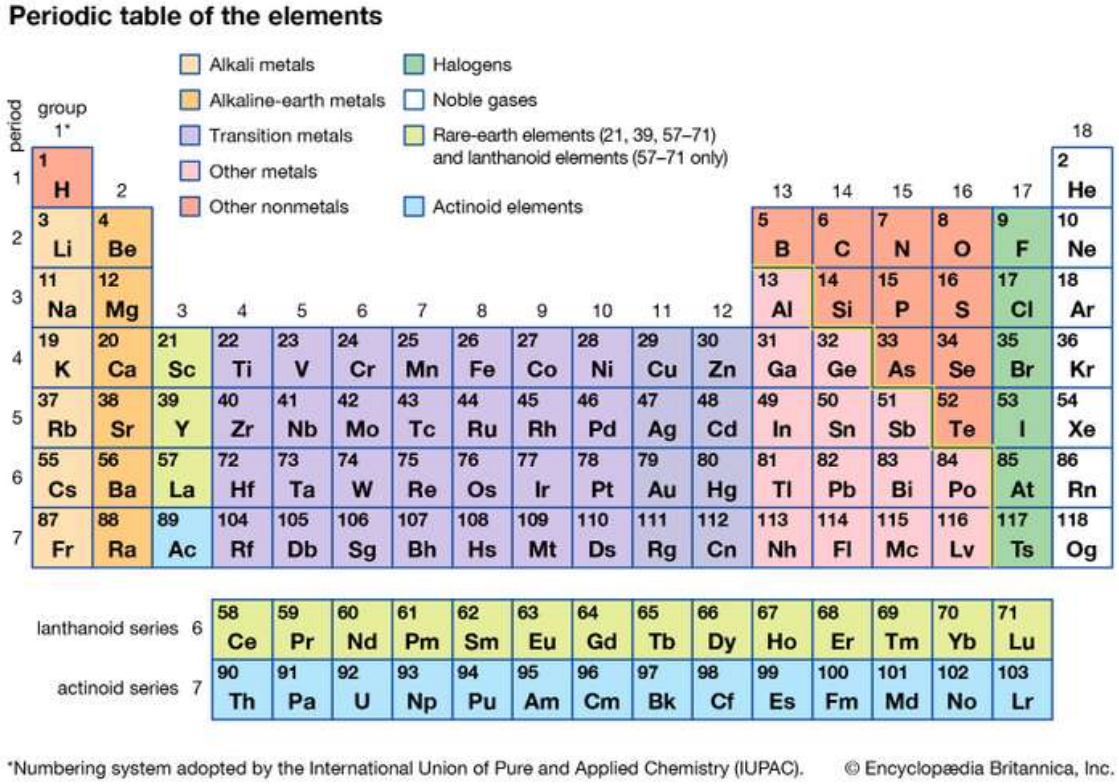

Fig. 5.3 Periodic Table

Any specific molecule is then a compound of two or more elements, for example, calciumphosphat: Ca3(PO4)2.

Moles bring substance to endurants. The physics attributes may ascribe weight and volume to endurants, but they do not explain what it is that gives weight, i.e., fills out the volume.

5.4.5 Special Attributes

We have already covered, in one form or another, some of the concepts of this section. Here we further comment on them. We shall treat the following attributes:

In Sect. 5.4.5.4 on page 86 give an example of their interplay with the *static, monitorable* and *programmable* attribute categories.

5.4.5.1 Modeling Attributes

The modeling attributes are represented, not as observable, by means of the **attr_** operator, but by means of **is_** prompts.

5.4.5.1.1 Manifestation.

We refer to Sect. 5.1.2 on page 61 and to its definitions of *manifest* and *structure*: 75 on page 61 and 76 on page 62. We also remind the reader of the respective prompts: 15 on page 62 and 16. We have earlier revealed that endurant parts can be transcendentally deduced into perdurant behaviours. These are the *behaviourable* manifest parts. It is the case that:

- $\forall p{:}P \bullet is\_endurant(p) \Rightarrow is\_manifest(p) \not\equiv is\_stationary(p)$

5.4.5.1.2 Behaviourability.

The domain analyser cum describers analyse some manifest parts into being *behaviourable*!

Definition 82 **Behaviourable Part.** By behaviourable parts we shall understand manifest parts which the domain analyser cum describers have analysed into being candidates for being trancendentally deducable into behaviours ▪

For the domain analyser cum describers to decide whether a manifest part is to be modeled as a behaviour is a choice. Parts chosen to not be behaviourable could as well be chosen to be behaviourable! Usually the choice is a pragmatic one: nothing is lost, the domain analyser cum describers decide, in the expressability of the domain, if chosen to not be behaviourable.

Analysis Predicate Prompt 17 is_behaviourable: The method provides the **domain analysis prompt**:

- is_behaviourable – where is_behaviourable(e) holds if e is behaviourable ▪

Example 64 **Behaviourable and Not Behaviourable Parts.** We have chosen, in this book, to model road net automobiles, hubs and links as behaviourable: automobiles, obviously so; hubs and links because the re-actively record the entering and leaving of automobiles! In a *goods container terminal port* [42] we, somewhat surprisingly to the reader, perhaps, have chosen to model *containers* as non-behaviours – allowing them to be attributes of container vessels, quay cranes, trucks and stacks! ▪

Definition 83 **Pro-Active Behaviour.** By a pro-active behaviour we shall understand a behaviour which, at its own volition, i.e., *internal non-deterministically*[88] [takes the initiative to] interact with other behaviours by offering messages to that other behaviour ▪

Analysis Predicate Prompt 18 is_proactive: The method provides the **domain analysis prompt**:

- is_proactive – where is_proactive(e) holds if e is proactive ▪

Definition 84 **Re-Active Behaviour**. By a re-active behaviour we shall understand a behaviour which [deterministically or external non-deterministically] accepts messages offered by other behaviors ▪

Example 65 **Re-Active Behaviours**. Links and hubs in the "ongoing" road net example are modeled as re-active behaviours: all they do, in our model, is to accept communications from automobiles as to their road net whereabouts ▪

Analysis Predicate Prompt 19 is_reactive: The method provides the **domain analysis prompt**:

- is_reactive – where is_reactive(e) holds if e is reactive ▪

Definition 85 **Active Behaviour**. By an active behaviour we understand a behaviour which is both pro- and re-active ▪

It is the case that: ∀p:P•is_endurant(p)⇒is_behaviourable(p)⇒is_proactive(p)∨is_reactive(p).

5.4.5.1.3 Mobility

Endurants are either **mobile** or **stationary**.

Definition 86 **Mobile**. An endurant is said to be mobile if it is capable of being moved – whether by its own volition, or otherwise ▪

Analysis Predicate Prompt 20 is_mobile: The method provides the **domain analysis prompt**:

- is_mobile – where is_mobile(e) holds if e is mobile ▪

Analysis Predicate Prompt 21 is_stationary: The method provides the **domain analysis prompt**:

- is_stationary – where is_stationary(e) holds if e is stationary ▪

It is the case that: ∀ p:P•is_part(p)⇒is_mobile(p)∨is_stationary(p).
Being stationary, or, vice-versa, being movable, is a static attribute.

Example 66 **Mobile Endurants**. Examples of mobile endurants are: (i) automobiles; (ii) container terminal vessels, containers, cranes and trucks; (iii) pipeline oil (or gas, or water, ...); (iv) sea, lake and river water ▪

[88] In CSP: a scenario where the system makes a choice between two or more behaviours automatically, completely independent of the environment or user input.

Example 67 Stationary Endurants. 64 on page 83 Examples of stationary endurants could be: (i) road hubs and links; (ii) container terminal stacks; (iii) pipeline units; and (iv) sea, lake and river beds.

Being stationary or mobile is an attribute of any manifest endurant. For every manifest endurant, *e*, it is the case that is_stationary(e)≡~is_mobile(e).

•••

Being stationary or, vice-versa, being mobile is often **tacitly assumed.** Having external or internal qualities of a certain kind is often also tacitly assumed. A major point of the domain analysis & description approach, of this primer, is to help the domain modeler – the domain engineer cum researcher – to unveil as many, if not all, these qualities. **Tacit understanding** would not be a common problem was it not for us to practice it "excessively"! It is not necessary that You do so. In that case manifestation – i.e., behaviourability, transpires from the domain description if a behaviour is defined for parts p:P.

5.4.5.1.4 Some Remarks.

The modeling attributes[89] are not used in calculating domain quantities. They are endurant [sometimes amalyser subjective] properties that determine [subsequent] modeling.

5.4.5.2 Universal Attributes

We consider 𝕊ℙ𝔸ℂ𝔼 and 𝕋𝕀𝕄𝔼 to be universal qualities: present in any universe. They are treated in the next section.

5.4.5.2.1 Spatial Notions

We refer to Sect. 5.5.1 on page 87. Entities, whether mobile or stationary, will, respectively may have spatial properties. Whether they must be recorded as attributes is a matter for the domain analysers to decide. 𝕊ℙ𝔸ℂ𝔼 is not a property of any one specific domain. It is a property of the universe. But specific attributes of parts may "involve" such aspects of 𝕊ℙ𝔸ℂ𝔼 as ℙ𝕆𝕀ℕ𝕋s, 𝔸ℝ𝔼𝔸s and 𝕍𝕆𝕃𝕌𝕄𝔼s.

5.4.5.2.2 Temporal Notions

We refer to Sect. 5.5.2 on page 88. Entities, whether mobile or stationary, will, respectively may have temporal properties. Whether they must be recorded as attributes is a matter for the domain analysers to decide. 𝕋𝕀𝕄𝔼 is not a property of any one specific domain. It is a property of the universe. But specific attributes of parts may "involve" such aspects of 𝕋𝕀𝕄𝔼 as annotated 𝕋𝕀𝕄𝔼 (as shown in Example 61's item 78 on page 75 and Example 63's item 82 on page 76) and 𝕋ime 𝕀ntervals.

5.4.5.2.3 Some Remarks

The universal attributes[90] are used in calculating domain quantities. Common to both is that their [constituent] types (𝕋𝕀𝕄𝔼,𝕊ℙ𝔸ℂ𝔼) are "measured" somewhat arbitrarily. 𝕋𝕀𝕄𝔼 is never represented and 𝕋Is, time-intervals is what we use: *"in the year 2026 after the birth of Jesus Christ The Lord"*. Similar for 𝕊ℙ𝔸ℂ𝔼: {we express spatial points relative to a ℙ𝕆𝕀ℕ𝕋 that, e.g., stand for a specific point at the Meridian: Greenwich, London, England: **Latitude**, **Longitude**, **Altitude**!

[89] Edit: I must find a better term

[90] Edit: I must find a better term

5.4.5.3 Ancillary Attributes

There is yet another 'class' of attributes. We name that class *ancillary* [91]. We have yet to make use of it. But it is there: People can speak, rationally, about endurants possessing some of these ancillary attributes.

Graphic:

maps:

- cadestral
- city,
- land,
- railway,
- sea, and
- aerial maps, etc.;

technical drawings, of:

- machines,
- buildings,
- computers,

photos,
[art] prints, etc.

Other:

- car colour
- curtain material, etc.

We leave the subject of endurant attributes here – leaving it to the reader to study that subject !

5.4.5.4 An Example

Example 68 An Interplay. We continue Examples 61 on page 75, 62 on page 75 and 63 on page 76.

92 Automobiles, hubs and links are manifest.
93 Automobiles, hubs and links have locations, here spelled LOCATION. They are not attributes of endurants and can, as such, not be observed from the endurant. They are "attributes" of the universe [around] the endurants and are recorded, **record_LOCATION**, "at the endurant" in the context of the universe. See Sect. 5.5.1 on the facing page.
For automobiles location is a programmable attribute; for hubs and links You might think locations are static attributes but the road net authority, which we do not model, may decide to alter, for example, their detailed layout – so we may have to consider them monitorable attributes !
[ι 78 π 75]. Hub traffic is a programmable attribute.
[ι 82 π 76]. Link traffic is a programmable attribute.
Similar to LOCATION, TIME is not an attribute of endurants and can, as such, not be observed from the endurant. They are "attributes" of the universe [around] the endurants and is recorded, record_TIME(), "at the endurant" in the context of the universe. See Sect. 5.5.2 on page 88.
94 Links and hubs have cartography. [92] You may think of their cartographies as graphic drawings.
95 Links and hubs have a cadrastra. [93]
96 Automobiles have maker, model, year, chassis number, engine number, etc.

type
93. LOCATION = $\mathcal{E}$(SPACE)
[[ι 82 π 76]] L_Traffic = A_UI $\xrightarrow{m}$ (TIME×(H_UI×Frac×H_UI))*
[[ι 78 π 75]] H_Traffic = A_UI $\xrightarrow{m}$ (TIME × VPos)*
94. Cartography
95. Cadastre
95. AutoInfo = Maker × Model × Year × Chassis# × Engine# × ...
value
93. **record_LOCATION**: **Unit** → LOCATION

[91] **Ancillary** is an adjective meaning providing necessary support to a primary activity, or serving as a secondary, supplementary addition. Something that is ancillary is auxiliary, subsidiary, or accessory to a larger operation [https://www.google.com/search?q=ancillary]

[92] **Cartography**: Cartography is the study and practice of making and using maps. Combining science, aesthetics and technique, cartography builds on the premise that reality (or an imagined reality) can be modeled in ways that communicate spatial information effectively [Wikipedia]

[93] **Cadastre**: A cadastre or cadaster is a comprehensive recording of the real estate or real property's metes-and-bounds of a country. Often it is represented graphically in a cadastral map [Wikipedia]

[[ι 78 π 75]] **attr**_H_Traffic: H → H_Traffic
[[ι 82 π 76]] **attr**_Traffic: L → L_Traffic
94. **attr**_Cartography: (L|H) → Cartography
95. **attr**_Cadastre: (L|H) → Cadastre
95. **attr**_AutoInfo: A → AutoInfo ▪

5.5 SPACE and TIME

The two concepts: **space**, here spelled SPACE, and **time**, here spelled TIME7, are not attributes of entities. In fact, they are not internal qualities of endurants. They are universal qualities of any world. endurants. [One may consider them attributes of the universe.] As argued in Sect. 2.4.9, SPACE and TIME are unavoidable concepts of any world. But we can ascribe spatial attributes to any concrete, manifest endurant. And we can ascribe attributes to endurants that record temporal concepts.

5.5.1 SPACE

Space is just there. So we do not define an observer, observe_space. For us – bound to model mostly artefactual worlds on this earth – there is but one space. Although SPACE, as a type, could be thought of as defining more than one space we shall consider these to be isomorphic ! SPACE is considered to consist of (an infinite number of) POINTs.

97 We can assume a point observer, observe_POINT, is a function which applies to endurants, e, and yield a point, pt : POINT

97. observe_POINT: E → POINT

At which "point" of an endurant, e, observe_POINT(e), is applied, or which of the (infinitely) many points of an endurant E, observe_POINT(e), yields we leave up to the domain modeler to decide !

We suggest, besides POINTs, the following spatial attribute possibilities:

98 EXTENT as a dense set of POINTs;
99 Volume, of concrete type, for example, m^3, as the "volume" of an EXTENT such that
100 PLANEs as dense sets of POINTs have no volume, but an
101 Area, of concrete type, for example, m^2, as the "area" of a dense set of POINTs;
102 LINE as dense set of POINTs with no volume and no area, but
103 Length, of concrete type, for example, m.

For these we have that

104 the *intersection*, ⋂, of two EXTENTs is an EXTENT of possibly nil Volume,
105 the intersection, ⋂, of two SURFACEs may be either a possibly nil SURFACE or a possibly nil LINE, or a combination of these.
106 the intersection, ⋂, of two LINEs may be either a possibly nil LINE or a POINT.

Similarly we can define

107 the *union*, ⋃, of two not-disjoint EXTENTs,
108 the *union*, ⋃, of two not-disjoint SURFACEs,
109 the *union*, ⋃, and of two not-disjoint LINEs.

and:

110 the *[in]equality*, ≠, =, of pairs of EXTENT, pairs of SURFACEs, and pairs of LINEs.

We invite the reader to first express the signatures for these operations, then their pre-conditions, and finally, being courageous, appropriate fragments of axiom systems. We leave it up to the reader to introduce, and hence define, functions that add, subtract, compare, etc., EXTENTs, PLANEs, LINEs and SPACE.

5.5.2 TIME

a moving image of eternity;
the number of the movement in respect of the before and the after;
the life of the soul in movement as it passes
from one stage of act or experience to another;
a present of things past: memory,
a present of things present: sight,
and a present of things future: expectations[94]

This thing all things devours:
Birds, beasts, trees, flowers;
Gnaws iron, bites steel,
Grinds hard stones to meal;
Slays king, ruins town,
And beats high mountain down.[95]

Concepts of time continue to fascinate philosophers and scientists [92, 135, 142, 146–151, 153, 170] and [94].

J.M.E. McTaggart (1908, [92,135,153]) discussed theories of time around the notions of "**A-series**": with concepts like "past", "present" and "future", and "**B-series**": has terms like "precede", "simultaneous" and "follow". Johan van Benthem [170] and Wayne D. Blizard [71] relates abstracted entities to spatial points and time. A recent computer programming-oriented treatment is given in [94, Mandrioli et al., 2013].

5.5.2.1 Time Motivated Philosophically

Definition 87 **Indefinite Time**. *We motivate, repeating from Sect. 2.4.9.2, the abstract notion of time as follows. Two different states must necessarily be ascribed different incompatible predicates. But how can we ensure so ? Only if states stand in an asymmetric relation to one another. This state relation is also transitive. So that is an indispensable property of any world. By a transcendental deduction we say that primary entities exist in time. So every possible world must exist in time* ▪

Definition 88 **Definite Time**. By a *definite time* we shall understand an abstract representation of time such as for example year, month, day, hour, minute, second, et cetera ▪

Example 69 **Temporal Notions of Endurants**. *By temporal notions of endurants we mean time properties of endurants, usually modeled as attributes. Examples are: (i) the time stamped link traffic, cf. Item 82 on page 76 and (ii) the time stamped hub traffic, cf. Item 78 on page 75*▪

[94] Quoted from [4, Cambridge Dictionary of Philosophy]
[95] J.R.R. Tolkien, The Hobbit

5.5.2.2 Time Values

We shall not be concerned with any representation of time. That is, we leave it to the domain modeler to choose an own representation [94]. Similarly we shall not be concerned with any representation of time intervals.[96]

111 So there is an abstract type $\mathbb{T}$ime,
112 and an abstract type $\mathbb{TI}$: $\mathbb{T}$ime $\mathbb{I}$nterval.
113 There is no $\mathbb{T}$ime origin, but there is a "zero" $\mathbb{T}$ime $\mathbb{I}$nterval.
114 One can add (subtract) a time interval to (from) a time and obtain a time.
115 One can add and subtract two time intervals and obtain a time interval – with subtraction respecting that the subtrahend is smaller than or equal to the minuend.
116 One can subtract a time from another time obtaining a time interval respecting that the subtrahend is smaller than or equal to the minuend.
117 One can multiply a time interval with a real and obtain a time interval.
118 One can compare two times and two time intervals.

type
111 $\mathbb{T}$
112 $\mathbb{TI}$
value
113 **0**:$\mathbb{TI}$
114 +,−: $\mathbb{T} \times \mathbb{TI} \to \mathbb{T}$
115 +,−: $\mathbb{TI} \times \mathbb{TI} \xrightarrow{\sim} \mathbb{TI}$
116 −: $\mathbb{T} \times \mathbb{T} \to \mathbb{TI}$
117 ∗: $\mathbb{TI} \times$ **Real** $\to \mathbb{TI}$
118 <,≤,=,≠,≥,>: $\mathbb{T} \times \mathbb{T} \to$ **Bool**
118 <,≤,=,≠,≥,>: $\mathbb{TI} \times \mathbb{TI} \to$ **Bool**
axiom
114 ∀ t:$\mathbb{T}$ • t+**0** = t

5.5.2.3 Temporal Observers

119 We define the signature of the meta-physical time observer.

type
119 $\mathbb{T}$
value
119 record_$\mathbb{TIME}$(): **Unit** $\to \mathbb{T}$

The time recorder applies to nothing and yields a time. record_$\mathbb{TIME}$() can only occur in action, event and behavioural descriptions.

5.5.3 A Domain Law

Attribute Names

A Domain Law 3 **Attribute Names:**

The attribute names of any part type is a static property. They do not change: increase, shrink or otherwise.

axiom ∀ p:P • determine_attr_type_names(p) = ans ⇒ □ ∀ p′:P • determine_attr_type_names(p′) = ans

[96] – but point out, that although a definite time interval may be referred to by number of years, number of days (less than 365), number of hours (less than 24), number of minutes (less than 60) number of seconds (less than 60), et cetera, this is not a time, but a time interval.

5.6 Intentional Pull

In the next section we shall encircle the 'intention' concept by extensively quoting from Kai Sørlander's Philosphy [161–164].

Intentionality[97] "expresses" conceptual, abstract relations between otherwise, or seemingly unrelated entities.

5.6.1 Issues Leading Up to Intentionality

5.6.1.1 Causality of Purpose

"If there is to be the possibility of language and meaning *then there must exist primary entities which are* not entirely encapsulated within the physical conditions*; that they are stable and can influence one another. This is only possible if such primary entities are subject to a* supplementary causality directed at the future*: a* causality of purpose*."*

5.6.1.2 Living Species

"These primary entities are here called living species. What can be deduced about them ? They are characterised by causality of purpose*: they* have some form they can be developed to reach; *and which* they must be causally determined to maintain; *this* development *and* maintenance *must occur in* an exchange of matter with an environment*. It must be possible that living species occur in one of two forms: one form which is characterised by* development, form *and* exchange*, and another form which, additionally, can be characterised by the ability of* purposeful movement*s. The first we call* plant*s, the second we call* animal*s."*

5.6.1.3 Animate Entities

"For an animal to purposefully move around there must be "additional conditions" for such self-movements to be in accordance with the principle of causality: they must have sensory organ*s sensing among others the immediate purpose of its movement; they must have* means of motion *so that it can move; and they must have* instinct*s,* incentive*s and* feeling*s as causal conditions that what it senses can drive it to movements. And all of this in accordance with the laws of physics."*

5.6.1.4 Animals

"To possess these three kinds of "additional conditions", these entities must be built from special units which have an inner relation to their function as a whole; Their purposefulness *must be built into their physical building units, that is their genomes. That is, animals are built from genomes which give them the* inner determination *to such building blocks for* instinct*s, incentives and* feeling*s. Similar kinds of deduction can be carried out with respect to plants. Transcendentally one can deduce basic principles of evolution but not its details."*

[97] The Oxford English Dictionary [131] characterises intentionality as follows: *"the quality of mental states (e.g. thoughts, beliefs, desires, hopes) which consists in their being directed towards some object or state of affairs".*

5.6.1.5 Humans – Consciousness and Learning

"The existence of animals is a necessary condition for there being language and meaning in any world. That there can be language means that animals are capable of developing language. *And this must presuppose that animals can* learn from their experience. *To learn implies that animals can feel pleasure and distaste and can learn. One can therefore deduce that animals must possess such building blocks whose inner determination is a basis for learning and consciousness."*

"*Animals with higher social interaction use signs, eventually develop a language. These languages adhere to the same system of defined concepts which are a prerequisite for any description of any world: namely the system that philosophy lays bare from a basis of transcendental deductions and the principle of contradiction and its implicit meaning theory. A human is an animal which has a* language."

5.6.1.6 Knowledge

"Humans must be conscious *of having knowledge of its concrete situation, and as such humans can have knowledge about what they feel and eventually that humans can know whether what they feel is true or false. Consequently* humans can describe their situation correctly."

5.6.1.7 Responsibility

"In this way one can deduce that humans can thus have memory and hence can have responsibility, be responsible. *Further deductions lead us into ethics."*

•••

We shall not further develop the theme of *living species: plants and animals*, thus excluding, most notably *humans*, in this chapter. We claim that the present chapter, due to its foundation in Kai Sørlander's Philosophy, provides a firm foundation within which we, or others, can further develop this theme: *analysis & description of living species.*

5.6.2 Intentionality

Intentionality as a philosophical concept is defined by the Stanford Encyclopedia of Philosophy[98] as *"the power of minds to be about, to represent, or to stand for, things, properties and states of affairs."*

5.6.2.1 Intentional Pull

Two or more artefactual parts of different sorts, but with overlapping sets of intents may excert an *intentional "pull"* on one another. This *intentional "pull"* may take many forms. Let $p_x : X$ and $p_y : Y$ be two parts of *different sorts* (X, Y), and with *common intent*, ι, of the same universe of discourse. *Manifestations* of these, their common intent, must somehow be *subject to constraints*, and these must be *expressed predicatively.*

Example 70 **Double Bookkeeping**. A classical example of intentional pull is found in double bookkeeping which states that every financial transaction has equal and opposite effects in at least two different accounts. It is used to satisfy the accounting equation: Assets = Liabilities + Equity. The intentional pull is

[98] Jacob, P. (Aug 31, 2010). *Intentionality*. Stanford Encyclopedia of Philosophy (https://seop.illc.uva.nl/entries/intentionality/) October 15, 2014, retrieved April 3, 2018.

then reflected in commensurate postings, for example: either in both debit and passive entries or in both credit and passive entries ▪

Example 71 **The The Henry George Theorem**. The Henry George Theorem states that under certain conditions, aggregate spending by government on public goods will increase aggregate rent based on land value (land rent) more than that amount, with the benefit of the last marginal investment equaling its cost [99,100] ▪

Example 72 **The Henry George Theorem at Work**. This example is based on research done between 1960 and 2000 by professor Gunnar Thorlund Jepsen and his students, Aarhus University. Every year they had a minor data-acquisition project: that of recording land prices along the main Danish freeway: # 1. Over the years – and with the *Storebælts Bridge* eventually completed in 1999 – they could show that the land value increase in 40 years – adjusted for general land price development – well exceeded the full cost of the bridge ▪

When a compound artefact is modeled as put together with a number of distinct sort endurants then it does have an intentionality and the components' individual intentionalities do, i.e., shall relate to that. The composite road transport system has intentionality of the road serving the automobile part, and the automobiles have the intent of being served by the roads, and vice versa, the roads of serving the automobiles.

Natural endurants, for example, rivers, lakes, seas[101] and oceans become, in a way, artefacts when mankind use them for transport; natural gas becomes an artefact when drilled for, exploited and piped; and harbours make no sense without artefactual boats sailing on the natural water.

5.6.2.2 The Type Intent

This, perhaps vague, concept of intentionality has yet to be developed into something of a theory. Despite that this is yet to be done, we shall proceed to define an *intentionality analysis function*. First we postulate a set of **intent designators**. An *intent designator* is really a further undefined thing. But let us, for the moment, think of them as simple character strings, that is, literals, for example ""transport", "eating", "entertainment", etc.

type Intent

5.6.2.3 Intentionalities

Observer Function Prompt 8 determine_intentionality: The domain analyser analyses an endurant as to the finite number of intents, zero or more, with which the analyser judges the endurant can be associated. The method provides the **domain analysis prompt**:

[99] Stiglitz, Joseph (1977). "The Theory of Local Public Goods". In Feldstein, M.S.; Inman, R.P. (eds.). The Economics of Public Services. Palgrave Macmillan, London. pp. 274–333. doi:10.1007/978-1-349-02917-4_12. ISBN 978-1-349-02919-8.

[100] Henry George (September 2, 1839 – October 29, 1897) was an American political economist and journalist. His writing was immensely popular in 19th-century America and sparked several reform movements of the Progressive Era. He inspired the economic philosophy known as Georgism, the belief that people should own the value they produce themselves, but that the economic value of land (including natural resources) should belong equally to all members of society. George famously argued that a single tax on land values would create a more productive and just society.

[101] Seas are smaller than oceans and are usually located where the land and ocean meet. Typically, seas are partially enclosed by land. The Sargasso Sea is an exception. It is defined only by ocean currents [oceanservice.noaa.gov/facts/oceanorsea.html].

- determine_intentionality directs the domain analyser to observe a set of intents.

  value determine_intentionality(e) ≡ {i_1,i_2,...,i_n}⊆Intent

Example 73 Intentional Pull – Road Transport. We simplify the link, hub and automobile histories – aiming at just showing an essence of the intentional pull concept.

120 With links, hubs and automobiles we can associate history attributes.

a Link history attributes are ordered lists of time-stamped entries; they record the presence of automobiles.
b Hub history attributes are ordered lists of time-stamped entries; they record the presence of automobiles.
c Automobile history attributes are ordered lists of time-stamped entries; time-stamped they record their visits to links and hubs.

type
120a. LHist = AI $\overrightarrow{m}$ 𝕋𝕀𝕄𝔼*
120b. HHist = AI $\overrightarrow{m}$ 𝕋𝕀𝕄𝔼*
120c. AHist = (LI|HI) $\overrightarrow{m}$ 𝕋𝕀𝕄𝔼*

value
120a. attr_LHist: L → LHist
120b. attr_HHist: H → HHist
120c. attr_AHist: A → AHist

5.6.2.4 Well-formedness of Event Histories

Some observations must be made with respect to the above modeling of time-stamped event histories.

121 Each τ_ℓ : 𝕋𝕀𝕄𝔼* is an indefinite list. We have not expressed any criteria for the recording of events: *all the time, continuously* ! (?)
122 Each list of times, τ_ℓ : 𝕋𝕀𝕄𝔼*, is here to be in decreasing *strictly decreasing times*.
123 Time intervals from when an automobile enters a link (a hub) till it first time leaves that link (hub) must not overlap with other such time intervals for that automobile.
124 If an automobile leaves a link (a hub), at time τ, then it may enter a hub (resp. a link) and then that must be at time τ' where τ' is some infinitesimal, sampling time interval, quantity larger that τ. Again we refrain here from speculating on the issue of sampling !
125 Altogether, ensembles of link and hub event histories for any given automobile define routes that automobiles travel across the road net. Such routes must be in the set of routes defined by the road net.

As You can see, there is enough of interesting modeling issues to tackle !

5.6.2.5 Formulation of an Intentional Pull

126 An *intentional pull* of any road transport system, *rts*, is then if:

a for any automobile, *a*, of *rts*, on a link, ℓ (hub, *h*), at time τ,
b then that link, ℓ, (hub *h*) "records" automobile *a* at that time.

127 and:

c for any link, ℓ (hub, *h*) being visited by an automobile, *a*, at time τ,
d then that automobile, *a*, is visiting that link, ℓ (hub, *h*), at that time.

axiom
126a. ∀ a:A • a ∈ *as* ⇒
126a. **let** ahist = attr_AHist(a) **in**
126a. ∀ ui:(LI|HI) • ui ∈ **dom** ahist ⇒
126b. ∀ τ:𝕋𝕀𝕄𝔼 • τ ∈ **elems** ahist(ui) ⇒
126b. **let** hist = is_LI(ui) → attr_LHist(retr_L(ui))(σ),
126b. _ → attr_HHist(retr_H(ui))(σ) **in**
126b. τ ∈ **elems** hist(uid_A(a)) **end end**
127. ∧
127c. ∀ u:(L|H) • u ∈ *ls*∪*hs* ⇒
127c. **let** uhist = attr(L|H)Hist(u) **in**
127d. ∀ ai:AI • ai ∈ **dom** uhist ⇒
127d. ∀ τ:𝕋𝕀𝕄𝔼 • τ ∈ **elems** uhist(ai) ⇒
127d. **let** ahist = attr_AHist(retr_A(ai))(σ) **in**
127d. τ ∈ **elems** uhist(ai) **end end**

Please note, that *intents* are not [thought of as] attributes. We consider *intents* to be a fourth, a comprehensive internal quality of endurants. They, so to speak, govern relations between the three other internal quality of endurants: the unique identifiers, the mereologies and the attributes. That is, they predicate them, "arrange" their comprehensiveness. Much more should be said about intentionality. It is a truly, we believe, worthy research topic of its own ▪

Example 74 Aspects of Comprehensiveness of Internal Qualities. Let us illustrate the issues "at play" here.

- Consider a road transport system uod.
  - Applying analyse_intentionality(uod) may yield the set {"transport", ...}.
- Consider a financial service industry, fss.
  - Applying analyse_intentionality(fss) may yield the set {"interest on deposit", ...}.
- Consider a health care system, hcs.
  - Applying analyse_intentionality(hcs) may yield the set {"cure diseases", ...}.

What these analyses of intentionality yield, with respect to expressing intentional pull, is entirely of the discretion of the domain analysis & description ▪

We bring the above example, Example 74, to indicate, as the name of the example reveals, "Aspects of Comprehensiveness of Internal Qualities". That the various components of artefactual systems relate in – further to be explored – ways. In this respect, performing domain analysis & description is not only an engineering pursuit, but also one of research. We leave it to the readers to pursue this research aspect of domain analysis & description.

5.6.3 Artefacts

Humans create artefacts – for a reason, to serve a purpose, that is, with **intent**. Artefacts are like parts. They satisfy the laws of physics – and serve a *purpose*, fulfill an *intent*.

5.6.4 Assignment of Attributes

So what can we deduce from the above, almost three pages ?

The attributes of **natural parts** and **natural fluids** are generally of such concrete types – expressible as some **real** with a dimension[102] of the International System of Units: https://physics.nist.gov/cuu/Units/units.html. Attribute values usually enter into *differential equations* and *integrals*, that is, classical calculus.

The attributes of **humans**, besides those of parts, significantly include one of a usually non-empty set of *intents*. In directing the creation of artefacts humans create these with an intent.

Example 75 Intentional Pull – General Transport. These are examples of human intents: they create *roads* and *automobiles* with the intent of *transport*, they create *houses* with the intents of *living, offices, production*, etc., and they create *pipelines* with the intent of *oil* or *gas transport*.

Human attribute values usually enter into *modal logic* expressions.

5.6.5 Galois Connections

Galois Theory was first developed by Évariste Galois [1811-1832] around 1830[103]. Galois theory emphasizes a notion of **Galois connections**. We refer to standard textbooks on Galois Theory, e.g., [168, 2009].

5.6.5.1 Galois Theory: An Ultra-brief Characterisation

To us, an essence of Galois connections can be illustrated as follows:

- Let us observe[104] properties of a number of endurants, say in the form of attribute types.
- Let the function $\mathcal{F}$ map sets of entities to the set of common attributes.
- Let the function $\mathcal{G}$ map sets of attributes to sets of entities that all have these attributes.
- $(\mathcal{F},\mathcal{G})$ is a Galois Connection:
  - if, when including more entities, the common attributes remain the same or fewer, and
  - if when including more attributes, the set of entities remain the same or fewer.
  - $(\mathcal{F},\mathcal{G})$ is monotonously decreasing.

Example 76 LEGO Blocks. We[105] have

- There is a collection of LEGO™ blocks.
- From this collection, A, we identify the **red** square blocks, e.
- That is $\mathcal{F}(A)$ is B = {attr_Color(e) = **red**,attr_Form(e)=**square**}.
- We now add all the **blue** square blocks.
- And obtain A'.
- Now the common properties are their **squareness**: $\mathcal{F}(A')$ is B' = {attr_Form(e)=**square**}.
- More blocks as argument to $\mathcal{F}$ yields fewer or the same number of properties.
- The more entities we observe, the fewer common attributes they possess.

Example 77 Civil Engineering: Consultants and Contractors. Less playful, perhaps more seriously, and certainly more relevant to our endeavour, is this next example.

[102] Basic units are *m*eter, *kilo*gram, second, *A*mpere, *K*elvin, *mol*e, and *cand*ela. Some derived units are: Newton: $kg{\times}m{\times}s^{-2}$, Weber: $kg \times m^2 \times s^{-2} \times A^{-1}$, etc.

[103] en.wikipedia.org/wiki/Galois_theory

[104] The following is an edited version of an explanation kindly provided by Asger Eir, e-mail, June 5, 2020 [60, 89, 90].

[105] The E-mail, June 5, 2020, from Asger Eir

- Let X be the set of civil engineering, i.e., building, consultants, i.e., those who, like architects and structural engineers, design buildings – of whatever kind.
- Let Y be the set of building contractors, i.e., those firms who actually implement those designs.
- Now a subset, $X_{bridges}$ of X, contain exactly those consultants who specialise in the design of bridges, with a subset, $Y_{bridges}$, of Y capable of building bridges.
- If we change to a subset, $X_{bridges,tunnels}$ of X, allowing the design of both bridges **and** tunnels, then we obtain a corresponding subset, $Y_{bridges,tunnels}$, of Y.
- So when
  - we enlarge the number of properties from 'bridges' to 'bridges and tunnels',
  - we reduce, most likely, the number of contractors able to fulfill such properties,
  - and vice versa,
- then we have a Galois Connection[106].

5.6.5.2 Galois Connections and Intentionality – A Possible Research Topic ?

We have a hunch[107] ! Namely that there are some sort of Galois Connections with respect to intentionality. We leave to the interested reader to pursue this line of inquiry.

5.6.6 Discovering Intentional Pulls

The analysis and description of a domain's external qualities and the internal qualities of unique identifiers, mereologies and attributes can be pursued systematically – endurant sort by sort. Not so with the discovery of a domain's possible intentional pulls. Basically *"what is going on"* here is that the domain modeler considers pairs, triples or more part "independent"[108] endurants and reflects on whether they stand in an *intentional pull* relation to one another. We refer to Sects. 5.6.2.2 – 5.6.2.3.

5.6.7 Domain Laws wrt. Intentional Pull

A Domain Law 4 :

128 For every *intent* there is [therefore] a domain law that expresses the upholding of that intent.

```
128. axiom ∀ i:Intent • ∃ domain_law_i
```

5.7 On Modeling "Soft" Parts, II

We refer to this section's "predecessor" section, Sect. 4.6 on page 52.

We repeat here that section's figure:

[106] This was, more formally, shown in Dr. Asger Eir's PhD thesis [89].

[107] Hunch: a feeling or guess based on intuition rather than fact.

[108] By "independent" we shall here mean that these endurants are not 'derived' from one-another !

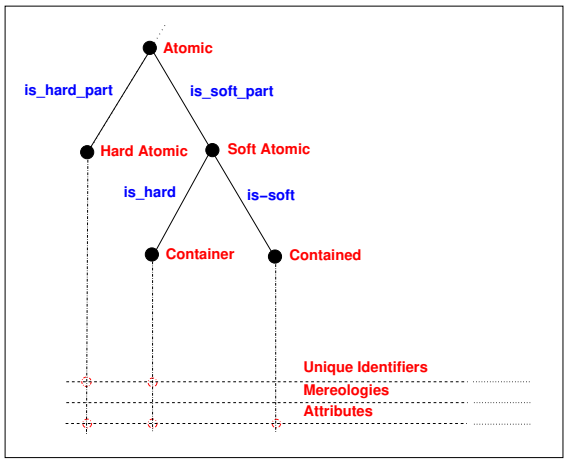

Fig. 5.4 Expanded segment of Fig. 4.1 on page 38 – same as Fig. 4.4 on page 53

In Sect. 4.6 on page 52 only the slanted lines of Fig. 4.4 on page 53 related to the text of that section. The text of this section now related to the vertical and horizontal lines of Fig. 5.4.

5.7.1 Unique Identification of Atoms

5.7.1.1 Hard Part Atoms

Hard part atoms are to be considered "ordinary" parts. As such they have unique identification.

5.7.1.2 Soft Part "Atoms"

[The use of double quotes signals that the soft part atoms are really Cartesians of a had container part and a soft contained endurant.]

Soft part atoms may, or may not be endowed with unique identification – as decided by the domain analyzer cum describer.

5.7.2 Mereology of Atoms

5.7.2.1 Hard Part Atoms

As ordinary parts they may be endowed with mereology.

5.7.2.2 Soft Part "Atoms"

Soft part atoms may, or may not be endowed with unique identification – as decided by the domain analyzer cum describer. If the analyzer cum describer decides to endow a soft part item, then it is the container part that "receives" this assignation.

5.7.3 Attributes of Atoms

5.7.3.1 Hard Part Atoms

As ordinary parts they may be endowed with attributes.

5.7.3.2 Soft Part "Atoms"

Soft parts attributes are assigned to both elements: the container and the contained.

Example 78 **A Model of Soft Part "Atoms": A Grocery Store, II.**

129 We decide to associate unique identifiers with *S* shelves, and *W*ares.
We are not interested in the mereologies of shelves and wares.
And we are also not interested in possible attributes of shelves.
130 Wares have attributes: *Name*, *Vol*ume, *Weight*, *Price*

```
type
129. SI, WI
value
129. uid_S: S→SI, uid_W: W→WI
type
130. Name, Vol, Weight, Price. ...
value
130. attr_Name: W→Name, attr_Vol: W→Vol, attr_Weight: W→Weight, attr_Price: W→Price, ...
```

5.8 A Domain Discovery Procedure, II

We continue from Sect. 4.9.

5.8.1 The Process

We shall again emphasize some aspects of the domain analysis & description method. A **method procedure** is that of *exhaustively analyse & describe* all internal qualities of the domain under scrutiny. A **method technique** implied here is that sketched below. The **method tools** are here all the analysis and description prompts covered so far.

Please be reminded of *Discovery Schema 0*'s declaration, Page 56, of *Notice Board* variables (Page 56). In this section we collect (i) the *description of unique identifiers* of all parts of the state; (ii) the *description of mereologies* of all parts of the state; (iii) the *description of attributes* of all parts of the state; and (iv) the *description of possible intentional pulls*. (v) We finally gather these into the *discover_internal_endurant_qualities* procedure.

Discovery Schema 2: An Internal Qualities Domain Modeling Process

```
value
    discover_uids: Unit → Unit
    discover_uids() ≡
        for ∀ v • v ∈ gen
            do txt := txt † [type_name(v)↦txt(type_name(v))⌢⟨describe_unique_identifier(v)⟩] end
    discover_mereologies: Unit → Unit
```

```
    discover_mereologies() ≡
        for ∀ v • v ∈ gen
            do txt := txt † [type_name(v)↦txt(type_name(v))⌢⟨describe_mereology(v)⟩] end
    discover_attributes: Unit → Unit
    discover_attributes() ≡
        for ∀ v • v ∈ gen
            do txt := txt † [type_name(v)↦txt(type_name(v))⌢⟨describe_attributes(v)⟩] end
    discover_intentional_pulls: Unit → Unit
    discover_intentional_pulls() ≡
        for ∀ (v′,v′′) • {v′,v′′} ⊆ gen
            do txt := txt † [type_name(v′)↦txt(type_name(v′))⌢⟨describe_intentional_pull()⟩]
                          † [type_name(v′′)↦txt(type_name(v′′))⌢⟨describe_intentional_pull()⟩] end
    describe_intentional_pull: Unit → ...
    describe_intentional_pull() ≡ ...


value
    discover_internal_qualities: Unit → Unit
    discover_internal_qualities() ≡
        discover_uids() ;
            axiom [ all parts have unique identifiers ]
        discover_mereologies() ;
            axiom [ all unique identifiers are mentioned in sum total of ]
                  [ all mereologies and no isolated proper sets of parts ]
        discover_attributes() ;
            axiom [ sum total of all attributes span all parts of the state ]
        discover_intentional_pulls()
```

5.8.2 A Suggested Analysis & Description Approach, II

Figure 5.5 on the next page possibly hints at an analysis & description order in which not only the external qualities of endurants are analysed & described, but also their internal qualities of unique identifiers, mereologies and attributes.

In Sect. 4.9 on page 56 we were concerned with the analysis & description order of endurants. We now follow up on the issue of (in Sect. 4.5.1.3 on page 48) on how compounds are treated: namely as both a "root" parts and as a composite of two or more "sibling" parts and/or fluids. The taxonomy of the road transport system domain, cf. Fig. 4.3 on page 49 and Example 36 on page 45, thus gives rise to many different analysis & description traversals. Figure 5.5 on the following page illustrates one such order.

Again, it is up to the domain engineer cum scientist to decide. If the domain modeler decides to not endow a compound "root" with internal qualities, then an 'internal qualities' traversal will not have to neither analyse nor describe those qualities.

5.9 Summary

Please consider Fig. 4.1 on page 38. This chapter has covered the horizontal and vertical lines to the left in Fig. 4.1.

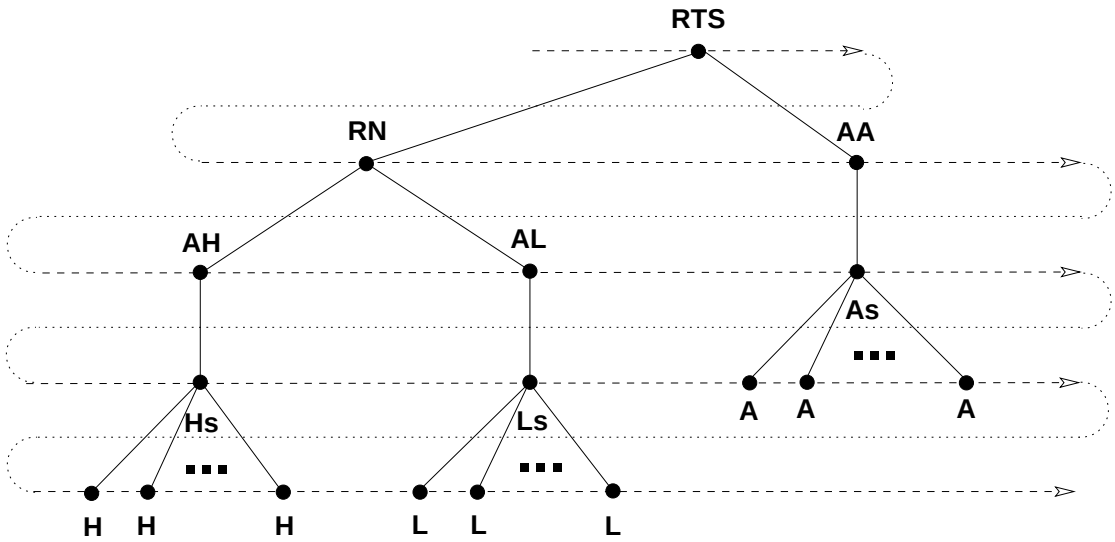

Fig. 5.5 A Breadth-First, Top-Down Traversal

Internal Qualities Predicates and Functions: Method Tools

- **Analysis Predicates**: As in Chapter 4 these predicates apply to endurants.
- **Attribute Analysis Predicates**: The predicates apply to attribute values.
- **Analysis Functions**: These functions yield appropriate values: unique identifiers and attribute type names.
- **Retrieval Function**: This function is generic. It applies to a unique part identifier and yields the part identified.
- **Description Functions**: There are three such functions: describing unique identifiers, mereologies and attributes.
- **Domain Discovery**: The procedure here being described, informally, guides the domain analyser cum describer to do the job!

Chapter 6
Perdurants

Contents

Please consider Fig. 4.1 on page 38. The previous two chapters covered the left of Fig. 4.1. This chapter covers the right of Fig. 4.1. 109

In this chapter we transcendentally "morph" manifest **parts** into **behaviours**, that is: **endurants** into **perdurants**. We analyse that notion, *perdurants*, and its constituent notions of **actors**, **channels** and **communication**, **actions** and **events** and **behaviours**.

Definition 89 **Behaviourable Parts.** By *behaviourable parts* we shall mean those parts for which the domain analyser cum describers have decided that they shall be transcendentally deduced into behaviours

▪

[109] **Editorial note:** – changes are made to the first paragraphs of this chapter

If a part, e:E, is designated 'behaviourable' then its sort E can be said to be a *behaviourable sort*.

We shall investigate the, as we shall call them, perdurants of domains. That is, state and time-evolving domain phenomena. The outcome of this chapter is that the reader will be able to model the perdurants of domains. Not just for a particular domain instance, but a possibly indefinite set of domain instances[110].

•••

In this chapter we shall analyse and describe *perdurants*, that is, the *entities* of domains for which only a fragment exists, in *space*, if we look at or touch them at any given snapshot in *time*.

This modeling will focus on the **actions**, **events** and **behaviours** of these perdurants and the means, here referred to as **channels**, by means of which the part behaviours interact.

On one hand there are the domain phenomena of perdurants. On the other hand there are means for analysing and describing these. The former are not formalized "before", or as, we analyse and describe them. The latter, 'the means', are assumed formalized.

•••

The **structure of this chapter** need be explained. The chapter attempts to motivate and explain the "morphing" of endurant parts into perdurant behaviours. The endurant parts were analysed and described in terms of RSL abstract types, i.e., sorts, and observers – of both external and internal qualities. The perdurant behaviours will be analysed and described in terms of tail-recursive functions, their signatures and 'body' definitions – and their ["output/input"] interaction by means of CSP output/input clauses and channels. To arrive at these analyses and descriptions we "move" from general motivation and text on behaviours, their actions and events and their reliance on channels, to increasingly more specific such text. Hence the seeming "repetition" of treatments of behaviours, actions, events and channels.

Primary Modeling Tool, II

The tool with which we describe perdurants will be the **tail recursive function** and the **channel** concepts of the formal specification language RSL [98]. A special focus will be on the *signature* of the action and behaviour function definitions.

Definition 90 Description, III. By a **description** of the perdurants of a domain we shall mean pairs of informal, narrative, and formal text which characterises the behaviour interaction channels, actors, actions, events and behaviours, i.e., the signatures, invocation and Initialisation of manifest part behaviours

This chapter explains what is meant by *channel*, *actor*, *behaviour*, *action*, *event*, *signature*, *invocation* and *initialisation*.

6.1 Parts and their Behaviours

By transcendental deduction we shall specifically me the "morphing"[111] of endurant parts into perdurant behaviours. We refer to Sect. 2.1.2's Example 2 on page 10.

6.1.1 General Notions

Parts are manifest entities of domains. Behaviours are likewise manifest entities of domains. Behaviours are domain notions. We shall express domain behaviours in terms of the RSL/CSP notions of *processes*.

[110] By this we mean: You are not just analysing a specific domain, say the one manifested around the corner from where you are, but any instance, anywhere in the world, which satisfies what you have described.

[111] Morph: change the form or character of ...

And we shall explain domain behaviours in terms of *domain actions*, *domain events*, and subsidiary, i.e., "embedded", *domain behaviours*.

Definition 91 Behaviour. We define domain behaviours as sets of sequences of *domain actions*, *domain events* and [subsidiary] *domain behaviours* ▪

Definition 92 Actor. An actor is anything that can invoke and sustain a behaviour, action or event ▪

Definition 93 Action. Actions are planned, purposeful state changes ▪

Definition 94 Event. Events are sporadic state changes ▪

By sporadic state changes we mean "something that happens or appears often, but not constantly or regularly" [OED] – something that is 'out of our control'. *Domain actions* are expressed in terms of the RSL notion of language clauses which prescribe state changes. *Domain events* are expressed in terms of the CSP notion of language clauses which prescribe interaction between CSP processes. *Domain behaviours*, to repeat, are expressed in terms of CSP processes, more specifically in terms of *tail recursive* function definitions.

Thus there are two notions: the domain notions of endurant parts and perdurant actions, events and behaviours, and the description language, here RSL/CSP, notions of part descriptions and expressions and statements, i.e., possibly state-changing processes.

6.1.2 An Aside: Behaviours versus Processes

In programming, in general, and in CSP in specific, we use the term *process* to characterize the execution of a set of sequences of [program] statements and processes[112].

Definition 95 Process. We define [computing] processes as sets of sequences of *state changes* (whether purposefully planned or accidental) ▪

In domains we use the term *behaviour*, in contrast, to characterize the *conduct*, the dynamics of the organization, as for endurants, and the *carrying out* of the *intentions* of a part.

6.1.3 Multiple, Communicating Behaviours

On the basis of Kai Sølander's Philosophy [161–165] we can reason that there is an indefinite number of parts; hence an indefinite number of "part" behaviours; and hence an indefinite number of CSP processes.

We shall further reason that these "part" behaviours interact.

There is the possibility that parts have dynamic attributes. For dynamic attribute values to change there must be an 'agent of change'. There is the possibility that two or more distinct parts interact.

Examples:
(i) *automobile*s enter and leave *hub*s and *link*s;
(ii) *retailer*s deposit funds in *bank*s;
(iii) *container vessel*s load and unload *container*s ▪

The part pairs: *(automobile,hub)*, *(automobile,link)*, *(retailer,bank)* and *(container vessel,container)*, are pairs of 'agents of change'.

This reasoning, by transcendental deduction, leads us to conclude that these mutual interactions can be seen as channel communications in the sense of, for example, CSP.

[112] Yes, we do mean a recursive definition.

6.1.4 Domain Behaviours and Domain Actions

What do we mean by behaviours and actions. First of all we must emphasize, as in Sect. 6.1.2 on the preceding page, that we are not dealing with with domains, not with computing, with domain behaviours, not with computing processes. Domains are not computers.[113] Then we focus of the conduct of domain actions of domain behaviours.

Generally speaking a **domain action**, of a part, is either updating the state, i.e., the mereology or dynamic attributes of the part of which the action is intended. Part actions are seen as "atomic", that is *"taking no time"* to "occur" — although they may be expressed in terms of *sub-actions*. We shall later elaborate on our notion of sub-actions.

Examples of transport domain actions were given above.

Correspondingly, a *domain behaviour*, of a part, is either a sequence of one or more *domain actions* or an *"alternative" set* of either internally, non-deterministically chosen (⨅) sequence of one or more *domain actions*, or externally, non-deterministically chosen (□) sequence of one or more *domain actions*, or combinations thereof. By *"alternative" set* we mean that each element of the set is one of the internally or externally, non-deterministically chosen sequences of domain actions.

Examples of domain behaviours are those of (a) an automobile which internally, non-deterministically (⨅) alternates between (i–v) above; or (b) a link which externally, non-deterministically (□) alternates between (b.i) welcoming incoming automobiles, (b.ii) accepting that automobiles remain on the link, (b.iii) "saying good bye" to outgoing automobiles; or (c) a hub which externally, non-deterministically (□) alternates between (c.i) welcoming incoming automobiles, (c.ii) accepting that automobiles remain at the hub, (c.iii) "saying good bye" to outgoing automobiles ▪

6.2 Channel Description

The CSP concept of *channel* is to be our way of expressing the "medium" in which behaviours interact. A channel is thus an abstract concept. Please do not think of it as a physical, an IT (information technology) device. As an abstract concept it is defined in terms of, roughly, the laws, the semantics, of CSP [114]. We write 'roughly' since the CSP we are speaking of, is "embedded" in RSL.

Definition 96 Channels. A channel is an abstract notion, not a physical "gadget". A channel is anything that allows behaviours to interact: to synchronise and communicate, i.e., exchange information ▪

We simplify the general treatment of channel declarations. Basically, all we can say, for any domain, is that any two distinct part behaviours may need to communicate. Therefore we declare a vector of channels indexed by sets of two distinct part identifiers.

Domain Description Prompt 7 describe_channels:

```
value
  describe_channels: Unit → RSL-Text
  describe_channels() ≡ ❝ channel { ch[{ij,ik}] | ij,ik:UI • {ij,ik}⊆ uid_σ ∧ ij≠ik } M ❞
```

Initially we shall leave the type, M, of messages over channels further undefined. As we, laboriously, work through the definition of behaviours, we shall be able to make M precise.

[113] Yes, we exclude from the domains, for which we put forward the calculi of this paper, such parts which acts like general computing devices.

6.3 Action and Event Description, I

The main tenet, i.e., "message", of this chapter, to recall, is that for every behaviourable part there is a behaviour.

For each [part] behaviour we identify the zero, one or more actions and events which that [part] behaviour initiates, respectively is subjected to. Actions, to recall, are planned, purposeful state changes. Events are surreptitious state changes. The actor, i.e., the [part] behaviour plan actions and await events.

Example 79 Road Transport – Actions.

- **Automobile Actions**:
  - 131 progress_around_hub,
  - 132 leave_hub_enter_link,
  - 133 disappears_from_road_net,
  - 134 progress_along_link,
  - 135 idle_on_link[114] and
  - 136 leave_link_enter_hub.
- **Hub Actions**: none !
- **Link Actions**: none !

We omit a number of actions: accellerate_auto, brake_auto, etc.

We continue our treatment of actions in Sect. 6.6 on page 109.

6.4 Perdurant Taxonomy

Perdurant taxonomies are suggested to be graphics that shows the relationships between behaviours and their interactions [by means of channel communications].

That is, a perdurant taxonomy can be rendered as a graph whose [rounded box] nodes designate [one or more] behaviours and whose [arrow pointed] edges designate channel communications between behaviours.

Example: Fig. 6.1 on the following page shows a perdurant taxonomy for a domain of multi-mode transport: The behaviours reflect customers who order the transport of merchandise from logistics and conveyor companies – with the latter managing conveyors: buses, trucks, trains, ship and aircraft – who move along the nodes and links of a transport net [a graph] . [115]

The domain analyser cum describers may wish to embellish perdurant taxonomy graphs with such texts and graphics that may, for example, "inform" of relations between communications: which may be triggered by others, and which trigger subsequent communications.

Figure 6.2 on page 107 suggests a perdurant taxonomy for a domain of customers, banks, portfolio managers, brokerages, [a] stock exchange and a financial industry "watchdog". The behaviour boxes are not rounded. These examples show the informality of perdurant taxonomy graphs.

6.5 Behaviour Signatures

Behaviours have to be described. Behaviour definitions are in the form of tail-recursive function definitions and are here expressed in RSL relying, very much, on its CSP component. The tail-recursion expresses that the behaviour goes on, potentially "forever" ! Behaviour definitions describe the type of the arguments that the function, i.e., the behaviour, accepts.

[114] We do not consider automobiles idling in hubs.

[115] The example of Fig. 6.1 on the next page is taken from [53, *Transport – A Domain Description*].

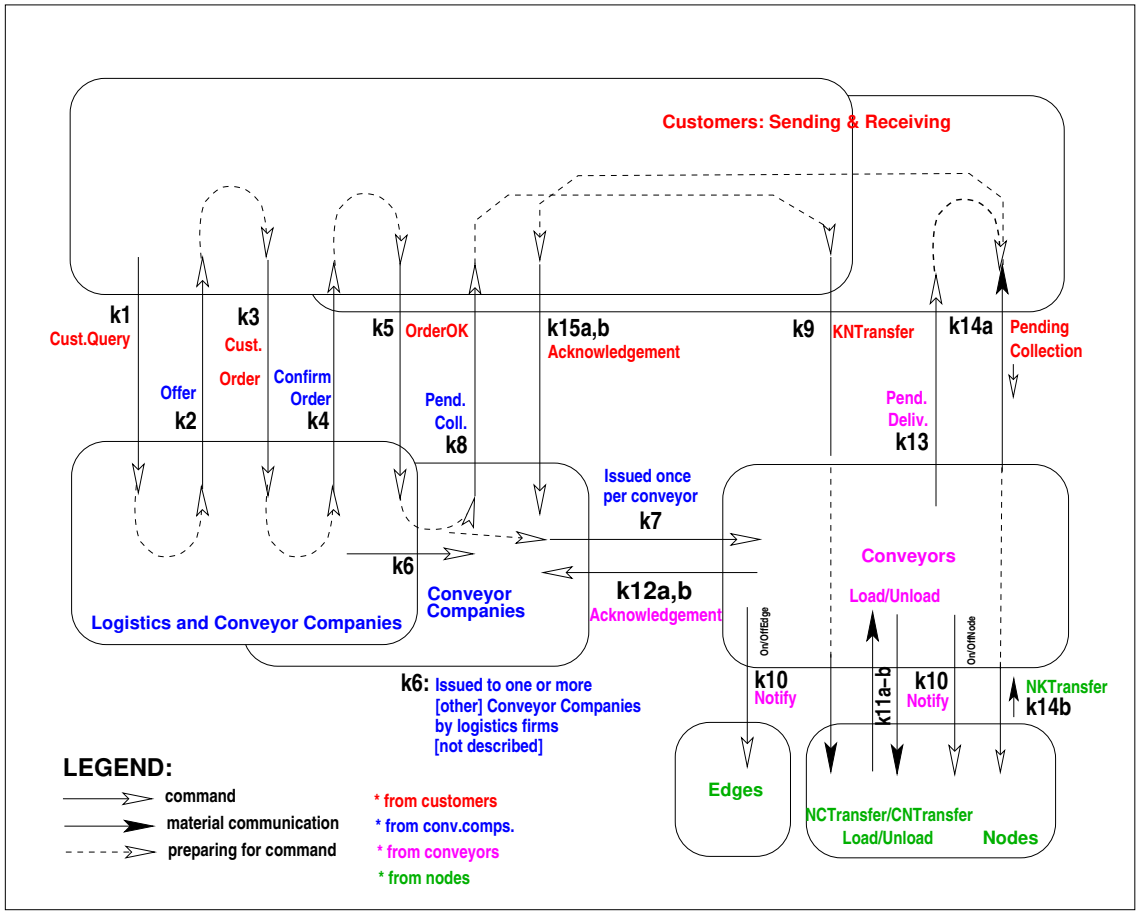

Fig. 6.1 A Multi-mode Transport Perdurant Taxonomy

Thus there are two elements to a behaviour definition: (i) the behaviour *signature* and (ii) the behaviour *body* definitions.

Behaviour signatures indicate that behaviours evolve around (a) the internal qualities of the part from which the behaviour is transcendentally deduced, and (b) the interaction with other [part] behaviours. Thus there are basically two elements to behaviour signatures: (α) the *unique identifier, mereology* and *attributes* element, and (β) the element of *channel* interface to potentially interacting other behaviours.

Definition 97 **Signature**. A signature is something that is associated with any behaviour and, hence, action. A signature consists of two parts: a name for the behaviour or action; and a function type expression, the latter is typically of the form $A \rightarrow B$, where A is the type of the behaviour or action input [arguments], and B is the type of the behaviour or action output, i.e., result ▪

The next **perdurant description functions** are those of the description of the [part] behaviour and [part] action signatures.

We shall develop a variety of possible behaviour and action signatures. From a very simple to a full-fledged "traditional"signatures.

In all cases the [part] behaviour (and [part] action) definitions — such as we have chosen them — evolve around the part *"from which they are transcendentally deduced"*, and the interaction with other [part] behaviours, what else could there be ?

We shall illustrate two kinds of signatures: simple part argument signatures which imply that the behaviour so-to-speak "carries" the whole part "with it"; and internal quality argument signatures which

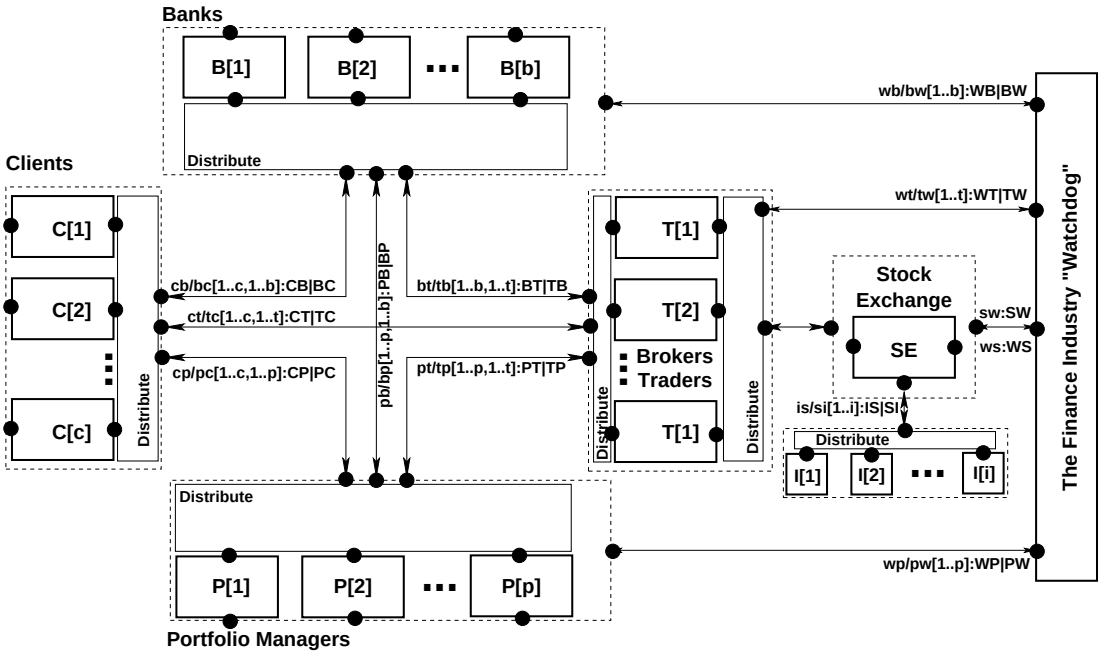

Fig. 6.2 A Stock Brokerage Perdurant Taxonomy

imply that the behaviour, in the conventional manner of program procedure definitions has a number of internal quality constant, or variable, or reference arguments.

6.5.1 Part Argument Behaviour Signatures

The simplest behaviour signature expresses that the behaviour, as a function definition, as argument, takes a part of the sort for which the behaviour is defined. For each manifest part sort there is one behaviour definition.

The behaviour definition for a sort applies to all instantiations of behaviours of that sort.

A very simplified part behaviour description is given in the "pseudo"-prompt – "pseudo" because it is really not a proper prompt in that it omits treatment of internal quality part argument.

Domain Description Prompt 8 pseudo-describe_behaviour_signature(p), **I:**
value
describe_behaviour_signature: "P → ... channels **Unit**"

channels is an expression of channels based on the mereology of P. **Unit** designates the value (), i.e., a state-to-state function.

Example 80 Signature. A schematic pseudo-example:

value
automobile: a:A → **out** {ch[ai,ri]|ai:AI•ai=**uid**_A(a)∧ri:(HI|LI)•ri ∈ *his*∪*lis*}.

6.5.2 Internal Quality Argument Behaviour Signatures

The internal quality argument behaviour signature "unfolds" the internal part qualities into separate arguments: together these arguments "substitute" for the simple behaviour and action part argument.

The internal quality arguments are: the unique part identifier, the part mereology, the static attributes argument, the monitorable attribute names argument, and the programmable attributes argument. The signature furthermore names the behaviour, the channels, and the state-to-state function designator **Unit**.

A schematic form of part (p) behaviour signatures is:

Domain Description Prompt 9 schematic_describe_behaviour_signature, **II:**

```
value
  schematic_describe_behavior_signature(p) ≡
    ❝ b: bi:BI→me:Mer→svl:StaV*→mvl:MonV*→prgl:PrgV* channels Unit ❞
```

We shall explain the general form of part behaviour, B, signatures, "step-by-step":

α.	b	the [chosen] name of part p behaviours.
β.	U→V→...→W→Z:	The function signature is expressed in the Schönfinkel/Curry[116] style – corresponding to the invocation form F(u)(v)...(w)
γ.	bi:BI:	a general value and the type of part p unique identifier
δ.	me:Mer:	a general value and the type of part p mereology
ϵ.	svl:StaV*:	a general (possibly empty) list of values and types of part p's (possibly empty) list of static attributes
ζ.	mvl:MonV*:	a general list of names of types of part p's (possibly empty) list of monitorable attributes
η.	prgl:PrgV*:	a general list of values and types of part p's (possibly empty) list of programmable attributes
θ.	channels:	are usually of the form: {ch[{i,j}]|(i,j)∈I(me)} and express the subset of channels over which behaviour Bs interact with other behaviors
ι.	**Unit**:	designates the single value, (), **Unit**

In detail:

α. **Behaviour name:** In each domain description there are many sorts, B, of parts. For each sort there is a generic behaviour, whose name, here b, is chosen to suitably reflect B.

β. **Currying** is here used in the pragmatic sense of grouping "same kind of arguments", i.e., separating these from one-another, by means of the →s.

γ. The **unique identifier** of part sort B is here chosen to be BI. Its value is a constant.

δ. The **mereology** is a usually constant. For same part sorts it may be a variable.

Example 81 Variable Mereologies. For a road transport system where we focus on the transport the mereology is a constant. For a road net where we focus on the development of the road net: building new roads: inserting and removing hubs and links, the mereology is a variable. Similar remarks apply to canal systems www.imm.dtu.dk/~dibj/2021/Graphs/Rivers-and-Canals.pdf, pipeline systems [34], container terminals [42], assembly line systems [47], etc. ▪

ϵ. **Static attribute values** are constants. The use of static attribute values in behaviour body definitions is expressed by an identifier of the svl list of identifiers.

ζ. **Monitorable attribute values** are generally, ascertainable, i.e., readable, cf. Sect. 5.4.3.1 on page 79. Some are *biddable*, can be changed by a, or the behaviour, cf. Sect. 5.4.3.2 on Page 79, but there is no

[116] Moses Schönfinkel (1888–1942) was a logician and mathematician accredited with having invented combinatory logic [https://en.wikipedia.org/wiki/Moses_Schönfinkel]. Haskell B. Curry (1900–1982) was a mathematician and logician known for his work in combinatory logic [https://en.wikipedia.org/wiki/Haskell_Curry]

guarantee, as for programmable attributes, that they remain fixed. The use of a[ny] monitorable attribute value in behaviour body definitions is expressed by a read_A_from_P(mv,bi) where mv is an identifier of the mvl list of identifiers and bi is the unique part identifier of the behaviour definition in which the read occurs. The update of a biddable attribute value in behaviour body definitions is expressed by a update_P_with_A(bi,mv,a).

η. **Programmable attribute values** are just that. They vary as specified, i.e., "programmed", by the behaviour body definition. Tail-recursive invocations of behaviour B_i "replace" relevant programmable attribute argument list elements with "new" values.

θ. **channels**: $I(\text{me})$ expresses a set of unique part identifiers different from bi, hence of behaviours, with which behaviour b(i) interacts.

ι. The **Unit** of the behaviour signature is a short-hand for the behaviour either **read**ing the value of a monitorable attribute, hence global state σ, or performing a **write**, i.e., an *update*, on σ.

6.6 Action Signatures and General Form of Action Definitions

Actions come in basically one signature form, like for behaviours, cf. Sect. 6.5.2 on the facing page:

137 likewise determine every action of that [part] behaviour.

And the action definition, i.e., the "body", is of the general form

138 first a description, act, of the action proper, one in which both the part mereology and the programmable attributes may be changed,

139 then the tail-recursive invocation of the possibly updated [part] behaviour.

Domain Description Prompt 10 describe_action:

```
value
137.  action: bi:BI → mer:Mer → svl:StaV* → mvl:MonV* → prgl:PrgV* channels Unit
138.  act: bi:BI → mer:Mer → svl:StaV* → mvl:MonV* → prgl:PrgV* channels Unit
137.  action(bi)(mer)(svl)(mvl)(prgl) ≡
138.    let (mer′,prgl′) = act(bi)(mer)(svl)(mvl)(prgl) in
139.    behaviour(bi)(mer′)(svl)(mvl)(prgl′) end
```

140 The act is of basically three forms:
- a either it is an "active" form in which it initiates an interaction with another behavior, bj,
- b or it is a "passive" form in which it awaits an interaction with another behavior, bj,
- c or it is neither, ie., it is some simple RSL clause.

```
140a.  act: ... ch[{bi,bj}] ! val ...
140b.  act: ... let id = ch[{bi,bj}] ? in ... end ...
140c.  act: ...
```

Example 82 Automobile, Hub and Link Signatures.

141 automobile:
- a There is the usual "triplet" or arguments: unique identifier, mereology, static (...) and monitorable (...) attributes;
- b programmable attributes: automobile location and automobile history;
- c and channel references allowing communication between the automobile and the hub and link behaviours.

142 Similar for hubs and link behaviours.

```
value
141a automobile: ai:AI → (_,uis):AM → ... → ...
141b       → (A_Loc × A_Hist)
141c         out {ch[{ai,ui}]|ui:(HI|LI) • ui ∈ hisUlis} Unit
142a    hub: hi:HI → (lis,ais,rni):HM → (HΩ × ...) → ...
142b       → (HΣ×H_Hist)
142c         in {ch[{hi,ui}]|ui:(LI|HI|RNI)-set • ui ∈ lisUhisUrni} Unit
142a    link: li:LI → (his,ais,rni):LM → (LEN × LΩ × ...)
142b       → (LΣ×L_Hist)
142c         in {ch[{li,ui}]|ui:(LI|HI|RNI)-set • li ∈ lisUhisUrni} Unit
```

6.7 Behaviour Invocation

Definition 98 Invocation. By action or behaviour, i.e., function, invocation we shall understand the act of initiating, invoking, that which is prescribed by the action or behaviour definition ▪

The general form of behaviour invocation is shown below. The invocation follows the "Currying" of the behaviour type signature. [Normally one would write all this on one line: b(i)(m)(s)(m)(p).]

```
behaviour
    (unique_identifier)
        (mereology)
            (static_values)
                (monitorable_attribute_names)
                    (programmable_variables)
```

When first "invoked", that is, transcendentally deduced, i.e., "morphed", from a manifest part, *p*, the invocation looks like:

Domain Description Prompt 11 describe_behaviour_signature, **II**:

```
value
    describe_behaviour_signature: P → RSL-Text
    describe_behaviour_signature(p) ≡
    ❝ behaviour: UId → Mereo → StaVL → MonVL → ProVL → channels Unit
      behaviour(uid_B(p))
                (mereo_B(p))
                    (types_to_values(static_attribute_types(p)))
                        (mon_attribute_types(p))
                            (types_to_values(programmable_attribute_types(p)))
        pre: is_manifest(p) ❞

    describe_behaviour_signatures: Unit → RSL-Text
    describe_behaviour_signatures() ≡
        { describe_behaviour_signature(p) | p ∈ σ ∧ is_manifest(p) }
```

6.8 Behaviour Definition Bodies

Definition 99 Behaviour Definition. By behaviour definition we shall understand the prescription that characterises the behaviour ▪

In general a behaviour alternates between a number, m, of actions, act_action_i, that either actively initiates interaction with other behaviours or do not engage in interactions, or a number, n, of actions, pas_action_j, that passively seek such interaction. The alternation between the former is **internal non-deterministic**, ⌈⌉, i.e., it is the behaviour that determines in which alternative to engage. The alternation between the latter is **external non-deterministic**, □, i.e., it is the behaviour that determines in which alternative to engage.

6.8.1 Behaviour Patterns

Experimental evidence, cf. [10], has shown that domain behaviours "come in a few variants", i.e., "follow a certain pattern". By a *behaviour definition pattern* we shall understand a structure, a schema, of RSL/CSP clauses that is used "repeatably" in order to express behaviours of domain behaviours ▪ We can also refer to the recent [133].

6.8.1.1 Behaviour Definition Schema I

In Schema I some lines designate non-deterministic actions, other deterministic actions.

```
value
  b(bi)(me)(svl)(mvl)(prl) ≡
      non−deterministic_action_1(bi)(me)(svl)(mvl)(prl)
    ⌈⌉ non−deterministic_action_2(bi)(me)(svl)(mvl)(prl)
       ...
    ⌈⌉ non−deterministic_action_n(bi)(me)(svl)(mvl)(prl)
    □ deterministic_action_1(bi)(me)(svl)(mvl)(prl)
    □ deterministic_action_2(bi)(me)(svl)(mvl)(prl)
       ...
    □ deterministic_action_d(bi)(me)(svl)(mvl)(prl)
```

6.8.1.2 Behaviour Definition Schema II

In Schema II we have made use of Domain Description Prompt 10's explication of action behaviours.

```
value
  b(bi)(me)(svl)(mvl)(prl) ≡
    let (me′,prl′) = non−deterministic_act_1(bi)(me)(svl)(mvl)(prl)
                   ⌈⌉ non−deterministic_act_2(bi)(me)(svl)(mvl)(prl)
                   ...
                   ⌈⌉ non−deterministic_act_n(bi)(me)(svl)(mvl)(prl)
                   □ deterministic_act_1(bi)(me)(svl)(mvl)(prl)
                   □ deterministic_act_2(bi)(me)(svl)(mvl)(prl)
                   ...
                   □ deterministic_act_d(bi)(me)(svl)(mvl)(prl) in
    b(bi)(me′)(svl)(mvl)(prl′) end
```

6.8.2 Describe Behaviour Definition Bodies

In other words, for current lack of a more definitive methodology for "describing" the bodies of behaviour definitions we resort to "..." ! To describe the behaviour of a set of parts we apply the above two schemes:

Domain Description Prompt 12 describe_behaviour_definition[s]:

```
value
    describe_behaviour_definition: P → RSL-Text
    describe_behaviour_definition(p) ≡ [Scheme I or Schema II]

    describe_behaviour_definitions: Unit → RSL-Text
    describe_behaviour_definitions() ≡
        { describe_behaviour_definition(p) | p ∈ σ ∧ is_manifest(p) }
```

6.9 Behaviour, Action and Event Examples

Example 83 **Automobile Behaviour**. We remind the reader of the main, running example of this primer, the of *the road transport system* Example[117].

Definitions: Automobile at a Hub

143 We abstract automobile behaviour at a Hub (hi).
144 Internally non-deterministically, an automobile
145 either progresses around the hub
146 or leaves the hub to enter a link.

```
143  automobile(ai)(aai,uis)(...)(apos:atH(fli,hi,tli),ahist) ≡
145,131     automobile_progress_around_hub(ai)(aai,uis)(...)(apos:atH(fli,hi,tli),ahist)
144         ⌈⌉
146,132     automobile_leave_hub_enter_link(ai)(aai,uis)(...)(apos:atH(fli,hi,tli),ahist)
```

147 [131] The automobile progresses around the hub:

a the automobile at that hub,
b informing ("first") the hub behaviour.

```
147,131  automobile_progress_around_hub(ai)(aai,uis)(...)(atH(fli,hi,tli),ahist) ≡
147      let τ = record_TIME() in
147b      ch[ai,hi] ! τ ;
147a      automobile(ai)(aai,uis)(...)(atH(fli,hi,tli),upd_hist(τ,hi)(ahist))
147      end

147a  upd_hist: (TIME×UI) → (AHist→AHist)|(HHist→HHist)|(LHist→LHist)
147a  upd_hist(τ,ui)(hist) ≡ hist † [ ui ↦ ⟨τ⟩⌢hist(ui) ]
```

[117] That is, for example, examples 28 on page 36, 35 on page 43, 36 on page 45, 37 on page 48, 44 on page 62, 45 on page 64, 48 on page 66, 49 on page 66, 50 on page 68, 51 on page 68, 61 on page 75, 62 on page 75, 63 on page 76, and 73 on page 93.

148 The automobile leaves the hub entering a link:

a tli, whose "next" hub, identified by thi, is obtained from the mereology of the link identified by tli;
b informs the hub it is leaving and the link it is entering,
c "whereupon" the vehicle resumes (i.e., "while at the same time" resuming) the vehicle behaviour positioned at the very beginning (0) of that link.

148 automobile_leave_hub_enter_link(ai)(aai,uis)(...)(apos:atH(fli,hi,tli),ahist) ≡
148a (**let** ({fhi,thi},ais) = mereo_L(retr_L(tli)(σ)) **in assert:** fhi=hi
148b (ch[ai,hi] ! τ ‖ ch[ai,tli] ! τ) ;
148c automobile(ai)(aai,uis)(...)(onL(tli,(hi,thi),0),upd_hist(τ,tli)(upd_hist(τ,hi)(ahist))) **end**)

149 [133] Or the automobile "disappears — off the radar" !

149,133 automobile_stop(ai)(aai,uis),(...)(apos:atH(fli,hi,tli),ahist) ≡ **stop**

Similar behaviour definitions can be given for *automobiles on a link*, for *link*s and for *hub*s. Together they must reflect, amongst other things: the time continuity of automobile flow, that automobiles follow routes, that automobiles, links and hubs together adhere to the intentional pull expressed earlier, et cetera. A specification of these aspects must be proved to adhere to these properties.

6.10 On Modeling "Soft" Parts, III

We can now reveal – what we have, in a sense, "known" from the outset of our investigation into the *modeling of "soft" parts,* namely – that we, in our search for and domain models of many domains [48], have **not** found a need to model soft parts as behaviours.

There is, however, an area in which soft parts are modeled. That is, as "carried', i.e., transported, by human, automobile, ship or aircraft, or other behaviours. We suggest to include such soft parts, as attributes of these "carriers".

Thus we can now conclude our investigation, in Sects. 4.6 on page 52, 5.7 on page 96 and in this section, Sect. 6.10. Perhaps a little tame, but then their existence is included !

6.11 Domain [Behaviour] Initialisation

Definition 100 Domain Initialisation. By behaviour initialisation we shall understand the [initial] invocation of all part behaviours ▪

For every manifest part sort there is a single description: signature and definition (i.e., its syntax). For every manifest part there is a behaviour (i.e., its semantics "realization"). For the total of all manifest domain parts there is their initialization: the parallel "execution" of the behaviour of each manifest part, properly initialized.

Domain Description Prompt 13 describe_domain_initialisation:

```
❝ ‖ { b
      (uid_P(p))
        (mereo_P(p))
          analyse_static_attribute_type_names_Cartesian(p)
            analyse_monitorable_attribute_type_names_Cartesian(p)
              analyse_programmable_attribute_type_names_Cartesian(p)
                | p:P • p ∈ σ } ❞
```

Example 84 The Road Transport System Initialisation. We "wrap up" the main example of this primer. We omit treatment of monitorable attributes.

150 Let us refer to the system initialisation as a behaviour.
151 All links are initialised,
152 all hubs are initialised,
153 all automobiles are initialised,
154 etc.

```
value
  rts_initialisation: Unit → Unit
  rts_initialisation() ≡
    || { link(uid_L(l))(mereo_L(l))(attr_LEN(l),attr_LΩ(l))(attr_L_Traffic(l),attr_LΣ(l))| l:L • l ∈ ls }
    || || { hub(uid_H(l))(mereo_H(l))(attr_HΩ(l))(attr_H_Traffic(l),attr_HΣ(l))| h:H • h ∈ hs }
    || || { automobile(uid_A(a))(mereo_A(a))(attr_RegNo(a))(attr_APos(a)) | a:A • a ∈ as }
    || ...
```

We have here omitted possible monitorable attributes. We refer to *ls*: Item 36 on page 55, *hs*: Item 37 on page 55, and *as*: Item 38 on page 55 ▪

6.12 Discrete Dynamic Domains

Up till now our analysis & description of a domain, has, in a sense, been *static:* in analysing a domain we considered its entities to be of a definite number. In this section we shall consider the case where the number of entities change: where new entities are *created* and existing entities are *destroyed*, that is: where new parts, and hence behaviours, arise, and existing parts, and hence behaviours, cease to exist.

6.12.1 Create and Destroy Entities

In the domain we can expect that its behaviours create and destroy entities.

Example 85 Creation and Destruction of Entities. In the *road transport* domain new hubs, links and automobiles may be inserted into the road net, and existing links, hubs and automobiles may be removed from the road net. In a *container terminal* domain [22,42] new containers are introduced, old are discarded; new container vessels are introduced, old are discarded; new ship-to-shore cranes are introduced, old are discarded; et cetera. In a *retailer* domain [46] new customers are introduced, old are discarded; new retailers are introduced, old are discarded; new merchandise is introduced, old is discarded; et cetera. In a *financial system* domain new customers are introduced, old are discarded; new banks are introduced, old are discarded; new brokers are introduced, old are discarded; et cetera ▪

The issue here is: When hubs and links are inserted or removed the mereologies of "neighbouring" road elements change, and so does the mereology of automobiles. When automobiles are inserted or removed the mereology of road elements have to be changed to take account of the insertions and removals, and so does the mereology of automobiles. And, some domain laws must be re-expressed: The domain part state, σ, must be updated[118], and so must the unique identifier state, uid_σ[119].

[118] Cf. Sect. 4.8.2 on page 56
[119] Cf. Sect. 46 on page 65

6.12.1.1 Create Entities

It is taken for granted here that there are behaviours, one or more, which take the initiative to and carry out the creation of specific entities. Let us refer to such a behaviour as the "creator". To create an entity implies the following three major steps [A.–C.] the step wise creation of the part and initialisation of the transduced behaviour, and [D.] the adjustment of all such part behaviours that might have their mereologies and attributes updated to accept such requests from creators.

A. To decide on the part sort – in order to create that part – that is
   - to obtain a unique identifier – one hitherto not used;
   - to obtain a mereology, one
     - according to the general mereology for parts of that sort,
     - and how the part specifically is to "fit" into its surroundings;
   - to obtain an appropriate set of attributes:
     - again according to the attribute types for that part sort
     - and, more specifically, choosing initial attribute values.
   - This part is then "joined" to the global part state, σ[120] and
   - its unique identifier "joined" to the global unique identifier state, uid_σ[121].

B. Then to transcendentally deduce that part into a behaviour:
   - initialised (according to Sect. 6.5) with
     - the unique identifier,
     - the mereology, and
     - the attribute values
   - This behaviour is then invoked and "joined" to the set of current behaviours, cf. Sect. 6.11 on page 113 – i.e., just above !

C. Then, finally, to "adjust" the mereologies of topologically or conceptually related parts,
   - that is, for each of these parts to update:
   - their mereology and possibly some
   - state and state space

   arguments of their corresponding behaviours.

D. The update of the mereologies of already "running" behaviours requires the following:
   - that, potentially all, behaviours offers to accept
   - mereology update requests from the "creator" behaviour.

The latter means, practically speaking, that each part/behaviour which may be subject to mereology changes externally non-deterministically expresses an offer to accept such a change.

Example 86 Road Net Administrator. We introduce the road net behaviour – based on the road net composite part, RN.

155 The road net has a programmable attribute: a *road net (development & maintenance) graph.*[122] The road net graph consists of a quadruple: a map that for each hub identifier records "all" the information that the road net administrator deems necessary[123] for the maintenance and development of road net hubs; a map that for each link identifier records "all" the information[124] that the road net administrator deems necessary for the maintenance and development of road net links; and a map from the hub identifiers to the set of identifiers of the links it is connected to, and the set of all automobile identifiers.

[120] Cf. Sect. 4.8.2 on page 56

[121] Cf. Sect. 46 on page 65

[122] The presentation of the road net Behaviour, rn, is simplified.

[123] We presently abstract from what this information is.

[124] See footnote 123.

156 This graph is commensurate with the actual topology of the road net.

type
155. G = (HI $\overrightarrow{m}$ H_Info) × (LI $\overrightarrow{m}$ L_Info) × (HI $\overrightarrow{m}$ LI-**set**) × AI-**set**
value
155. attr_G: RN → G
axiom
155. ∀ (hi_info,li_info,map,ais):G •
155. **dom** map = **dom** hi_info = *his* ∧ ∪ **rng** map = **dom** li_info = *lis* ∧
156. ∀ hi:HI • hi ∈ **dom** hi_info ⇒
156. **let** h:H • h ∈ σ ∧ uid_H(h)=hi **in**
156. **let** (lis′,...) = mereo_H(h) **in** lis′ = map(hi)
156. ais ⊆ $a_{ui}s$[125] ∧ ...
156. **end end**

Please note the fundamental difference between the *road net (development & maintenance) graph* and the road net. The latter pretends to be "the real thing". The former is "just" an abstraction thereof!

157 The road net [administrator] behaviour,
158 amongst other activities (...)
159 internal non-deterministically decides upon

a either a hub insertion,
b or a link insertion,
c or a hub removal,
d or a link removal;

These four sub-behaviours each resume being the road net behaviour.

value
157. rn: RNI → RNMer → G → **in**,**out**{ch[{i,j}]|{i,j}⊆uid_σ}
157. rn(rni)(rnmer)(g) ≡
158. ...
159a. ∏ insert_hub(g)(rni)(rnmer)
159b. ∏ insert_link(g)(rni)(rnmer)
159c. ∏ remove_hub(g)(rni)(rnmer)
159d. ∏ remove_link(g)(rni)(rnmer)

Details on the insert and remove actions are given below.

160 These road net sub-behaviours require information about

a a hub to be inserted: its initial state, state space and [empty] traffic history, or
b a link to be inserted: its length, initial state, state space and [empty] traffic history, or
c a hub to be removed: its unique identifier, or
d a link to be removed: its unique identifier.

type
160. Info == nHInfo | nLInfo | oHInfo | oLInfo
160. nHInfo :: HΣ × HΩ × H_Traffic
160. nLInfo :: LEN × LΣ × LΩ × L_Traffic
160. oHInfo :: HI
160. oLInfo :: LI ▪

[125] The $a_{ui}s$ was defined in Sect. 5.2.3, Item 49 on Page 65.

Example 87 Road Net Development: Hub Insertion. Road net development alternates between design, based on the *road net (development & maintenance) graph*, and actual, "real life", construction taking place in the real surroundings of the road net.

161 If a hub insertion then the road net behaviour, based on the hub and link information and the road net layout in the *road net (development & maintenance) graph* selects

- a an initial mereology for the hub, h_mer,
- b an initial hub state, hσ, and
- c an initial hub state space, hω, and
- d an initial, i.e., empty hub traffic history;

162 updates its *road net (development & maintenance) graph* with information about the new hub,
163 and results in a suitable grouping of these.

```
value
161.  design_new_hub: G → (nHInfo×G)
161.  design_new_hub(g) ≡
161a.     let h_mer:HMer = M_ih(g),
161b.         hσ:HΣ = S_ih(g),
161c.         hω:HΩ = O_ih(g),
161d.         h_traffic = [],
162.          g′ = MSO_ih(g) in
163.      ((h_mer,hσ,hω,h_traffic),g′) end
```

We leave open, in Items 161a–161c, as to what the initial hub mereology, state and state space should be initialised, i.e., the $\mathcal{M}_{ih}$,$\mathcal{S}_{ih}$,$\mathcal{O}_{ih}$ and $\mathcal{MSO}_{ih}$ functions.

164 To insert a new hub the road net administrator

- a first designs the new hub,
- b then selects a hub part
- c which satisfies the design,
  whereupon it updates the global state
- d of parts σ,
- e of unique identifiers, and
- f of hub identifiers –

in parallel, and in parallel with
165 initiating a new hub behaviour
166 and resuming being the road net behaviour.

```
164.  insert_hub: G×RNI×RNMer → Unit
164.  insert_hub(g,rni,rnmer) ≡
164a.     let ((h_mer,hσ,hω,h_traffic),g′) = design_new_hub(g) in
164b.     let h:H • h∉σ •
164c.               mereo_H(h)=h_mer ∧ hσ=attr_HΣ(h) ∧
164c.               hω=attr_HΩ(h) ∧ h_traffic=attr_HTraffic(h) in
164d.     σ := σ ∪ {h}
164e.  ‖ uid_σ := uid_σ ∪ {uid_H(h)}
164f.  ‖ his := his ∪ {uid_H(h)}
165.   ‖ hub(uid_H(h))(attr_HΣ(h),attr_HΩ(h),attr_HΩ(h))
166.   ‖ rn(rni)(rnmer)(g′)
164.    end end ▪
```

Example 88 Road Net Development: Link Insertion.

167 If a link insertion then the road net behaviour based on the hub and link information and the road net layout in the *road net (development & maintenance) graph* selects

a the mereology for the link, h_mer[126],
b the (static) length (attribute),
c an initial link state, lσ,
d an initial link state space lω, and
e and initial, i.e., empty, link traffic history;

168 updates its *road net (development & maintenance) graph* with information about the new link,
169 and results in a suitable grouping of these.

```
value
167.  design_new_link: G → (nLInfo×G)
167.  design_new_link(g) ≡
167a.    let l_mer:LMer = 𝓜il(g),
167b.        le:LEN = 𝓛il(g),
167c.        lσ:LΣ = 𝓢il(g),
167d.        lω:LΩ = 𝓞il(g),
167e.        l_hist:L_Hist = [ ]
168.         g′:G = 𝓜𝓛𝓢𝓞il(g) in
169.     ((l_mer,le,lσ,lω,l_hist),g′) end
```

We leave open, in Items 167a–167d, as to what the initial link mereology, state and state space should be initialised.

170 To insert a new link the road net administrator

a first designs the new link,
b then selects a link part
c which satisfies the design,
  whereupon it updates the global states
d of parts, σ,
e of unique part identifiers, and
f of link identifiers –

in parallel, and in parallel with
171 initiating a new link behaviour and
172 updating the mereologies and possibly the state and the state space attributes of the connected hubs.

```
value
170.  insert_link: G → Unit
170.  insert_link(rni,l) ≡
170a.    let ((l_mer,le,lσ,lω,l_traffic_hist),g′) = design_new_link(g) in
170c.    let l:L • l∉σ • mereo_L(l)=l_mer ∧
170c.                le=attr_LEN(l) ∧ lσ=attr_LΣ(l) ∧
170c.                lω=attr_LΩ(l) ∧ l_traffic_hist=attr_HTraffic(l) in
170d.    σ := σ ∪ {l}
170e.  ‖ uidσ := uidσ ∪ {uid_L(l)}
170f.  ‖ lis := list ∪ {}
171.   ‖ link(uid_L(l))(l_mer)(le,lω)(lσ,l_traffic)
172.   ‖ ch[{rni,hi1}] ! updH(𝓜il(g),Σil(g),Ωil(g))
172.   ‖ ch[{rni,hi2}] !
170.     end end ▪
```

We leave undefined the mereology and the state σ and state space ω update functions.

[126] that is, the two existing hub identifiers between whose hubs the new link is to be inserted

6.12.1.2 Destroy Entities

The introduction to Sect. 6.12.1.1 on page 115 on the *creation of entities* outlined a number of creation issues ([A, B, C, D]). For the *destruction of entities* description matters are a bit simpler. It is, almost, simply a matter of designating, by its unique identifier, the entity: part and behaviour to be destroyed. Almost ! The mereology of the destroyed entity must be such that the destruction does not leave "dangling" references !

Example 89 Road Net Development: Hub Removal.

173 If a hub removal then the road net design_remove_hub behaviour, based on the *road net (development & maintenance) graph*, calculates the *unique hub identifier* of the "isolated" hub to be removed – that is, is not connected to any links,
174 updates the *road net (development & maintenance) graph*, and
175 results in a pair of these.

```
value
173.  design_remove_hub: G → (HI×G)
173.  design_remove_hub(g) as (hi,g′)
173.    let hi:HI • hi ∈ his ∧ let (_,lis) = mereo_H(retr_part(hi)) in lis={} end in
174.    let g′ = M_rh(hi,g) in
175.    (hi,g′) end end
```

176 To remove a hub the road net administrator

a first designs which old hub is to be removed
b then removes the designated hub,
whereupon it updates the global states
c of parts σ,
d of unique identifiers, and
e of hub identifiers –

in parallel, and in parallel with
177 stopping the old hub behaviour
178 and resuming being a road net behaviour.

```
value
176.  remove_hub: G → RNI → RNMer → Unit
176.  remove_hub(g)(rni)(rnmer) ≡
176a.   let (hi,g′) = design_remove_hub(g) in
176b.   let h:H • uid_H(h)=hi ∧ ... in
176c.   σ := σ \ {h}
176d.  ‖ uid_σ := uid_σ \ {hi}
176e.  ‖ his := his \ {hi}
177.   ‖ ch[{rni,hi}] ! mkStop()
178.   ‖ rn(rni)(rnmer)(g′)
176.    end end ▪
```

6.12.2 Adjustment of Creatable and Destructable Behaviours

When an entity is created or destroyed its creation, respectively destruction affects the mereologically related parts and their behaviours. their mereology and possibly their programmable state attributes need to be adjusted. And when entities are destroyed their behaviours are **stop**ped ! These entities are "informed"

so by the creator/destructor entity – as was shown in Examples 87–89. The next example will illustrate how such 'affected' entities handle such creator/destructor communication.

Example 90 Hub Adjustments. We have not yet illustrated hub (nor link) behaviours. Now we have to !

179 The mereology of a hub is a triple: the identification of the set of automobiles that may enter the hub, the identification of the set of links that connect to the hub, and the identification of the road net.
180 The hub behaviour external non-deterministically (□) alternates between
181 doing "own work",
182 or accepting a stop "command" from the road net administrator, or
183 or accepting mereology & state update information,
184 or other.

```
type
179.  HMer = AI-set × LI-set × RNI
value
179.  mereo_H: H → HMer
180.  hub: hi:HI → (auis,lis,rni):HMer → hω:HΩ → (hσ:HΣ×ht:HTraffic) →
180.        {ch[hi,ui]|ui:(RNI|AI) • ui=rni∨ui ∈ auis}  Unit
180.  hub(hi)(hm:(auis,lis,rni))(hω)(hσ,ht) ≡
181.      ...
182.    □ let mkStop() = ch[hi,rni] ? in stop end
183.    □ let mkUpdH(hm′,hσ′,hσ′) = ch[{rni,hi}] ? in
183.      hub(hi)(hm′)(hω′)(hσ′,ht) end
184.      ...
```

Observe from formula Item 182 that the hub behaviour ends, whereas "from" Item 183 it tail recurses ! ▪

6.12.3 Summary on Creatable & Destructable Entities

We have sketched how we may model the dynamics of creating and destroying entities. It is, but a sketch. We should wish for a more methodological account. So, that is what we are working on – amongst other issues – at the moment.

6.13 Domain Engineering: Description and Construction

There are two meanings to the term 'Domain Engineering'.

- the construction of *descriptions* of domains, and
- the construction of *domains*.

Most sections of Chapters 4–6 are "devoted" to the former; the previous section, Sect. 6.12 to the latter.

6.14 A Domain Discovery Procedure, III

The predecessors of this section are Sects. 4.9.2 on page 57 and 5.8 on page 98.

6.14.1 Review of the Endurant Analysis and Description Process

The describe_... functions below were defined in Sects. 4.9.2 on page 57 and 5.8 on page 98.

```
value
  endurant_analysis_and_description: Unit → Unit
  endurant_analysis_and_description() ≡
      discover_sorts();                              [Page 57]
      discover_internal_endurant_qualities()  [Page 98]
```

We are now to define a perdurant_analysis_and_description procedure – to follow the above endurant_analysis_and_description procedure.

6.14.2 A Domain Discovery Process, III

We define the perdurant_analysis_and_description procedure in the reverse order of that of Sect. 5.8 on page 98, first the full procedure, then its sub-procedures.

A Domain Endurant Analysis and Description Process

```
value
  perdurant_analysis_and_description: Unit → Unit
  perdurant_analysis_and_description() ≡
      describe_state();                          axiom ...   [ Note (a) ]
      describe_channels();                       axiom ...   [ Note (b) ]
      describe_behaviour_signatures(); axiom ...   [ Note (c) ]
      describe_behaviour_definitions(); axiom ...   [ Note (d) ]
      describe_initial_system()                  axiom ...   [ Note (e) ]
```

Notes:

- (a) **The States: σ and ui_σ** We refer to Sect. 4.8.2 on page 56 and Sect. 46 on page 65. The state calculation, as shown on Page 55, must be replicated, i.e., re-described, in any separate domain analysis & description. The purpose of the state, i.e., σ, is to formulate appropriate axiomatic constraints and domain laws.
- (b) **The Channels:** We refer to Sect. 6.2 on page 104. Thus we indiscriminately declare a channel for each pair of distinct unique part identifiers whether the corresponding pair of part behaviours, if at all invoked, communicate or not.
- (c) **Behaviour Signatures:** We refer to Sect. 6.5 on page 105. We find it more productive to first settle on the signatures of all behaviours – careful thinking has to go into that – before tackling the far more time-consuming work on defining the behaviours:
- (d) **Behaviour Definitions:** We refer to Sect. 6.8 on page 111.
- (e) **The Running System:** We refer to Sect. 6.11 on page 113.

6.15 Summary

Perdurants: Analysis & Description: Method Tools

- **Domain Discovery**: The procedures being described here, informally, guides the domain analyser cum describer to do the job!

  We have basically finished our listings of the procedural steps of the domain engineering methodology of this primer!

#	Name	pg.
	Description Functions	
	describe_channels	**104**
	describe_behaviour_signatures	**110**
	describe_behaviour_definitions	**112**
	describe_initial_system	**113**
	perdurant_analysis_and_description	**121**

•••

Please consider Fig. 4.1 on page 38. This chapter has covered the right-hand side of Fig. 4.1.
127

[127] Observe "insertion" of 'Domain Method' and 'Method' Domain' index range commands.

Chapter 7
The DOMAIN Method

Contents

7.1 Introduction

By a **method** we shall understand a *set of* **principles** and **procedures** for *selecting* and *applying* a *set of* **techniques** *using* a *set of* **tools** in order to *construct* an *artefact* [DB].

- Major **principles** [128] of DOMAIN are (i) abstraction, (ii) the use of mathematics, and (iii) a combination of rigorous narration and formalisation.
- The DOMAIN analyser cum describer has, as its major **tool**[129], two languages: Domain Analysis Language and Domain Description Language; Domain Analysis Language is a domain analysis language, Domain Description Language a domain description language.
- Their "deployment", i.e., use, is based on the **procedure**[130] which is implied by the domain analysis & description ontology shown graphically in Fig. 3.1 on page 27
- Major **techniques**[131] of DOMAIN are the deployment "stackings" and uses of a notion of dashboard.

•••

In this chapter we present a rigorous, but not entirely formal, description of the DOMAIN method. The chapter is short of examples. Once the reader has reached the end of the previous chapter, that person is able to read the Appendix A and B examples. Therefore we refer, in this chapter, for several of the procedure stages, to respective sections of the example of Appendix A. We also refer, for each of the stages of the method procedure, to the section, in chapters 4–6, which first treat the subject.

7.2 The DOMAIN Procedure

The DOMAIN notion of *procedure* — implied by Fig. 3.1 on page 27 — is that of the domain analyser cum describers proceed in *stages* and *steps*, "traversing", as it were, (i) the domain, and (ii) the graph as well as the taxonomy of the specific domain.

Definition: By a **stage** of domain analysis & description we shall loosely understand, i.e., informally mean a sequence of one or more *steps* of domain analysis & description – with each stage being focused on a node in the analysis & description ontology graph, i.e., on a well-delineated facet of the domain[132] ▪

Definition: By a **step** of domain analysis & description we shall loosely understand, i.e., informally mean a small sequence of either analysis or description actions (e.g., a prompt) – with each step being focused on a simple component within a stage[133] ▪

While the domain analyser cum describers are analysing & describing the domain they commit the state of their work to a "dashboard", here referred to as θ:Θ, an informal, conceptual (if not real) state.

Definition: This **dashboard** shall be understood as follows: It represents, in "abbreviated" form, various summaries of the evolving domain description. These summaries can either be written down, say on a chalkboard (slate, board, green board, white board, black board, writing board) or be deduced, all along, from the evolving domain description – which is [similarly to the dashboard] available to all domain analyser cum describers ▪

- **Please Note**: The formulas of this entire section are either those of the procedure stages and steps, and they are in a variant of RSL [which we omit to name], or they are the RSL$^+$Text being generated. Although they "look alike" they are different, informal, respectively [only] rigorously, i.e., not formally, formal languages – easy to, but should not be confused !

Stages [and, within stages, steps] of the above can be given a rigorous, although not quite formal specification. In the rendition below we assume just one domain analyser cum describer.

[128] **Principle**: a fundamental truth or proposition that serves as the foundation for a system of belief or behaviour or for a chain of reasoning [Wikipedia].

[129] **Tool**: A tool is an object that can extend an individual's ability to modify features of the surrounding environment or help them accomplish a particular task [Wikipedia].

[130] **Procedure**: an established or official way of doing something [Oxford Languages and Google: https://languages.oup.-com/google-dictionary-en/].

[131] **Technique**: a way of carrying out a particular task, especially the execution or performance of an artistic work or a scientific procedure [Oxford Languages and Google].

[132] Edit: This definition must be improved upon !

[133] Edit: This definition must be improved upon !

7.2.1 Stage 0: Dashboard Initialization

The Dashboard Initialization stage consists of four steps.

185 There is, thus, a stage, say, discover_domain, which takes no argument, but delivers a domain description of a domain which that procedure elicits !

186 To "perform" discover_domain properly the dashboard state θ:Θ must be initialized.

- a $\theta_{esn} : \Theta_{ESN}$ is to hold a list of parts and their type under examination. This list will grow during "ontology traversal" examination of the of the domain.
- b $\theta_{dsu} : \Theta_{DSU}$ is to hold the list of all domain specification units as they are issued during the analysis procedure. The list grows.
- c $\theta_{vde} : \Theta_{VDE}$ is to hold the list of RSL$^+$Texts of value definitions.
- d $\theta_{beh} : \Theta_{BEH}$ is to hold the set of sort (names) of those [parts] which are to be "morphed" into behaviours. The set is set during procedure State 4, pages 127–129.

type
186a. Θ_{ESN} = (P×S)* [Pairs of values and endurant names]
186b. Θ_{DSU} = DSU* [Domain Description Units]
186c. Θ_{VDE} = RSL$^+$Text* [Value definitions]
186d. Θ_{BEH} = S-**set** [Behaviour Sorts]
variable
186a. $\theta_{esn} : \Theta_{ESN}$:= ⟨⟩
186b. $\theta_{dsu} : \Theta_{DSU}$:= ⟨⟩
186c. $\theta_{vde} : \Theta_{VDE}$:= ⟨⟩
186d. $\theta_{beh} : \Theta_{BEH}$:= {}

Inquiry and update of the θ state is expressed below as if there is just one domain analyser cum describer. If indeed thee are more than one, a careful transaction processing protocol[134] – e.g., a la Jim Gray [101].

7.2.2 Stage 1: The *discover_domain* Procedure

The *discover_domain* stage consists of three steps.

187 The discover_domain procedure takes no explicit arguments and "delivers" its result deposited in $\theta_{dsu} : \Theta_{DSU}$. That is: the domain analyser cum describer, of course, work in the environment of the domain.

- a First the domain must be selected, that is, the universe of discourse must be chosen.
- b Then the external qualities must be analysed and described.
- c Then the internal qualities must be analysed and described.
- d Finally the perdurants must be analysed and described.

187. discover_domain: **Unit** → **Unit**
187. discover_domain() ≡
187a. initialize_domain();
187b. discover_external_qualities() ;
187c. discover_internal_qualities() ;
187d. discover_perdurants()

What "connects" these procedural phases is the dashboard state: θ:Θ.

[134] Transaction processing is information processing that is divided into individual, indivisible operations called transactions. Each transaction must succeed or fail as a complete unit; it can never be only partially complete.

7.2.3 Stage 2: The *initialize_domain* Procedure

[135]

The *initialize_domain* stage consists of three steps:

188 We define the analysis procedure.

a The domain analyser cum describer choose a domain, and names it, say UoD.
b A display line **The Universe-of-Discourse**: prefixes the domain specification unit, and
c the dashboard state θesn is initialised.
d And that is it, for now !

```
value
188.  discover_initialization_of_domain: Unit → Unit
188.  discover_initialization_of_domain() ≡
188a.     let (text,uod:S) [ domain analyser cum describer choices ] in
188b.           { θdsu := ⟨ (“ The Universe-of-Discourse:, text, value uod:$ ”) ⟩
188c.           ‖ θesn := ⟨ (text, value uod:$) ⟩ } ”
188d.     end
```

7.2.4 Stage 3: The *discover_external_qualities* Procedures.

[136]

The *discover_external_qualities* stage consists of three sets of stages:

189 The discover_external_qualities

a starts by discovering endurants;
b then goes on to describe a domain endurant-taxonomy;
c and to discover endurant states.

```
value
189.  discover_external_qualities: Unit → Unit
189.  discover_external_qualities() ≡
189a.     discover_endurant_descriptions() ;
189b.     describe_taxonomy() ;
189c.     describe_endurant_states()
```

7.2.4.1 Stage 4: The discover_endurant_descriptions

[137]

The *discover_endurant_description[s]* stage consists of two sets of stages:

190 The examination (i.e., analysis) of the domain wrt. external qualities takes place in the/a current state $\theta : \Theta$.

a θ_{esn} “holds” the list of names of the sorts yet to be examined. For every sort examination that list either grows or shrinks. It grows when examining compound sorts. It shrinks when examining atomic sorts. So as long as there are names in the list there is examination to do !
b (step 1) The endurant part, p, to be examined is selected.
c (step 2) It is then described – into RSL$^+$Text, and

[135] We refer to Sect. 4.1 on page 34 and Example Sect. A.1 on page 161.
[136] We refer to Sect. 4 on page 33 and Example Sect. A.2 on page 161.
[137] We refer to Sect. 3.4.2 on page 27 and Example Sect. A.2 on page 161.

d (step 3) p is removed from θ_{esn}.

```
190.  discover_endurant_descriptions: Unit → Unit
190.  discover_endurant_descriptions() ≡
190a.     for i=1 to len co θesn do
190b.        let (p,E) = co θesn[i] in
190c.        traverse(p,E) end
190a.     end
```

The **co** θ_{esn} in the **for** i=1 **to len co** θ_{esn} **do** loop is to be evaluated "every time around the loop" as θ_{esn} changes within the loop ! Now we get to the "real" work.

191 The traverse step, as the name suggests, figuratively traverses the larger, **blue** lined box of Fig. 3.1 on page 27 – while observing a part p of type E. We read that traversal:

a If p is an *entity*,
b then
c if p is an *endurant*
d then
e if p is a *solid*
f then
g if p is a *part*,
h then
i if p is *atomic*
j then *handle* that *atomic*;
k else [it is a *compound*]
l if it is a *Cartesian*
m then *handle* the *Cartesian*
n else [it is a *Part set*] *handle* the *Part set*;
o or else it is a *living species*,
p or else it is a *fluid*,
q or else it is a *perdurant* !

We do not handle *living species* and *fluids* in this book, and we do not "reach" the *perdurant* alternative since we initially invoked traverse with an endurant part !

```
value
191.  traverse: (P×E) → Unit
191.  traverse(p,E) ≡
191a.     if is_entity(p)
191b.        then
191c.           if is_endurant(p)
191d.              then [axiom: is_entity(p)]
191e.                 if is_solid(p)
191f.                    then [axiom: is_endurant(p)]
191g.                       if is_part(p)
191h.                          then
191i.                             if is_atomic(p)
191j.                                then handle_atomic(p,E)
191k.                                 else [axiom is_compound(p)]
191l.                                   if is_Cartesian(p)
191m.                                      then handle_Cartesian(p,E)()
191n.                                      else [axiom is_Part_set(p)] handle_Part_set(p,E)()
191l.                                   end
191i.                             end
191o.                          else [axiom is_living_species(p)] skip
191g.                       end
191p.                    else [axiom is_fluid(p)] then skip
191c.                 end
191q.              else [axiom: is_perdurant(p)] skip
191c.           end
191a.     end
```

We mark in **red** the functions, i.e., the handle_atom(p,E) and handle_Cartesian functions:

191j. The handle_atom(p,E) function joins its sort to those of the behaviourals.

```
value
191j.  handle_atomic(p,E) ≡ θ_beh := co θ_beh ∪ {E},
```

138

192 ,191k. The handle_Cartesian function behaves as follows.

a The domain analyser cum describer observes/decides that one can observed n distinct parts, each of it own sort and
b generates/supplies narrative text "explaining" these parts, and
c updates the dashboard state accordingly – prefixing the domain specification unit with a display line: **Endurant Sorts:**.
d The domain analyser cum describer finally decides on the possibly empty, possibly full, subset of the sorts of the n Cartesian components are behavioral.

```
value
191m.   handle_Cartesian: (P × E) × Unit → Unit
191m.   handle_Cartesian(p,E)() ≡
192a.     let ((p1,E1),(p2,E2),...,(pn,En)) = observe(p) in
192b.     let (txt1,txt2,...,txtn) = analyser_cum_describer_choice(p) in
192c.     θ_esn := co θ_esn ⌢⟨ (p1,E1),(p2,E2),...,(pn,En) ⟩ ;
192c.     θ_dsu := co θ_dsu ⌢⟨" Endurant Sorts:
192c.                          Narrative:
192c.                            1. txt1, ...,  n. txtn.
192c.                          Formalisation:
192c.                            type
192c.                              1. E1, ..., En
192c.                            value
192c.                              1. obs_E1: E→E1, ..., n. obs_En: E→En " ⟩ ;
192d.     θ_beh := co θ_beh ∪ select_Cartesian_endurants({E1,E2,...,En}) ,
192.      end end

192d.  select_Cartesian_endurants: E-set → E-set
192d.  select_Cartesian_endurants({E1,E2,...,En}) ≡
192d.    {Ei,Ej,...,Ek}[139] where {Ei,Ej,...,Ek} ⊆ {E1,E2,...,En}
```

193 ,191l If the part is considered a part set then actions very similar to those for Cartesian parts are "performed" by the domain analyser cum describer.

140

```
value
191n.   handle_Part-set: (P × E) × Unit → Unit
191n.   handle_Part-set(p,E)() ≡
193.    let (ps:PS) = observe(p) axiom ps ≠ {}  in
193.    let p:P • p ∈ ps  in
193.    let (text_ps,text_p) = analyser_cum_describer_Choice(ps,p)  in
193.    θ_esn := co θ_esn ⌢⟨(p,P)⟩ ;
193.    θ_dsu := co θ_dsu ⌢⟨ " Endurant Sorts:
193.                          Narrative:
193.                            1. text_ps, 2. text_p
193.                          Formalisation:
193.                            type
```

[138] We refer to Sect. 4.5.1.2.1.2 on page 45 and Example Sect. A.2 on page 161:

[139] The choice/selection of {Ei,Ej,...,Ek} is one that the domain analyser cum describer must decide upon.

[140] We refer to Sect. 4.5.1.2.2 on page 46 and Example Sect. A.2 on page 161:

193. 1. PS = P-**set**[141], 2. P
193. **value**
193. 1. **obs_**PS: E → PS ” ⟩ ;
193. θ_{beh} := **co** θ_{beh} ∪ select_Part_set_endurant(P) ;
193. **end end end**

193. select_Part_set_endurant: P → P-**set**
193. select_Part_set_endurant(P) ≡ **if** **is_behavioural(P)** **then** {P} **else** {} **end**

7.2.4.2 Termination ?

Does the discover_endurant_descriptions procedure terminate ? Yes. The reasons are as follows:

- Endurants are not recursive ! That is, no endurant must be describable/definable in terms of itself. The reader may object ! How about trees, such as we see them in nature ? Well, these trees have roots below ground and trunks, branches and leaves above ground. The “above ground” latter are not trees. So we model trees as specially restricted graphs.
- For two distinct endurants, say two distinct Cartesians, E1 and E2, the same sort of part, p:E, does not occur in both. If the domain analyser cum describers think so, they are wrong. These two endurants sorts should be distinctly named, for example E1_E and E2_E or E_E1 and E_E2. They may be “alike”, but somehow their, perhaps some or their “not enunciated” internal qualities differ.

So the discover_endurant_descriptions procedure terminate: eventually all examined endurants are atomic parts.

7.2.4.3 Stage 5: Describe Endurant Taxonomy.

The Describe Endurant Taxonomy stage consists of two steps. [142]
[We also refer to Carl von Linneaeus, the originator of the term Taxonomy[143].]

A visual domain model endurant taxonomy of the structure of parts do not add to the meaning of the domain model. It is merely of practical help in comprehending the possible complexity of the domain.

194 An endurant taxonomy domain specification unit starts with a **Taxonomy** display line, usually prefixed by the name, E, of the “overall”, i.e., a “main” part that is “at the root” of the taxonomy, i.e., the usually compound part whose siblings – and again their siblings, etc., till we “reach”, as leaves of the taxonomy tree, the atomic parts.

a The TAX_DSU tree-like diagram has as its root a box labeled with E.
b In general now, if the taxonomy box stands for an atom then we leave it there: no taxonomy sub-tree emanating from that box.
c If the taxonomy box, E, stands for a Cartesian endurant with components E1, E2, ..., En then we draw, on the same horizontal line below box E, n boxes, connected to box E by straight, usually slanted lines, n boxes, left-to-right, labeled E1, E2, ..., En.
d If the taxonomy box, E, stands for a part set, then we draw, below it, a box labeled with the part set name, f.ex., PS, observed from E – connected to box E by a vertical line.
e This is followed by drawing two or three boxes, “immediately” below box PS and on the same horizontal line and all identically labeled by the same endurant sort name, say P, where P is the type occurring in PS = P-**set** observed from E.

[142] We refer to Sect. 4.5.1.3 on page 48 and Example Sect. A.4 on page 162.

[143] https://en.wikipedia.org/wiki/Carl_Linnaeus and to his main work: https://ia600508.us.archive.org/9/items/-mobot31753000798865/mobot31753000798865_bw.pdf

f The above diagram construction instructions are followed till all domain endurants "derivable" from E have been followed.

```
value
194. describe_endurant_taxonomy: Unit → Unit
194. describe_endurant_taxonomy() ≡
194.   let endurant_taxonomy = design_endurant_taxonomy() in
194.   θdsu := co θdsu ⌢ ⟨ " Endurant Taxonomy: endurant_taxonomy " ⟩
194.   end
```

Here design_endurant_taxonomy is a "function" which is performed by the domain analyser cum describer.

7.2.4.4 Stage 6: Describe Endurant State.

144

The Describe Endurant State stage consists of two steps.

195 The describe_endurant_state stage

a initially appends an **Endurant State:** prefix and
b the RSL⁺Text **value** to θ_{dsu};
c then proceeds, for all (part,sort)s that have been "discovered" and therefor are in θ_{ves},
d to select these elements of θ_{ves},
e and append them, as domain specification units to θ_{dsu}.

196 Then a notion of "global" σtates is introduced:

a σ_{parts} – the union of all parts, and
b $\sigma_{behavioural_{parts}}$ the union of behavioural all parts.

```
195. describe_endurant_state: Unit → Unit
195. describe_endurant_state() ≡
195a.   θdsu := co θdsu ⌢ " ⟨ Endurant State: value ⟩ ";
195c.   for i=1 to len θves do
195d.     let (p,E) = θves[i] in
195e.     θdsu := co θdsu ⌢ " ⟨ ev:E = p ⟩ " ;
196.      θves := co θves ⌢ " ⟨ ev:E = p ⟩ " ;
196a.     σall_parts = [ ∪ of ∀ parts ] ,
196b.     σall_behavioural_parts = [ ∪ of ∀ hehaviourable parts ] "
195c.   end end
```

Here ev is a suitable, distinct value name chosen by the domain analyser cum describer.

7.2.5 Stage 7: The *discover_internal_qualities* Procedures.

The *discover_internal_qualities* stage consists of four sets of stages.

197 The discover_internal_qualities procedure generates domain specification units for the

a unique identification,
b mereologies,

[144] We refer to Sect. 4.8 on page 54 and Example Sect. A.3 on page 162.

c attributes and
d for possible intentional pull descriptions/

– and in that order !

197. discover_internal_qualities: **Unit** → **Unit**
197. discover_internal_qualities() ≡
197a. describe_identification() ;
197b. discover_mereologies() ;
197c. discover_attributes() ;
197d. discover_intentional_pulls()

7.2.5.1 Stage 8: The *unique_identification* Procedure.

145

The *unique_identification* stage consists of three sets of stages.

198 The describe_identification procedure generates domain specification units for the

a unique identifiers,
b unique identifier states,
c uniqueness of parts,

– and in that order !

198. describe_identification() ≡
198a. describe_unique_identifiers() ;
198b. describe_unique_identifier_states() ;
198c. describe_part_uniqueness() ;

7.2.5.1.1 Stage 9: The *unique_identifiers* Procedure.

The *unique_identifiers* stage consists of three sets of stages.
Note: UI stands for the sort of all unique identifiers.

199 The *describe_unique_identifiers* procedure is to generate

a a domain specification unit **Unique Identification**: display line and
b records the **type** and **uid**_E observers for all "behavioral" parts, that is: those parts which the domain analyser cum describer considers candidate to be "morphed" into behaviours – those which are dashboard recorded in θ_{beh}:Θ_{BEH}.
  i For all E noted in the dashboard as behavioral
  ii the**type** of the unique identifier and
  iii the **value** and its observer
  iv is added to the emerging domain description θ_{dsu}.

199. describe_unique_identifiers() ≡
199a. θ_{dsu} := **co** θ_{dsu} ⌢ ⟨ **Unique Identification**: ⟩ ;
199a. **for** ∀ E **in co** θ_{beh} **do**
199(b)iv. θ_{dsu} :=
199(b)iv. **co** θ_{dsu} ⌢
199(b)ii. ⟨ { " **type**
199(b)ii. EI

[145] We refer to Sect. 5.2 on page 62 and Example Sect. A.6 on page 164.

199(b)iii. **value**
199(b)iii. **uid**_E: E → EI ”
199(b)i. | (_,E) ∈ **elems** θ_{esn} } ⟩
199. **end**

Since there will usually be many such unique identifier definitions, each with their **type** and **value** literals, these can be summarised into one such set of definitions.

7.2.5.1.2 Stage 10: Unique Identifier States.

The Unique Identifier States stage consists of five steps.

200 The describe_unique_identifier_states stage

a augments θ_{dsu} with a **Unique Identifier States** display line; followed by a
b for all (p,E) known such in θ_{esn}
c with “their” **value**
d contribution to the unique identifier states; followed by
e a $\sigma_{uid_{all\_parts}}$ definition, and
f a $\sigma_{uid_{all\_behav\_parts}}$ definition.

describe

200. describe_unique_identifier_states() ≡
200a. θ_{dsu} := **co** θ_{dsu} ⌢⟨ **Unique Identifier States**: ⟩ ;
200b. **for** ∀ (p,E) **in co** θ_{esn} **do**
200a. θ_{dsu} :=
200c. **co** θ_{dsu} ⌢⟨ “ **value** ” ⟩
200d. ⌢⟨ { “ p_{uid}:EI = **uid**_E(p), ” | (p,E) ∈ **elems** θ_{esn} } ⟩
200e. ⌢⟨ “ $\sigma_{uid_{all\_parts}}$:UI-set = [...] ” ⟩
200f. ⌢⟨ “ $\sigma_{uid_{all\_behav\_parts}}$:UI-set = [...] ” ⟩
200b. **end**

7.2.5.1.3 Stage 11: Uniqueness.

146

The Uniqueness stage consists of one step.

201 The all parts are unique **axiom** is simple: the number of all parts equals the number of all part [unique] identifiers.

axiom
201. **card** σ_{all_parts} = **card** $\sigma_{uid_{all\_behav\_parts}}$

7.2.5.2 Stage 12: The *mereology* Procedure.

147

The *mereology* stage consists of one step.

[146] We refer to Sect. 5.2.1 on page 63 and Example Sect. A.8 on page 165.
[147] We refer to Sect. 5.3 on page 67 and Example Sect. A.9 on page 165.

202 The *mereology* procedure is to generate a domain specification unit which records the type and **obs**_E observers for all "behavioral" parts, that is: those parts which the domain analyser cum describer considers candidate to be "morphed" into behaviours – those which are dashboard recorded in θ_{beh}:Θ_{BEH}.

a The θ_{dsu} is updated,
b For all E noted in the dashboard as behavioral the
c **type** and
d **value**
e of the mereology definition and its observer is added to the emerging domain description.

```
202.  discover_mereologies: Unit → Unit
202.  discover_mereologies ≡
202a.    θdsu := co θdsu ⌢ ⟨ Mereologies: ⟩ ;
202b.    for ∀ E in co θbeh do
202e.      θdsu :=
202e.          co θdsu ⌢
202c.            ⟨ " type
202c.                MereoE = ℳE(UI1,UI2,...,UIn)
202d.              value
202d.                mereo_E: E → MereoE  " ⟩
202.   end
```

Since there will usually be many such mereology definitions, each with their **type** and **value** literals, these can be summarised into one such set of definitions.

The **red** text shall convey to the reader that this part of the description procedure requires the "creative" thinking on the part of the domain analyser cum describer.

The $\mathcal{M}$(UI1,UI2,...,UIn) expression is over unique identifiers. It may, for example, be expressed in terms of a triplet:

- ($\mathcal{M}_i$(iUI1,iUI2,...,iUIn),$\mathcal{M}_{io}$(ioUI1,ioUI2,...,ioUIn),$\mathcal{M}_o$(oUI1,oUI2,...,oUIn))

where the

- $\mathcal{M}_i$ part $\mathcal{M}_i$(iUI1,iUI2,...,iUIn) designate behaviours from which the "e:E" behaviour offers to accept inputs,
- $\mathcal{M}_{io}$ part, $\mathcal{M}$(ioUI1,ioUI2,...,ioUIn), designate behaviours from which the "e:E" behaviour accepts input and to which it offers communication, and
- $\mathcal{M}_o$ part $\mathcal{M}$(oUI1,oUI2,...,oUIn) designate behaviours to which the "e:E" behaviour offers communication.

The $\mathcal{M}$(UI1,UI2,...,UIn) are typically of the form UI-**set**.

7.2.5.3 Stage 13: The *discover_attributes* Procedure.

148

The *discover_attributes* stage consists of several steps.

203 The **attr***ibutes* procedure is to generate
204 a domain specification unit which records the type and **attr**_A observers for all "behavioral" parts, that is: those parts which the domain analyser cum describer considers candidate to be "morphed" into behaviours – those which are dashboard recorded in θ_{beh}:Θ_{BEH}.

a **Attributes**: prefixes the individual attribute type and observer definitions.
b For all E noted in the dashboard as behavioral the domain analyser cum describer
c analyses that type to have a number of attributes (A1,A2,...,An);

[148] We refer to Sect. 5.4 on page 71 and Example Sect. A.10 on page 165.

d with these possessing respective properties (AttrType_1,AttrType_2,...,AttrType_n).
AttrType_i are of the form:
- AttrType_i = AttrType_Expr_i × Attr_Cat [× SI_Unit]
- Attr_Cat = **sta**|**mon** | **prg**
- SI_Unit = ...

The [...] in [× SI_Unit] mean that this term is optional. Not all attributes can be given an SI Unit[149]

sta refers to static attributes, **mon** refers to static attributes, and **prg** refers to programmable attributes.

e θ_{dsu} is therefore extended with a domain specification unit with the list element containing
f attribute **type** definitions and
g **value** definitions of the **attr**ibute observers.

```
203.  discover_attributes: Unit → Unit
203.  discover_attributes() ≡
204a.     θdsu := co θdsu ⌢⟨ Attributes: ⟩ ;
204b.     for ∀ E ∈ co θbeh do
204c.        let (A1,A2,...,An) = observe_attribute_names(p:E) in
204d.        let (AttrType_1,AttrType_2,...,AttrType_n) = observe_attribute_types(p:E) in
204e.        θdsu := co θdsu ⌢
204f.           ⟨ " type
204f.                 A1 = AttrType_1, A2 = AttrType_2, ..., An = AttrType_n
204g.              value
204g.                 attr_A1: E → AttrType_1,
204g.                 attr_A2: E → AttrType_2,
204g.                 ...
204g.                 attr_An: E → AttrType_n  " ⟩
203.     end end end
```

Attribute types AttrType_i are type expressions over "standard" RSL definable types, unique identifier sorts and mereo types.

Red "formulas" indicate work to be done by the domain analyser cum describer.

•••

Important attributes, ones that can be ascribed to [almost all] parts, E, is the **E_history** attributes. They may not be observable, not physically measurable, but they can be *"remembered"*, can be *"talked about"* – say by humans. They therefore objectively, indisputably records the occurrence of **events**. In our domain models we record E_history attributes as **sequences** *of* TIME*-stamped events*. Examples are the events of automobiles deciding to remain at hubs or on links, or stopping, or leaving or entering these, etc.

7.2.5.4 Stage 14: The *discover_intentional_pull* Procedure.

150

The *discover_intentional_pull* stage consists of several sub-stages and steps.

Whereas the expression [i.e., definition of] unique identification, mereologies and attributes are specific to individual part types, intentional pulls usually range over several part types: *"roads are meant for automobiles and automobiles are meant for roads, etc."*.

[149] The International System of Units for physical, observably measurable phenomena:
- International System of Units - Wikipedia: https://share.google/kckTszukBHJWt4Nwm
- SI Units, the US NIST: https://share.google/4lkLUKbdHheQce1nT
- Physical Quantities, Units, and Measurements –STEM for Educators: https://share.google/uUs65ejqDO1RUqJ3n.

[150] We refer to Sect. 5.6 on page 90 and Example Sect. A.11 on page 166.

Intentional pulls are expressed in the form of $\mathcal{B}_{p_i}(...)$ implies $\mathcal{B}_{q_i}(...)$ if-and-only-if $\mathcal{B}_{x_j}(...)$ implies $\mathcal{B}_{y_k}(...)$: *"if an automobile, a, is on road r, at time τ, then road r observes automobile a on that road at that time* **and** *if road, r, observes automobile a, at time τ, then automobile a is on road r on that road at that time".*

205 The *discover_intentional_pulls* procedure [possibly] generates an "Intentional Pull" domain specification unit with zero, one or more, n, intentional pulls.

a θ_{dsu} is extended
b by an **axiom** and
c n, $n \geq 0$, intentional pull predicates.

The $\mathcal{B}(...)$ terms are $\mathcal{B}$oolean predicates.

```
205.  discover_intentional_pulls: Unit → Unit
205.  discover_intentional_pulls() ≡
205a.    θdsu := co θdsu ⌢⟨ "Intentional Pulls:
205b.          axiom
205c.            Bp1(...) ⇒ Bq1(...)
205c.            ≡
205c.            Bx1(...) ⇒ By1(...) ,
205c.
205c.            Bp2(...) ⇒ Bq2(...)
205c.            ≡
205c.            Bx2(...) ⇒ By2(...) ,
205c.
205c.            ...
205c.
205c.            Bpn(...) ⇒ Bqn(...)
205c.            ≡
205c.            Bxn(...) ⇒ Byn(...)  " ⟩
```

The **red** formulae designates work to be done by the domain analyser cum describer. [Hard work !]

7.2.6 Stage 15: The *discover_perdurant* Procedures

151

The *discover_perdurant* stage consists of several sub-stages and steps.

206 The *discover_perdurant* stage has four sub-stages:

a the description of channels,
b the characterisation of behaviour actions,
c the description of behavioural taxonomy,
d the discovery of behaviours, and
e the description of domain initialization;

```
206.  discover_perdurants: Unit → Unit
206.  discover_perdurants() ≡
206a.     describe_channel() ;
206b.     characterisation_of_actions() ;
206c.     describe_behavioural_taxonomy() ;
206d.     discover_behaviours() ;
206e.     describe_initialization()
```

[151] We refer to Chapter 6.

7.2.6.1 Stage 16: The *describe* channel Procedure.

152

The describe **channel** stage consists of one step.

207 The *describe* **channel** procedure is very straightforward:

a θ_{dsu} is extended with the declaration of a **channel** array
b named ch, where the distinct array indexes, ui, uj [ui ≠ uj] range over the unique identifiers of all behavioral parts.

207. describe_channel: **Unit** → **Unit**
207. describe_channel() ≡
207a. θ_{dsu} := **co** θ_{dsu} ⌢ " **Channels:**
207b. **channel** { ch[{ui,uj}] | ui,uj:UI • {ui,uj}⊆$\sigma_{uid_{parts}}$ }: **M** "

M is a type expression referring to values other than sort names, variables, etc. We leave it undefined ! The channels are referenced in CSP input/output clauses:

- ch[{ui,uj}] ?: input to behaviour ui from behaviour uj – with ch[{ui,uj}] ? denoting a[ny] value. ch[{ui,uj}] ? is an expression.
- ch[{ui,uj}] ! expr: output from behaviour ui to behaviour uj of he value denoted by the expression. ch[{ui,uj}] ! expr is a clause, i.e., like a statement.
- [] { ch[{ui,uj}] ? | uj:UI • ui ∈ set_of_uis }: expresses the **external** non-deterministic choice of input to behaviour ui from either one of the behaviours designated by uj.
- ⨅ { ch[{ui,uj}] ! expr | uj:UI • uj ∈ set_of_uis } expresses the **internal** non-deterministic output of the value of an expression from behaviour ui to one of the behaviours in set_of_uis.
- || { ch[{ui,uj}] ! expr | uj:UI • set_of_uis }: expresses the simultaneous deterministic output to all behaviours designated by uj in the from behaviour uj.

7.2.6.2 On Behaviour Signatures.

153

Behaviours transpire, as I say, by transcendental deduction, that is, are "morphed" from behaviourable parts. There is one behaviour for each such part. Their signatures follows "straight" from the composition of the internal qualities of the endurant parts from which they are "morphed":

- Their name is usually chosen to relate to the name of the part sort from which they emerge.
- A first argument, by convention, is the unique identifier of "the part".
- A second argument is the mereology of the part.
- A third argument is the zero, one or more static attribute values.
- A possible fourth argument the names zero, one or more monitorable attribute.
- A "final" argument is one or more programmable attribute values.
- The result "value" of a behaviour is the **Unit** value, ().

7.2.6.3 Stage 17: Characterisation of Actions Stage

154

The Characterisation of Actions stage consists of several steps.

Behaviours, when observed, generally consists of sets of sequences of actions, events and behaviours. We shall "restrict" this characterization to not allow "embedded" behaviours. That is: In our understanding of the kind of domains that we have "limited' ourselves – i.e., the DOMAIN method – to deal with:

- (i) any behaviour is exclusively the result of a transcendental deduction of a specific part sort,

[152] We refer to Sect. 6.2 on page 104 and Example Sect. A.12 on page 167.
[153] We refer to Sect. 6.5 on page 105 and Example Sect. A.14 on page 168.
[154] We refer to Sect. 6.3 on page 105.

- (ii) and such behaviours do not invoke other part behaviours.

In the definition of part behaviours we let the behaviour tail-recursively invoke "itself" [in order to go on, and on, ...].

I have skipped, till now, an important procedural stage: it is that of analysing each "emerging" part behaviour for its actions. It is the composition of these actions that determine the specification of a behaviour, i.e., its definition.

So, as a procedural stage, perhaps right after the specification of channels and thus before the specification of behaviour signatures we suggest that the domain analyser cum describers, as a group, sketch a list of behaviour actions: a list for each behaviour, noting "all" of its actions, and noting whether such actions only involve themselves, are specific to that behaviour, or communicate with other behaviours – and then which !

7.2.6.4 Stage 18: Perdurant Taxonomy

155

The Perdurant Taxonomy stage consists of several steps.

208 A perdurant taxonomy is a pictorial rendition, a two-dimensional graph, which shows "all" the channels (as pointed arrows) and the behaviours, say as circles or [rounded] boxes with arrows between these. The arrows designate communications between behaviours.

```
value
208. describe_perdurant_taxonomy: Unit → Unit
208. describe_perdurant_taxonomy() ≡
208.     let perdurant_taxonomy = design_perdurant_taxonomy() in
208.     θdsu := co θdsu ⌢ ⟨ " Perdurant Taxonomy: perdurant_taxonomy "⟩
208.     end
```

7.2.6.5 Stage 19: Behaviour Signatures.

156

The Behaviour Signatures stage consists of several steps.

Behaviours transpire, as I say, by transcendental deduction, that is, are "morphed" from behaviourable parts. There is one behaviour for each such part. Their signatures follows "straight" from the composition of the internal qualities of the endurant parts from which they are "morphed":

- Their name is usually chose to relate to the name of the part sort from which they emerge.
- A first argument, by convention, is the unique identifier of "the part".
- A second argument is the mereology of the part.
- A third argument is the zero, one or more static attribute values.
- A possible fourth argument the names zero, one or more monitorable attribute.
- A "final" argument is one or more programmable attribute values.
- The result "value" of a behaviour is the **Unit** value, ().

7.2.6.6 Stage 20: Behaviour Definitions.

157

The Behaviour Definitions stage consists of sub-stages (one per behavioural sort) and several steps.

209 The discover_behaviours stage "takes" the state of the dashboard and updates that dashboard.

[155] We refer to Sect. 6.4 on page 105 and Example Sect. A.13 on page 167.
[156] We refer to Sect. 6.5 on page 105 and Example Sect. A.14 on page 168.
[157] We refer to Sect. 6.8 on page 111 and Example Sect. A.14 on page 168.

210 First a **Behaviour Definitions** display line; then
211 for all behavioural parts

a the domain analyser cum describer analyses and describes the [thus transcendentally "morphed"] part into a behaviour definition.
b The general "pattern" of behaviour definitions start with a signature – [where we often find that we can omit the monitorable attributes] – and and a "token invocation", followed by ≡ – i.e., the definition.
c The "body" of behavioural definitions is either of a simple form:
  i B is a function, e.g., a behaviour which terminates delivering as its result a value, called state.
  ii From state one can "extract" updated values and the mereology of behaviour ui – for the case that behaviour introduces new, or removes existing behavioural parts – and one or more of the programmable attributes.
  iii whereupon behaviour resumes being the behaviour, but with an updated "state" !
d Or B follows the patterns illustrated above.

```
209.   discover_behaviours: Unit → Unit
209.   discover_behaviours() ≡
210.     θdsu := co θdsu ⌢ ⟨ "Behaviour Definitions:" ⟩ ;
211.     for ∀ E ∈ co θbeh do
211.       θdsu := co θdsu ⌢
211.         ⟨ let p:E • [where p ≡ recorded in θ as being of type E] in
211a.          "behaviour: UI → Mereo → StatAttrs → [MoniAttrs →] ProgrAttrs → Unit
211b.           behaviour(ui)(mereo)(sta_atts)(moni_attrs)(prgr_attrs) ≡[158]
211(c)i.        let state = B(ui)(mereo)(sta)[(mon)](prgr) in
211(c)ii.       let (mereo′,prgr′) = analyse(state) in
211(c)iii.      behaviour(ui)(mereo′)(sta_atts)(moni_attrs)(prgr′) end end"  end ⟩
211.     end
211d.    or: [...other !...]
```

7.2.6.7 Stage 21: Domain Initialisation.

159

The (final) Domain Initialisation stage consists of several steps (one per behavioural sort).

212 The describe_initialisation procedure stage first generates

a a display line: **Domain Initialisation:**
b then, for each behavioural part, p:E, that has been encountered, i.e., each part in $\sigma_{uid_{parts}}$
c the parallel composition of behaviour invocations, be(uid)(mereo)(sta)(moni)(prg), where the name, be, of the behaviour is a suitable name that has some connotation to the part type E,
d where the unique identification, uid, is obtained from p:E,
e the mereology, mereo, likewise,
  and so
f the static attribute values, sta,
g the monitorable attribute names, moni, and
h the programmable attribute values, prgr.

```
value
212.   describe_initialisation: Unit → Unit
```

[158] The domain analyser cum describer must, based on the internal qualities defined for E "fill in" the details of Mereo, StaticAttrs, MoniAttrs and ProgrammableAttrs. It should be obvious (!) how to do that. It is too cumbersome to lay that out here.

[159] We refer to Sect. 6.11 on page 113 and Example Sect. A.15 on page 170.

```
212. describe_initialisation() ≡
212a.     θdsu := co θdsu ⌢⟨ " Domain Initialisation: " ⟩
212c.     ⌢⟨ "  || { be
212d.                  (uid_E(p))
212e.                     (mereo_E(p))
212f.                        ( { attr_A(p) | A ∈ static_attrs(p) } )
212g.                           ( { ηA[160]| A ∈ moni_attrs(p) } )
212h.                              ( { attr_A(p) | A ∈ prgr_attrs(p) } ) "
212b.              | (p,E) ∈ elems θves ∧ E ∈ co θbeh } ⟩
```

The functions static_attrs, moni_attrs and prgr_attrs follow from the category of the attribute definitions for p:E.

7.2.7 Procedure Iteration.

The enunciation of the 21 stages – as a sequential, deterministic set, i.e., list, of procedural stages – shall not be understood as the only way to analyse & describe domains. "Mistakes", that is, unsuitable choices of description, do occur. The role of the preparatory domain modeling effort, the one that should precede any seriously full-scale modeling such as the one for which these 21 procedure stages are prescribed, is to "minimize" the likelihood of "mistakes". Anyway, "mistakes" do occur. In such situations, where domain analyser cum describer[s] decide that the model they are developing must be re-oriented/re-done [in a different direction], an iteration, a re-analysis/description, as from some stage, must take place – replacing domain specification units.

A "roll-back" to some earlier step, must take place.

We do not prescribe the stages involved in such a roll-back. But it should be obvious that the more the procedure stages are followed and documented, the better an analysis & description iteration can succeed.

7.3 Some 𝔻𝕆𝕄𝔸𝕀ℕ Techniques

We remind the reader of what we consider techniques to be: *a way of carrying out a particular task, especially the execution or performance of an artistic work or a scientific procedure [Oxford Languages and Google].* Some 𝔻𝕆𝕄𝔸𝕀ℕ techniques are: the technique of deploying [stacking and using] "the" dashboard; MORE TO COME ...

7.4 Some 𝔻𝕆𝕄𝔸𝕀ℕ Tools

The foremost tools of 𝔻𝕆𝕄𝔸𝕀ℕ are the domain analysis and the domain description languages, DAL and DDL. Not much more to say about that, other than citing [54, 55]. Computerised tools in the form of support for the use of DDL do not exist – yet!? But would be a much needed tool in commercial developments based on he use of the 𝔻𝕆𝕄𝔸𝕀ℕ method.

[160] All but the monitorable arguments are *"called-by-value"*,the monitorable arguments are *"called-by-name"*.

7.5 A DOMAIN Model of The Domain Modeling Procedure

The rigorous, but informal, description, Δ, that has been given in the previous section is an "ordinary", informal description given in an informal, RSL-like language. It is not a DOMAIN description. But one could "lift" Δ into a domain model: One behaviour is that of the domain, uod:UoD; another behaviour is that of the dashboard state, θ:Θ; and there are then n behaviours, one for each domain analyser cum describer. The DOMAIN "domain" model would then explicate how the domain analyser cum describers interact amongst themselves, with the domain, and with the dashboard state $\theta : \Theta$, ![161]

We leave it to You to further contemplate, etc., such a model !

7.6 Skills Assessment: The DOMAIN Method

We have "laid bare" 1+21 stages of a domain analyser cum describer process. How difficult are, i.e., what does it take to perform these stages !

Here is my assessment:

- **Stages 0-3** (7.2.1 on page 125– 7.2.4 on page 126; Initialization, Discover Domain, Initialize Domain, External Qualities) are straightforward.
- **Stage 4** (7.2.4.1 on page 126; Endurants) is a crucial stage: it calls for the experienced analytical skills of the domain analyser cum describers. The crucial are the observe [Item 192a] and analyser_cum_describer_choice [Item 192b on page 128].
- **Stages 5-6** (7.2.4.3 on page 129– 7.2.4.4 on page 130; Endurant Taxonomy, Endurant State) are straightforward, respectively crucial: The choices, Items 192d and 193 page 128, of the Endurant State are seemingly easy to make, but a first such may be regretted, but it is "easy" to fix !
- **Stages 7-11** (7.2.5 on page 130– 7.2.5.1.3 on page 132; Internal Qualities, Unique Identifiers, Unique Identifier State, Uniqueness of Parts) are straightforward.
- **Stage 12-14** (7.2.5.2 on page 132– 7.2.5.4 on page 134; Mereology, Attributes, Intentional Pull) are crucial stages: they call for the experienced analytical skills of the domain analyser cum describers. The crucial tasks are those of determining $\mathcal{M}_E$(UI1,UI2,...,UIn) Item 202c on page 133, observe_attribute_names Item 204c on page 133, observe_attribute_types Item 204d on page 134, and the discover_intentional_pulls Item 205 on page 135.
- **Stages 15–16** (7.2.6 on page 135– 7.2.4.3 on page 129; Perdurants, Channel) are straightforward.
- **Stage 17** (7.2.6.3 on page 136; Actions) is a crucial stage: it calls for the experienced analytical skills of the domain analyser cum describers. I am presently not satisfied with this section !
- **Stages 18-19** (7.2.6.4 on page 137– 7.2.6.5 on page 137; Behaviourable Taxonomy, Signatures) are, on the background of the completion of the previous stages, relatively straightforward.
- **Stage 20** (7.2.6.6 on page 137; Behaviour Definitions) is a crucial stage: it calls for the experienced specification, "CSP programming" skills of the domain analyser cum describers. I am presently – likewise – not satisfied with this section !
- **Stage 21** (7.2.6.7 on page 138; Domain Initialization) is straightforward.

7.7 Our Characterisation of the Term 'Method'

Is our characterisation[162] of the term 'method' commensurate with "other" characterisations of that term ? I refer to Descartes: [86]:

[161] The model would have to allow concurrent updates to θ, also those which "mess-up: matters. A requirements prescription would/might then sort out such things through appropriate *transaction protocols* – i.e., a la James Gray [101].

[162] By a **method** we shall understand a *set of* **principles** and **procedures** for *selecting* and *applying* a *set of* **techniques** *using* a *set of* **tools** in order to *construct* an *artefact* [DB].

- "**Le Discours de la méthode** *(1637) de René Descartes est une œuvre fondatrice de la philosophie moderne, prônant l'usage de la raison pour trouver la vérité scientifique. Il propose une approche rigoureuse en quatre règles: (i)* **ne rien accepter pour vrai**, *(ii)* **diviser les difficultés**, *(iii)* **ordonner les pensées du simple au complexe**, *et (iv)* **faire des dénombrements complets**.".

Also from the Internet: Descartes thus proposed a simple, four-step process (i–iv) to ensure that the mind never accepts a falsehood as truth:

- **The Four Fundamental Rules**:

  (i) **The Rule of Evidence**: Never accept anything as true unless it is so "clear and distinct" that there is no reason to doubt it. This is a rejection of prejudice and jumping to conclusions.
  (ii) **The Rule of Analysis**: Break down large, complex problems into as many smaller parts as possible. Solving many small problems is easier than tackling one giant one.
  (iii) **The Rule of Synthesis**: Organize your thoughts by starting with the simplest objects and gradually "climbing" to the most complex, assuming an order even where none naturally exists.
  (iv) **The Rule of Enumeration**: Make thorough lists and reviews to ensure that nothing has been omitted. This is essentially a final "audit" of your logic.

7.8 A Critical Review of the DOMAIN Method

We risk a critical assessment of the DOMAIN method. The itemized list below is not prioritized.

- **Part Sets**: Part sets are endurants of the form PS = P-**set**. We have, not arbitrarily, but from experimental evidence [10], decided to limit parts p:P of part sets to be atomic. Is that limitation well-founded: not really necessary ? We must admit that it is not ! Let us give an **example**:

  > Let us take a domain of companies, say like A. P. Møller Maersk[163] https://www.maersk.com/. It has many divisions, typically one could model these many divisions as a part set. That is: a division is a part p:P where PS = P-**set**. Now these individual divisions would in the present DOMAIN method be modeled as atoms. No chance for divisions to have sets of subdivisions, that is: being [non-atomic] compounds !

  So, there really is no good reason to mandate that p:P in PS = P-**set** be atomic.
  So why have we, so far, restricted p:P in PS = P-**set** to be atomic. The simple, but perhaps embarrassing answer is: to keep some matters simple[164].
  An example of where the present limitation makes the presentation of the method simpler is in the Describe Endurant State stage of the procedure, Sect. 7.2.4.4 on page 130. See items 195d– 195e on page 130. "Lifting" the restriction from atoms to parts in general would mean that, say a recursively defined exploration of sub-parts in line 195e on page 130 would have to be specified – and all the domain examples [10] have "survived" with the restriction ! We leave it to the reader to lift the restriction !
- **Sequentiality of DOMAIN Modeling Procedure**: A "backbone" of the procedure of Sect. 7.2 (pages 124–139) is that it suggests a sequential domain modeling process.
  But is it always possible to model domains sequentially ?
  No: domain analyser cum describers may find that they have "come across" a "more correct", a more elegant, a shorter, ..., way for a domain model than the one they are currently pursuing. So what to do ? Stick to the less desirable, "past/current" approach – or shift to the more "correct" model ! We suggest, of course, that they shift.
  Such shifts imply that the modeling retrace: "go back to an earlier stage" – the one marking point where the currently less desirable model was "mis-directed" and redo the modeling stages from there.

[163] Maersk is a Danish company. From its homepage: *It is an integrated logistics company that offers supply chain solutions for managing shipments and cargo*

[164] Einstein: *"Everything Should Be Made as Simple as Possible, But Not Simpler"* https://quoteinvestigator.com/2011/-05/13/einstein-simple.

The question is then, what, if any of the current description can be salvaged, that is: edited into "a more desirable" model ?
The current procedure does not prescribe such domain modeling iterations !

- **Weak Treatment of Actions**: The treatment of actions, Sect. 7.2.6.3 on page 136 is presently not satisfactory. It lacks a systematic, convincingly "line of thought". The aim of Sect. 7.2.6.3 should include specific guidelines so that domain analyser cum describers can rigorously formulate the "tasks" of corresponding behaviour specifications.
- **Weak Treatment of Behaviour Patterns**: Similar to the treatment of actions, the treatment of behaviour patterns, that is the way behaviour definitions are structured, in Sect. 7.2.6.5 on page 137, is presently not satisfactory. It lacks a systematic, convincingly "line of thought". The aim of Sect. 7.2.6.5 should include specific guidelines so that domain analyser cum describers can rigorously formulate behaviour specifications.
- **Possible Formal Relations Between Taxonomies and The Formal Descriptions**: Sections 7.2.4.3 on page 129 and 7.2.6.4 on page 137 presented issues of endurant taxonomy, respectively behavioural taxonomy. It was suggested that these graphical renditions of domain facets in no way could be claimed to augment or detract from the [mathematical] meaning of the [emerging] domain model.
But suppose the could ! That is, can one give a semantics to these graphical renditions and formally relate them to the semantics of "the rest" of the [emerging] domain model. Or, put it differently, can these graphical renditions be automatically "derived" from that [emerging] model ?
- **Not Quite Satisfactory Treatment of Monitorables**: Monitorable attributes are such whose values are beyond "control" of "their" behaviours. Examples are *the velocity of an automobile, its acceleration, the temperature of an endurant, etc.*
The way the current version of the DOMAIN method treats monitorable attributes is as follows. First, monitorables, A, "appear" as arguments in behaviour definitions "by name", that is as ηA. Then, whenever encountered, as a, during the elaboration of a behaviour invocation, where the unique identifier of the behaviour is ui, that elaboration proceeds as follows:

  (i) The part, p, with unique identifier ui in state value σ_{parts} is "retrieved".
  (ii) If the elaboration is for the purposes of obtaining the current value, v of attribute a:A, then **attr**_A(p) is obtained and is the value of a.
  (iii) If the elaboration is for the purposes of changing the value of a, then its occurrence in behaviour ui is [somehow, say through a function ϕ] "paired" with a value, w with which to update attribute A of p. The new value, in p of σ_{parts}, is set to ϕ(v,w)[165] – leaving all other internal qualities unchanged !

  We are not quite happy with this model of monitorable attributes – but have not spent much time of finding other models. The reason for this "omission" is that we have not been confronted with domains where we have had to model parts with monitorable attributes. Having and original, engineering background in control theory, i.e., in its application[166], it appears that we have "shied" away and focused the DOMAIN method on discrete, rather than continuous, dynamics !
  But it might be worthwhile to reformulate the role of monitorables !
- **Precise Syntax of** DDL: There is, as yet, no document which specifies the syntax of DDL and RSL$^+$Text; but see [54, 55].

There will undoubtedly appear more critical items. [This list was first made up February 26, 2026.]

7.9 Conclusion

We have presented a method for analysing & describing domains – such as we have "defined" that term. The method entails a linear procedure, outlined here in 1+21 stages. It is, we suggest, an interesting

[165] We leave the details of ϕ to the reader's imagination !

[166] (i) Sandviken Steel: 18-9 Chrome-Nickel steel production automation [https://www.home.sandvik/en/]; (ii) Billerud: paper production automation [https://www.billerud.com/] – at the IBM Nordic Laboratories, Stockholm, Sweden in the early 1960s

"feature" that it is linear ! At each stage we need not refer to issues that are first introduced in later stages.

There is, however, more to domain modeling than the procedural model presented here !

There is, for example, the modeling of *domain facets* [57, *Chapter 8*]. We leave it to others to systematize their pursuit.

Chapter 8
Closing

Contents

8.1 What has been Achieved and Not Achieved?

8.1.1 What has been Achieved?

An initial phase of software development has been suggested, one that is also independent of whether software is indeed intended: that of domain engineering. A calculus of domain inquiry and a calculus of domain description have been put forward — calculi that are presently focused on domain endurants.

8.1.2 What has Not been Achieved?

The next section elucidates on both what has been, and what has not been achieved.

8.2 Related Issues

A number of issues related to domain modeling need be briefly addressed.

8.2.1 A Calculus of Perdurants

Chapters 4–5 uncovered what we would like to refer to as *calculus of endurants*. We should have liked to be able to also present, in Chapter 6, a *calculus of perdurants*. But I have, so far, not been able to uncover the mysteries of such a calculus of domain perdurants, if it exists ! ? Our treatment, for example, of the "patterns" of behaviour definitions is, cf. Sect. 6.8.1 on page 111, to us, *not* satisfactory.

8.2.2 Axioms, Well-formedness and Proof Obligations

The reader may have noticed that this primer hardly mentions the notion of verification, yet domain descriptions, as possibly any specification related to software, may require some form of verification. Yet this primer appears to skirt the issue. Indeed, we have, regrettably, omitted the issue. So we must refer the reader to relevant literature. We cannot, July 17, 2026, point to any definitive book on the topic. The field is under intense research. Instead we refer to such diverse papers as: [69, 96] as well as the seminal book [84].

In *Endurant Description Prompt* 2 on page 45 we mention the concept of proof obligation. They are also mentioned in *Attribute Description Prompt* 6 on page 72. In numerous other places we mention the concept of *axiom*: 5.2.4 on page 65 (uniqueness of part identifiers), 5 on page 67 (mereology), 114 on page 89 (property of time), And in some places we mention the concept of *well-formedness,* for example, Sect. 5.6.2.4 on page 93,

- **Axioms** express properties of endurants, whether external or internal qualities, that hold – as were they laws of the domain.
- **Well-formedness** predicates are defined where external or internal qualities of endurants are defined by concrete types in such ways as to warrant such predicates.
- **Proof obligations** are usually warranted where distinct sort definitions need be separated.

8.2.3 From Programming Language Semantics to Domain Models

In 1973–1974, at the *IBM Vienna Laboratory*, we, Peter Lucas, Hans Bekič, Cliff Jones, Wolfgang Henhapl and Dines Bjørner researched & developed a formal description of the PL/I programming language [7]. In 1979–1984, at the *Dansk Datamatik Center, DDC*, under Dines Bjørner's leadership and with invaluable help from his colleague, Dr. Hans Bruun, and based on Dines Bjørner's MSc. lectures, seven M.Sc. students[167] developed formal descriptions of (and later full compilers for) the programming languages CHILL [104] and Ada, the latter under the informed management of Dr. Ole N. Oest [66].

In a domain model we describe essential nouns and verbs of the "language spoken" by practitioners of the domain. The "extension" from the language "spoken by programmers" to that "spoken by domain practitioners" should be obvious.

In both cases, the descriptions, for realistic programming languages and for realistic domains, are not trivial. They are sizable. The PL/I, CHILL and Ada descriptions span from a hundred pages to several hundred pages ! Similarly, their implementation, in terms of interpreters and compilers, took many man-years. For the *DDC* Ada Compiler [80, 103] it took 44 man-years !

From a description of realistic facets of a domain one can develop a number of more-or-less distinct requirements, and from these one can develop computing systems software and we can expect similar size efforts.

[167] Jørgen Bundgaard, Ole Dommergaard, Peter L. Haff, Hans Henrik Løvengreen, Jan Storbank Petersen, Søren Prehn, Lennart Schulz

8.2.4 Domain Specific Languages

A domain specific language, generically referred to as a DSL, is a language whose basic syntactic elements directly reflect endurants and perdurants of a specific domain. *Actulus*, a language in which to express calculations of actuarian character [78], is a DSL.

The semantics of a DSL, obviously, must relate to a model for the domain in question. In fact, we advice, that DSLs be developed from the basis of relevant domain models.

8.2.5 The RAISE Specification Language, RSL

We refer to Sect. 1.10 on page 5. So we have used RSL in two ways in this primer: (i) informally, to explain the domain analysis & description method – in RSL$^+$, and (ii) formally, to present [fragments of] specific domain specifications. The latter always in enumerated examples.[168] Appendices A–B both exemplify formal uses of RSL. All the functions listed in Index Sects. D.5–D.7 and their explication are using the informal RSL$^+$.

8.2.6 Rôle of Algorithms

In all of the function formulation of domain phenomena, in this primer, You have not seen a single, interesting algorithm ![169] We need not apologize for that. There is a reason. The reason is that we almost only describe properties. To that end we make use of classical mathematical notions such as set comprehension, for example: { a | a:A • $\mathcal{P}$(a) }. The search for an appropriate *a* such that $\mathcal{P}(a)$ holds is often what requires, often beautiful algorithms. We refer to [108, 128, *Knuth and Harel*]. The need for clever algorithms, usually, first arise when designing software. Not in requirements engineering (cf. Sect. 8.2.9 on the next page), but in software design. Then requirements prescriptions, also usually expressed in terms of set, list or map comprehension, or corresponding quantifications, need efficient implementations; hence clever algorithms.

8.2.7 CSP versus PDEs

To model the behaviour of discrete dynamic domains, such as are the main focus of this primer, we use the CSP process concept [114]. To model the behaviour of continuous dynamic domains, which we really have not, we suggest that You use methods of analysis, to wit: *[Partial] Differential Equations, PDEs.* Perhaps also some *Fuzzy Logic* [124, 179]. That is: We see this as the "dividing line" between discrete and continuous dynamic systems modeling: *CSP versus DPEs.* Appendix B, pages 173–195, puts forward a domain whose continuous dynamics need be formalised, for example using PDEs [83]. Mathematical modeling such as based on *Adaptive Control Theory* [3], *Stochastic Control Theory* [126] or maybe *Fuzzy Control* [137], like algorithmics, first be required as possible techniques when issues of correct continuous dynamics and optimisation arise, as when implementing certain requirements.

[168] These are: Examples 36 on page 45, 37 on page 48, 38 on page 48, 43 on page 55, 45 on page 64, 47 on page 65, 48 on page 66, 49 on page 66, 50 on page 68, 51 on page 68, 54 on page 70, 61 on page 75, 62 on page 75, 63 on page 76, 73 on page 93, 83 on page 112, 84 on page 114, 86 on page 115, 87 on page 117, 88 on page 117, 89 on page 119, and 90 on page 120

[169] Algorithm: a process or set of rules to be followed in calculations or other problem-solving operations, especially by a computer.

8.2.8 Domain Facets

There are other, additional methodological domain modeling steps. In [45, Chapter 8, Pages 205–240] we cover the notion of *domain facets*. By a domain facet we shall understand one amongst a finite set of generic ways of analysing a domain: a view of the domain, such that the different facets cover conceptually different views, and such that these views together cover the domain. We here list:

- intrinsics,
- support technologies,
- rules & regulations, including
  - scripts,
  - license languages,
- management & organisation, and
- human behaviour.

as such facets. The referenced chapter ([45, Chapter 8, Pages 205–240]) is traditional, programming methodological, in the sense that there is no [semi-]formal calculi involved, as in this primer's Chapters 4–5. We could wish for that !

8.2.9 Requirements Engineering

Domain modeling, to repeat, can be pursued for two different, but related, reasons. (i) Simply, without any concern for, or idea of possible software, in order to "just" understand a domain, or (ii) for reasons of subsequent software development. In the later case a step of *requirements engineering* need be pursued. [45, Chapter 9, Pages 243–298] covers a notion of *requirements engineering*. In that chapter we show three stages of requirements development: (α) *domain requirements*, (β) *interface requirements*, and (γ) *machine requirements*. But first a definition of the term *'machine'*.

Definition 101 **Machine**. By *machine* we shall understand a, or the, combination of hardware and software that is the target for, or result of the required computing systems development.

Definition 102 **Requirement**. By a *requirement* we shall understand (cf., IEEE Standard 610.12): *"A condition or capability needed by a user to solve a problem or achieve an objective."* ▪

Definition 103 **Domain Requirements**. By *domain requirements* we shall understand those requirements which can be expressed sôlely using terms of the domain ▪

Definition 104 **Interface Requirements**. By *interface requirements* we shall understand those requirements which can be expressed only using technical terms of both the domain and the machine ▪

Definition 105 **Machine Requirements**. By *machine requirements* we shall understand those requirements which, in principle, can be expressed sôlely using terms of the machine ▪

The *domain requirements* stage of requirements development starts with a basis in the domain engineering's domain description. It is, so-to-speak, a first step in the development of a requirements prescription.[170] From there follows, according to [45, Chapter 9] a number of (five) steps:

[170] The "passage" from domain description to requirements prescription marks a transcendental deduction. Domain descriptions designate that which is being described. Requirements prescriptions designate what is intended to be implemented by computing. Please note the distinction: At the end of the development of a domain description we have just that: a domain description. At the beginning of the development of a requirements prescription we consider the domain description to be the initial requirements prescription: Thus, seemingly bewildering, in one instance a document is considered a domain description, in the next instance, without that document having been textually changed, it is now considered a requirements prescription. The transition from domain description to requirements prescription also marks a transition from "no-design mode" description to "design-mode" prescription.

Definition 106 Projection. By projection is meant a subset of the domain description, one which projects out all those endurants: parts and fluids, as well as perdurants: actions, events and behaviours that the stake-holders do not wish represented or relied upon by the machine.

Definition 107 Instantiation. By instantiation we mean a refinement of the partial domain requirements prescription (resulting from the projection step) in which the refinements aim at rendering more concrete, more specific the endurants: parts and fluids, as well as the perdurants: actions, events and behaviours of the domain requirements prescription.

Definition 108 Determination. By determination we mean a refinement of the partial domain requirements prescription, resulting from the instantiation step, in which the refinements aim at rendering less non-determinate, more determinate the endurants: parts and fluids, as well as the perdurants: functions, events and behaviours of the partial domain requirements prescription.

Definition 109 Extension. By extension we understand the introduction of endurants and perdurants that were not feasible in the original domain, but for which, with computing and communication, and with new, emerging technologies, for example, sensors, actuators and satellites, there is the possibility of feasible implementations, hence the requirements, that what is introduced becomes part of the unfolding requirements prescription.

Definition 110 Fitting. By requirements fitting we mean a harmonisation of two or more domain requirements that have overlapping (shared) not always consistent parts and which results in n partial domain requirement, and m shared domain requirement, that "fit into" two or more of the partial domain requirements.

[45, Chapter 9] then goes on to outline interface and machine requirements steps.

So domain engineering is a sound basis, we claim, for software development.

How that basis harmonises with the approaches taken by *Axel van Lamsweerde* [129] and *Michael A. Jackson* [122] is, really, a worthwhile study in-and-by itself!

8.2.10 Possible [PhD] Research Topics

We list here a number of possible (PhD) research topics:

1. **Intentional Pull:** This topic is not treated to the depth it deserves in this primer. Try think of intentional pulls in several domains: (i) *money flow in financial institutions* (while domain modeling a fair selection of such: banks, credit card companies, brokers, stock exchanges [32], etc.); (ii) *railway systems* (study, for example, [17, 18, 58, 62, 63, 67, 140, 152, 169]); and (iii) *container terminals* (see [42]).
2. **Discrete vs. Continuous Endurants and Perdurants:** Take the example of (oil, gas, water) *pipelines*. See Appendix B. Try model the dynamic flow of liquid in pipes, valves, pumps, etc., that is "mix", as may be expected, differential equations with RSL formulas. Some have tried. No real progress seems attained. Se however [177, 178]. The pipeline example should illustrate the use of monitorable attributes, their "reading" and their "biddable updates".
   The challenge here is threefold: (i) first the PDE etc. modeling of the flow for each kind of unit, including curved pipe units; (ii) then for their composition – for a specific layout, for example that hinted at in Fig. 4.1 on page 38; and (iii) finally for the infinite collection of pipeline systems such as defined by the "abstract syntax" of Appendix. B Item 289 on page 177 (including its wellformedness).
3. **Towards a Calculus of Perdurants:** This primer has unveiled the beginnings of a *Calculus of Endurants*. (Yet, its real "calculus-orientation" has yet to emerge: its laws, etc.) Sect. 6.8 hints at what we have in mind. A systematic analysis which aims at uncovering a fixed number of behaviour patterns such as sketched in Sect. 6.8.

4 **Modeling Human Interaction**: The "running example", summarised in Appendix A, illustrated a road net "populated" with automobiles driving "hither & dither". The current primer has not treated the interaction between humans and man-made artifacts, like, for example, drivers and their automobiles. You are to model, for example, such human actions as starting an automobile, accelerating, braking, turning left, turning right, and stopping. Doing so You will have to try out, experiment with the rôles of monitorable, including biddable automobile attributes. An aim, besides such a domain model, is to research method issues of modeling human interaction. Please disregard modeling issues of sentiments, feelings, etc.

5 **Transcendental Deduction**: In the philosophy of Kai Sørlander such as, for example, explained in Chapter 2, transcendental deduction is appealed to repeatably. In this primer, as in [45], transcendental deduction is appealed to only once ! Maybe research into possible calculi for perdurants, cf. Research Challenge 3, might yield some more examples of transcendental deductions.

6 **Formal Models of Domain Modeling Calculi**: In [36] an attempt is made at "the" first formal model of the domain analysis & description calculi. With [45] and, especially, this primer as a background, perhaps a more thorough attempt should be made to bring the model of [36] up-to-date and complete !

7 **Kai Sørlander's Philosophy**: We refer to Chapter 2. It is here strongly suggested that this research project be based on [165], Kai Sørlander's most recent book.[171] The challenge, in a sense, has two elements: (i) the identification of Sørlander's use of transcendental deduction: painstakingly identifying **all** it uses, analysing each of these, studying whether one can characterise these uses into more than one common kind of deduction, or whether one might claim *"classes of deductions"*, not necessarily disjoint, but perhaps structured in some kind of taxonomy; and (ii) the analysis of this primer's presentation of Sørlander's metaphysics.

8.3 Closing Remarks

8.3.1 Endurants versus Perdurants

The number of concepts pertaining to endurants versus the number of concepts pertaining to perdurants appears to signify something ! The number of concept definitions that relate to endurants is around 80. Those of perdurant concepts is around 10 ! How can that large difference be understood ?

8.3.2 Domain Science & Engineering

The present primer represents, at the moment, a long line of development. As mentioned in Sect. 1.5.2.1 on page 3 this grew out of a series of works: [21, 29, 33, 40, 44, 45]. The first inklings — in our work on what is now the *Domain Science & Engineering* of this primer— appeared in [11–14, 61, 1995-1996]. The *UN University's International Institute for Software Technology,* UNU/IIST conducted several domain engineering-based research & development projects, most of them under the leaderships of (the late) Søren Prehn and Chris W. George [97]. [28, 2008] touched upon the concept of *Domain Facets*, not covered in this primer, but in [45, 2021]. Two papers [29, 33, 2010] suggested reasonably relevant properties of domain descriptions. It was not until [40, 2017] that the analysis & description calculi of this primer emerged, and were refined in [44, 2019].

The iteration from [21] via [29, 33, 40, 44, 45] to the present primer reminds us of the French author *Anatole France's* story of the history of the mankind.[172]

[171] All of Sørlander's books [161–165, 1994–2022] are in Danish – so the researcher would either be able to read Danish, or, more preferably to us, to have a suitable (German, English [166], French, ...) translation at hand.

[172] On acceding to the throne of Persia, a young king assembled all the academicians of his realm and charged them with writing a detailed history of mankind, that he may learn from it to become a wise ruler.

8.4 Acknowledgments

In [45, *Preface/Acknowledgments, Page xiv*] the first author acknowledged the very many who, over his professional life, has inspired him. In "rewriting" this primer from [45] the first author has, again, attempted to "capture" Kai Sørlander's Philosophy, cf. Chapter 2. And again we wish to deeply acknowledge that work and, hence, Kai Sørlander. Here the first author, additionally,wishes to acknowledge, with pleasure, Laura Kovacs, TU Wien. Laura invited him to lecture, in the fall of 2022[173], at TU Wien. This primer is the result of that invitation. Dr. Mikhail Chupilko[174] together with colleagues, Dr. Anastasia Oleynik etc., and Dr. Yang ShaoFa[175] are currently translating this primer into Russian, respectively Chinese. The first author acknowledges, with many thanks, their ongoing comments.

The wise men deliberated and returned after twenty years with twelve camels, each carrying five hundred volumes. But the king could not find the time to read so many volumes, and tasked them with reducing the number of volumes "to the brevity of human existence".

The academicians worked for another twenty years and returned with fifteen hundred volumes. But the king said, "I am getting old and cannot read all these volumes".

The academicians returned after ten years with five hundred volumes but the king asked them to shorten it further so that he could learn, before dying, human history.

After five years, a lone academician carrying a single volume arrived at the palace. "Hurry up", an officer told him, "the king is dying". The king looked at the academician and said, "So I shall die without knowing human history".

"Sir", replied the academician, "I can summarize it for you in three words: –they are born, they suffer, they die." [http://profzeki.blogspot.com/2012/05/anatole-france-and-reductionism.html]

[173] Well, an invitation for Covid-19 year 2021 had to be postponed !

[174] ISP/RAS: Institute of Systems Programming, The Russian Academy of Sciences, Moscow

[175] IoS/CAS: Institute of Software, The Chinese Academy of Sciences, Beijing

Chapter 9
Bibliography

Contents

9.1 Bibliographical Notes

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    - *The "7 Seas"*,
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    - *Credit Cards*,
    - *Weather Information*,
    - *Documents*,
    - *Urban Planning*,
    - *Swarms of Drones*,
    - *Container Terminals*,
    - *A Retailer Market*,
    - *Double-entry Bookkeeping*,
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Appendix A
An Example: A Simple Road Transport

Contents

A.1 Universe of Discourse

213 The universe of discourse is that of
214 road traffic, RT –
215 whose narrative is then presented.

213. **universe of discourse**:
214. **type** RT
215. **narrative**: The road traffic domain consists of a road net aggregate and an automobile aggregate.
Road net aggregates consists of aggregates of hubs (street [link] intersections) and links.
The aggregates of hubs and links consists of sets of hubs and sets of links.
Hubs and links are here considered atomic.
Automobile aggregates ″are″ a set **of** automobiles −− that are here considered atomic.
Hubs, links and automobiles have unique identification, mereologies and attributes.
Automobile ride along hubs and links; leave hubs [links] to enter links [hubs], etc.

A.2 Compound Endurants

216 In the road traffic domain we can observe a road net aggregate and an automobile aggregate.

217 In a road net aggregate we can observer an aggregate of hubs and an aggregate of links.
218 In an aggregate of automobiles we can observe a set of [atomic] automobiles.
219 Aggregates of hubs ad links are sets of [atomic] hubs and links.

type
216. RNA, AA
217. AH, AL
218. AS = A-**set**
219. HS = H-**set**, LS = L-**set**

value
216. **obs**_RNA: RT → RNA, **obs**_AA: RT → AA
217. **obs**_AH: RNA → AH, **obs**_AL: RNA → AL
218. **obs**_AS: AA → AS
219. **obs**_HS: AH → RS, **obs**_LS: AL → LS

A.3 Endurant Values

220 There is the road transport [universe of discourse].
221 There is the road net aggregate.
222 There is the automobile aggregate.
223 There is the hub aggregate.
224 There is the link aggregate.
225 There is the set of automobiles.
226 There is the set of hubs.
227 There is the set of links.
228 And there is the entire set of all of these.
229 And there is the entire set of just all automobiles, hubs and links.

value
220. rt:RT
221. rna:RNA = **obs**_RNA(rt)
222. aa:AA = **obs**_AA(rt)
223. ah:AH = **obs**_AA(rna)
224. al:AL = **obs**_AL(rna)
225. as:AS = **obs**_AS(ah)
226. hs:HS = **obs**_HS(ah)
227. ls:LS = **obs**_LS(al)
228. σ_{parts} = {rt,rna,aa,ah,al} ∪ aa ∪ as ∪ hs ∪ ls
229. σ_{atoms} = aa ∪ hs ∪ ls

A.4 Endurant Taxonomy

For the simple automobile/hub/link domain Fig. A.1 on the next page may serve as a graphic rendition of its endurant taxonomy.

For a slightly more sophisticated, multi-mode transport domain — with customers, logistics & conveyor companies and [truck, train, ship and aircraft] conveyor and roads, rails, water and air transport of merchandise (including passengers) — Fig. A.2 on the facing page may serve as a graphic rendition of "its" endurant taxonomy.

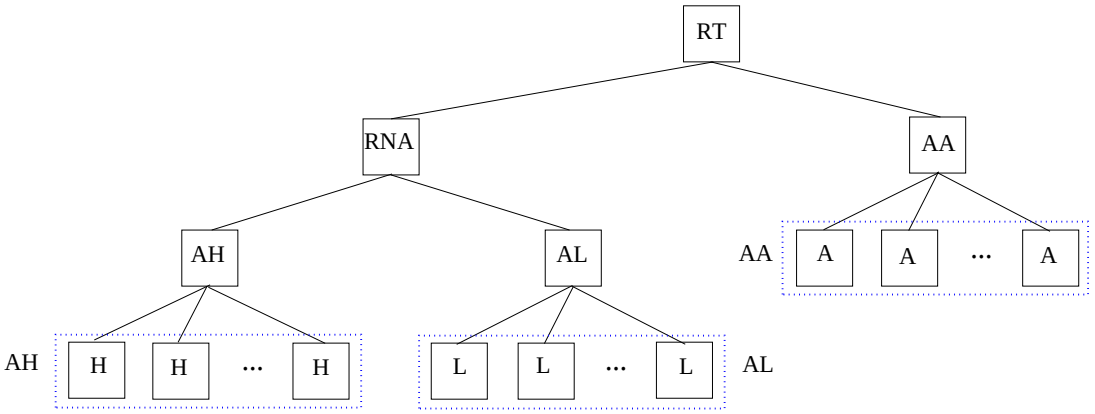

Fig. A.1 A Road Transport Domain Endurant Taxonomy

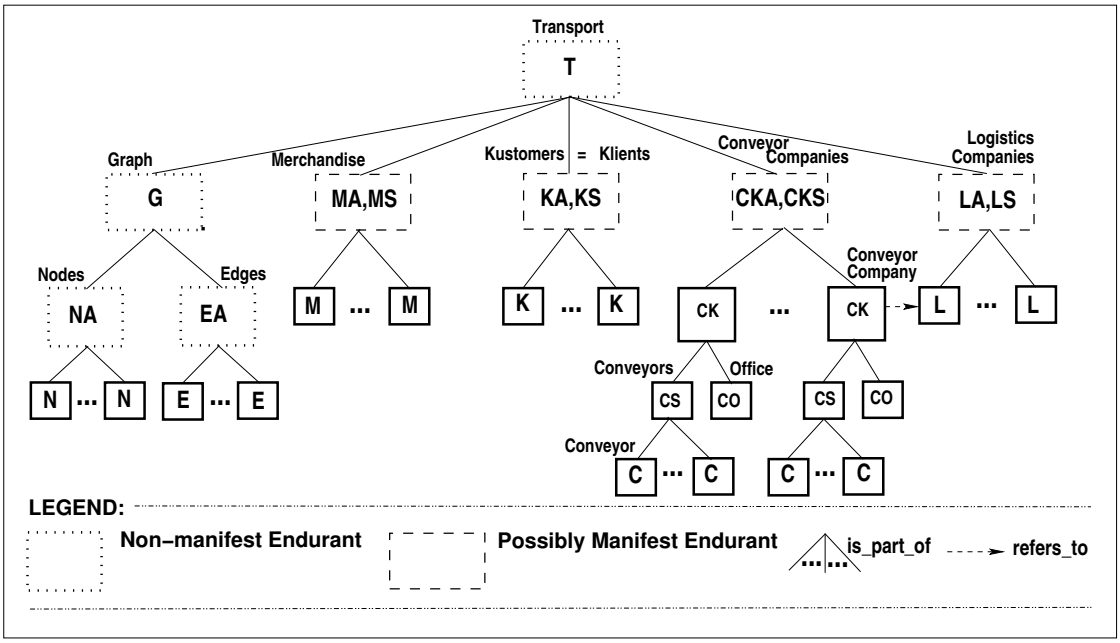

Fig. A.2 Multi-mode Transport Domain Endurant Taxonomy

We refer to [53, *Technical report*, https://www.imm.dtu.dk/~dibj/2025/transport/main.pdf.] and Sect. A.13 on page 167.

A.5 Manifestation and Mobility

230 Hubs, links and automobiles are manifest.
231 Hubs and links are stationary.
232 Automobiles are mobile.

axiom

230. ∀ e:(H|L|A) • **is**_manifest(e)
231. ∀ r:(H|L) • **is**_stationary(r) [≡ ~ **is**_mobile(r)]
232. ∀ a:A • **is**_mobile(e)

A.6 Unique Identification

The unique identifierss of a road transport, rt:RT, is here limited to just hubs, links and automobiles:

233 The universe of discourse has a unique identifier.
234 The road net agregate has a unique identifier.
235 The automobile aggregate has a unique identifier.
236 The hub aggregate has a unique identifier.
237 The link aggregate has a unique identifier.
238 Each hub has a unique identifier.
239 Each link has a unique identifier.
240 Each automobile has a unique identifier.

type
233. RTI
234. RNAI
235. AAI
236. HAI
237. LAI
238. HI
239. LI
240. AI

value
233. **uid**_RT: RT → RTI
234. **uid**_RNA: RNA → RNAI
235. **uid**_AA: AA → AAI
236. **uid**_HA: HA → HAI
237. **uid**_LA: LA → LAI
238. **uid**_H: H → HI
239. **uid**_L: L → LI
240. **uid**_A: A → AI ▪

A.7 Unique Identifier Values

241 There is the unique identifier of the road transport.
242 There is the unique identifier of the road net aggregate.
243 There is the unique identifier of the autombile aggregate.
244 There is the unique identifier of the hub aggregate.
245 There is the unique identifier of the link aggregate.
246 There are the unique identifiers of the automobiles of the set of autombiles.
247 There are the unique identifiers of the hubs of the set of hubs.
248 There are the unique identifiers of the links of the set of links.
249 And there is the entire set of all of these.
250 And there is the entire set of all identifiers of the automobiles, hubs and links.

value
241. rt_{uid}RT: RT = **uid**_RT(rt)
242. rna_{uid}RNA: RNA = **uid**_RNA(rt)
243. aa_{uid}AA: AA = **uid**_AA(rt)
244. ah_{uid}AH: AH = **uid**_AA(rna)
245. al_{uid}AL: AL = **uid**_AL(rna)
246. as_{uid}AS: AS = {**uid**_A(a)|a:A•a∈as}
247. hs_{uid}:HS = {**uid**_H(h)|h:H•h∈hs}
248. ls_{uid}:LS = {**uid**_L(l)|l:L•a∈ls}
249. $\sigma_{parts_{uid}}$ = {rt,rna,aa,ah,al} ∪ aa ∪ hs unon ls

250. $\sigma_{atoms_{uids}}$ = aa ∪ hs ∪ ls

A.8 Uniqueness of Endurants

251 Parts are uniquely identfied.

axiom
251. **card** σ_{atoms} = **card** $\sigma_{atoms_{uids}}$

A.9 Mereology

We shall be concerned only with the mereology of some manifest parts.

252 The mereology of links is a 2 element set of hub identifiers of the road net[1].
253 The mereology of a hub is a possibly empty set of hub identifiers of the road net.
254 The mereology of an automobile is [some subset of] a set of hub and link identifiers[2]

type
252. ML = LI-**set** **axiom** ∀ ml:MK • **card** ml = 2 ∧ ml ⊆ ls_{uis}
253. MH = HI-**set** **axiom** ∀ mh:MH • mh ⊆ hs_{uis}
254. MA = (HI|LI)-**set** **axiom** ∀ ma:MA • ma ⊆ as_{uis}
value
252. **mereo_**L: L → ML
253. **mereo_**H: H → MH
254. **mereo_**A: A → MA ▪

A.10 Attributes

Example attributes are:

- **Hubs**:

255 Hubs have states, hσ:HΣ: the set of pairs of link identifiers, ($f$li,$t$li), of the links $f$rom and to which automobiles may enter, respectively leave the hub.
256 Hubs have state spaces, hω:HΩ: the set of hub states "signaling" which states are open/closed, i.e., **green**/**red**.

- **Links**:

257 Links that have lengths, LEN;
258 Links have states, lσ:LΣ: the set of pairs of link identifiers, ($f$li,$t$li), of the links $f$rom and to which automobiles may enter, respectively leave the hub.
259 Links have state spaces, lω:LΩ: the set of link states "signaling" which states are open/closed, i.e., **green**/**red**.

[1] This is a simplified version: it allows for automoblie traffic in both directions of the link. We leave it to the reader to "cook" up othe such traffic possibilities.
[2] – a full set means that the specific automobile is allowed to travel all over the net.

- **Automobiles:**

260 Automobiles have road net positions, APos,
  a either *at a hub*, atH,
  b or *on a link*, onL, some fraction, f:**Real**, down a link, identified by li, from a hub, identified by fhi, towards a hub, identified by thi.
261 Automobiles have velocity;
262 Automobiles have acceleration;
263 Etc.[3]

- **Hubs, Links, Automobiles:**

264 Hubs, links and automobiles have *histories:* time-stamped, chronologically ordered sequences of automobiles entering and leaving links and hubs, with automobile histories similarly recording hubs and links entered and left.
265 Link positions have well-defined identifiers and fractions.

type
255. HΣ = (LI×LI)-**set** [**prg**]
256. HΩ = HΣ-**set** [**sta**]
257. LEN = **Nat** [**sta**] [m]
258. LΣ = (HI×HI)-**set** [**prg**]
259. LΩ = LΣ-**set** [**sta**]
260. APos = atH | onL [**prg**]
260a. atH :: HI
260b. onL :: LI × (fhi:HI × f:**Real** × thi:HI)
261. Veloc = **Real** [**mon**] [km/h]
262. Accel = **Real** [**mon**] [m/sec]
263. ...
264. HHis,LHis = (𝕋𝕀𝕄𝔼×AI)* [**prg**]
264. AHis = (𝕋𝕀𝕄𝔼×(HI|LI))* [**prg**]

value
255. attr_HΣ: H → HΣ
256. attr_HΩ: H → HΩ
257. attr_LEN: L → LEN
258. attr_LΣ: L → LΣ
259. attr_LΩ: L → LΩ
260. attr_APos: A → APos
261. attr_Veloc: A → Veloc
262. attr_Accel: A → Accel
264. attr_HHis: H → HHis
264. attr_LHis: L → LHis
264. attr_AHis: A → AHis

axiom
265. ∀ mk_onL(li,(fhi,f,thi)):onL • 0<f<1 ∧ li∈ls_{uids} ∧ {fhi,thi}⊆hs_{uids} ∧ ...▪

A.11 Intentional Pull

266 An *intentional pull* of any road transport system, *rts*, is then:

  a if for any automobile, *a*, of *rts*, on a link, ℓ (hub, *h*), at time τ,
  b then that link, ℓ, (hub *h*) "records" automobile *a* at that time.

267 and:

  c if for any link, ℓ (hub, *h*) being visited by an automobile, *a*, at time τ,
  d then that automobile, *a*, is visiting that link, ℓ (hub, *h*), at that time.

axiom
266a. ∀ a:A • a ∈ *as* ⇒
266a. **let** ahist = attr_AHist(a) **in**
266a. ∀ ui:(LI|HI) • ui ∈ **dom** ahist ⇒
266b. ∀ τ:𝕋𝕀𝕄𝔼 • τ ∈ **elems** ahist(ui) ⇒
266b. **let** hist = is_LI(ui) → attr_LHist(retr_L(ui))(σ),

[3]

266b. _ → attr_HHist(retr_H(ui))(σ) **in**
266b. τ ∈ **elems** hist(uid_A(a)) **end end**
267. ∧
267c. ∀ u:(L|H) • u ∈ *ls*∪*hs* ⇒
267c. **let** uhist = attr(L|H)Hist(u) **in**
267d. ∀ ai:AI • ai ∈ **dom** uhist ⇒
267d. ∀ τ:𝕋𝕀𝕄𝔼 • τ ∈ **elems** uhist(ai) ⇒
267d. **let** ahist = attr_AHist(retr_A(ai))(σ) **in**
267d. τ ∈ **elems** uhist(ai) **end end**

A.12 Channel

268 There is a set of channels between hubs, links and automobiles.
269 These channels communicate messages, M. M will "transpire" frm the behaviour definitions.

channel
268. { ch[{ui,uj}] | {ui,ij}:(HI|LI|AI)-**set** • ui≠uj∧{ui,uj}⊆σ_{uids} } M
type
268. M ▪

A.13 Perdurant Taxonomy

For the simple automobile/hub/link domain Fig. A.3 may serve as a graphic rendition of its perdurant taxonomy.

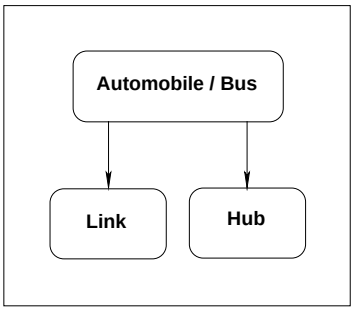

Fig. A.3 A Road Transport Domain Perdurant Taxonomy

For a slightly more sophisticated, multi-mode domain — with customers, logistics & conveyor companies and [truck, train, ship and aircraft] conveyor and roads, rails, water and air transport of merchandise (including passengers) — Fig. A.4 on the next page may serve as a graphic rendition of "its" perdurant taxonomy.

We refer to [53, *Technical report*, https://www.imm.dtu.dk/~dibj/2025/transport/main.pdf.] and Sect. A.4 on page 162.

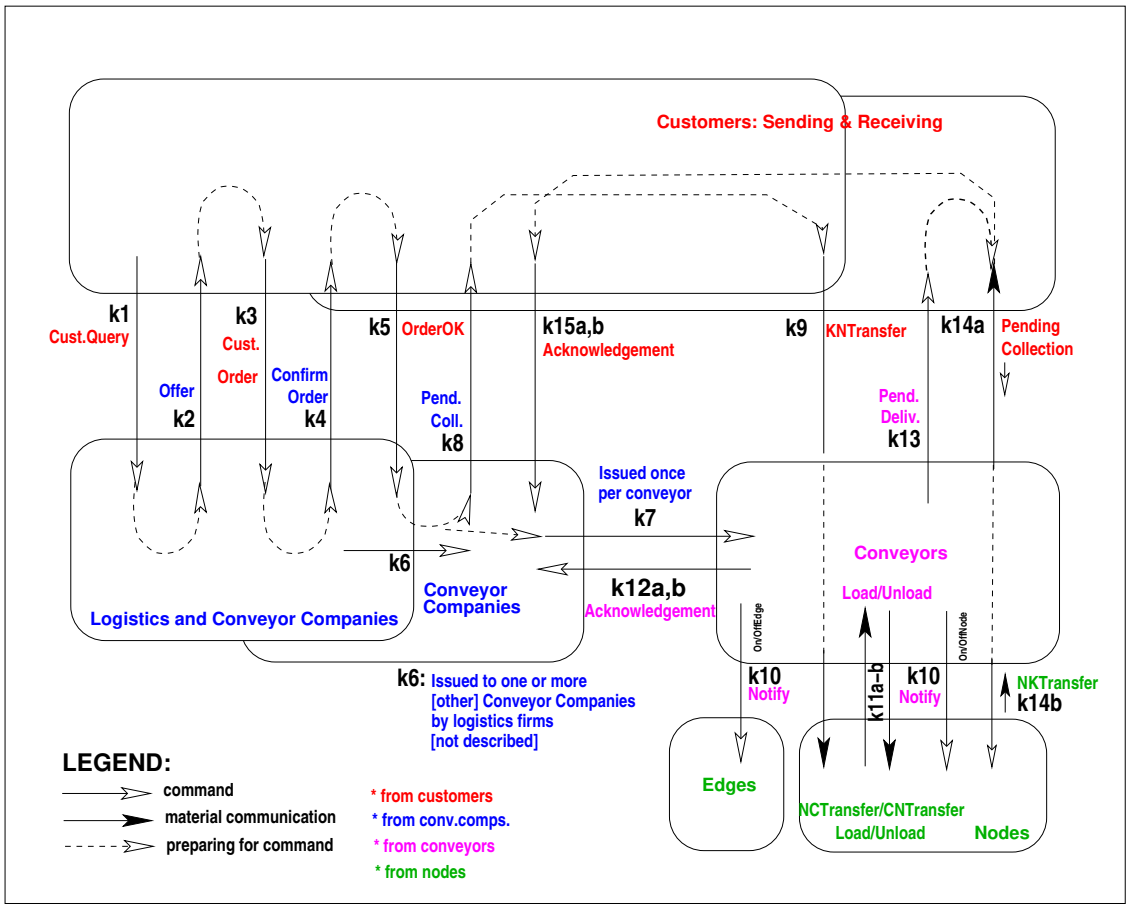

Fig. A.4 A Multi-mode Transport Domain Perdurant Taxonomy

A.14 Behaviours

There are three behaviours:

270 automobile, corresponding to endurants a:A, and with appropriate argument types,
271 hub, corresponding to endurants h:H, and with appropriate argument types, and
272 link, corresponding to endurants l:L, and with appropriate argument types.

value
270. automobile: AI → AM → ... → (Apos × AHist) → **Unit**
271. hub: HI → HM → (HΩ × ...) → (HΣ × HHist) → **Unit**
272. link: LI → LM → (LEN × LΩ × ...) → (LΣ × LHist) → **Unit**

273 The **automobile** behaviour is either *at a hub* or *on a link* – and **communicates** with the *hub* and *link* behaviours as to its entering, leaving, or remaining at the hub, respectively on the link.
274 Any **hub** behaviour is "passively" awaiting **communication** from automobile behaviours as to their entering, leaving, or remaining at the hub.
275 Any **link** behaviour is "passively" awaiting **communication** from automobile behaviours as to their entering, leaving, or remaining on the link.

273. automobile(ai)(ris)(...)(atH(hi),ahis)
273. automobile(ai)(ris)(...)(onL(li,(fhi,f,thi)),ahis)

274. hub(hi)(mh)(hω,...)(hσ,hhist)
275. link(li)(ml)(len,lω,...)(lσ,lhist)

276 We abstract automobile behaviour at a Hub (hi).

a Either the automobile remains at the hub,
b or, **internally non-deterministically**,
c leaves the hub entering a link,
d or, **internally non-deterministically**,
e stops.

```
276  automobile(ai)(ris)(...)(atH(hi),ahis) ≡
276a     automobile_remain_at_hub(ai)(ris)(...)(atH(hi),ahis)
276b   ⌈⌉
276c     automobile_leaving_hub(ai)(ris)(...)(atH(hi),ahis)
276d   ⌈⌉
276e     automobile_stop(ai)(ris)(...)(atH(hi),ahis)
```

where we leave it to the reader to fill in the signature of these three behaviours.

277 [276a] The automobile **remains** at a hub:

a time is recorded,
b informing the hub behaviour, whereupon
c the automobile remains at that hub, "idling",

```
277  automobile_remain_at_hub(ai)(ris)(...)(atH(hi),ahis) ≡
277a   let τ = record_TIME()() in
277b   ch[{ai,hi}] ! τ ;
277c   automobile(ai)(ris)(...)(atH(hi),⟨(τ,hi)⟩⌢ahis) end
```

278 [276c] The automobile **leaves** the hub entering link li:

a time is recorded;
b hub is informed of automobile leaving and link that it is entering;
c "whereupon" the vehicle resumes (i.e., "while at the same time" resuming) the vehicle behaviour positioned at the very beginning (0) of that link.

```
278  automobile_leaving_b(ai)({li}∪ris)(...)(atH(hi),ahis) ≡
278a   let τ = record_TIME()() in
278b   (ch[{ai,hi}] ! τ ‖ ch[{ai,li}] ! τ) ;
278c   automobile(ai)(ris)(...)(onL(li,(hi,0,_)),⟨(τ,li)⟩⌢ahis) end
278    pre: [hub is not isolated]
```

279 [276e] Or the automobile **stops**, "disappears — off the radar" !

```
279  automobile_stop(ai)(ris),(...)(atH(hi),ahis) ≡ stop ▪
```

280 We abstract automobile behaviour on a Link li:

a Either (*internally non-deterministically*) the automobile **remains** on the link, advancing a fraction or halts [temporarily],
b or, *internally non-deterministically*,
c **leaves** the link [when reaching its end] and enters the connected hub,
d or, *internally non-deterministically*,
e **stops** [leaves the road net altogether (!)].

```
280  automobile(ai)(ris)(...)(onL(li(fhi,f,thi)),lhis) ≡
280a      automobile_remains_on_link(ai)(ris)(...)(onL(li(fhi,f,thi)),lhis)
280b   ⌈⌉
280c      automobile_leaving_linl(ai)(ris)(...)(onL(li(fhi,f,thi)),lhis)
280d   ⌈⌉
280e      automobile_stops_on_link(ai)(ris)(...)(onL(li(fhi,f,thi)),lhis)
```

We leave it to the reader to complete the definitions of automobile_remains_on_link, automobile_-leaving_link and automobile_stops_on_link.

281 Hubs
282 external non-deterministically receives time-stamped, *t*, messages,*ai*, from automobiles as to their entering, remaining or leaving the hub.
283 They update their hub history accordingly and resume being a hub.

```
value
281.  hub(hi)(hm)(hω,...)(hσ,hhist) ≡
282.     let (t,ai) = □ { ch[{hi,ai}]? | ai ∈ hm } in
283.     hub(hi)(hm)(hω,...)(hσ,⟨(t,ai)⟩⌢hhist) end
```

284 Links
285 external non-deterministically receives time-stamped, *t*, messages,*ai*, from automobiles as to their entering, remaining or leaving the link.
286 They update their link history accordingly and resume being a link.

```
value
284.  link(li)(lm)(len,lω,...)(lσ,lhist) ≡
285.     let (t,ai) = □  { ch[{li,ai}]? | ai ∈ lm } in
286.     link(li)(lm)(len,lω,...)(lσ,⟨(t,ai)⟩⌢lhist) end
```

A.15 Initialization

287 Let us refer to the system initialization as parallel composition of behaviours:

a All hubs are initialized,
b and
c all links are initialized,
d and
e all automobiles are initialized.

```
value
287.  initialisation: Unit → Unit
287.  initialisation() ≡
287a.     ‖ { hub(uid_H(h))
287a.                 (mereo_H(h))
287a.                    (attr_HΩ(h),...)
287a.                       (attr_HΣ(h),attr_HΣ(h),attr_HHist(h))
287a.        | h:H • h ∈ hs }
287b.     ‖
287c.     ‖ { link(uid_L(l))
287c.                 (mereo_L(l))
287c.                    (attr_LEN(l),...)
287c.                       (attr_LΣ(l),attr_LHist(l))
287c.        | l:L • l ∈ ls }
287d.     ‖
287e.     ‖ { automobile(uid_A(a))
```

287e. (**mereo**_A(a))
287e. (**attr**_APos(a),**attr**_AHist(a))
287e. | a:A • a ∈ *as* }

A.16 Comments on the Example

The example of this appendix is an abstraction.
It fails to model:

- **Position of Automobiles**: We model the relative position, by fraction, of automobiles wrt. the end-points of links. Given the link length attributes an absolute position can be modeled.
- **Collision of Automobiles**: Two or more automobiles on a link, moving in either direction, may collide. This is not modeled – but is, of course a possibility. A road traffic domain model must include collisions – subsequent requirements prescriptions may then stipulate what is to be done about that !
- **Multi-lane Links**: Links, as modeled, have at least one lane. Automobiles move along that one, or several lanes. No attempt has been made to model multi-7, right and left, lane links.
- **Speed/Velocity and Acceleration**: No attempt has been made to model automobile drivers: their use of the accelerator and brake pedals – and the consequences (speed and acceleration) "thereof" !
- **Errands**: Automobile drivers may have errands at a hub or along a link: parking for shopping or other, filling up gasoline at a petrol station [similar for electric vehicles], etc.
- **Etc.**:

Appendix B
Pipelines, A Draft, Incomplete Example

Contents

Fig. B.1 The Planned Nabucco Pipeline: http://en.wikipedia.org/wiki/Nabucco_Pipeline

B.1 Illustrations of Pipeline Phenomena

The Nabucco pipeline was, for many years, a planned pipeline involving Austria, Turkey, Iran and other states and companies.

Fig. B.2 Pipeline Construction

An example pipeline construction. It shows the linking of pipe segments.

B.1.1 Pipes

A pipe segment is a straight "tube"-like unit.

B.1.2 Valves

A pipe valve allows for the control of flow in pipes.

Fig. B.3 Pipe Segments

Fig. B.4 Valves

B.1.3 Pumps

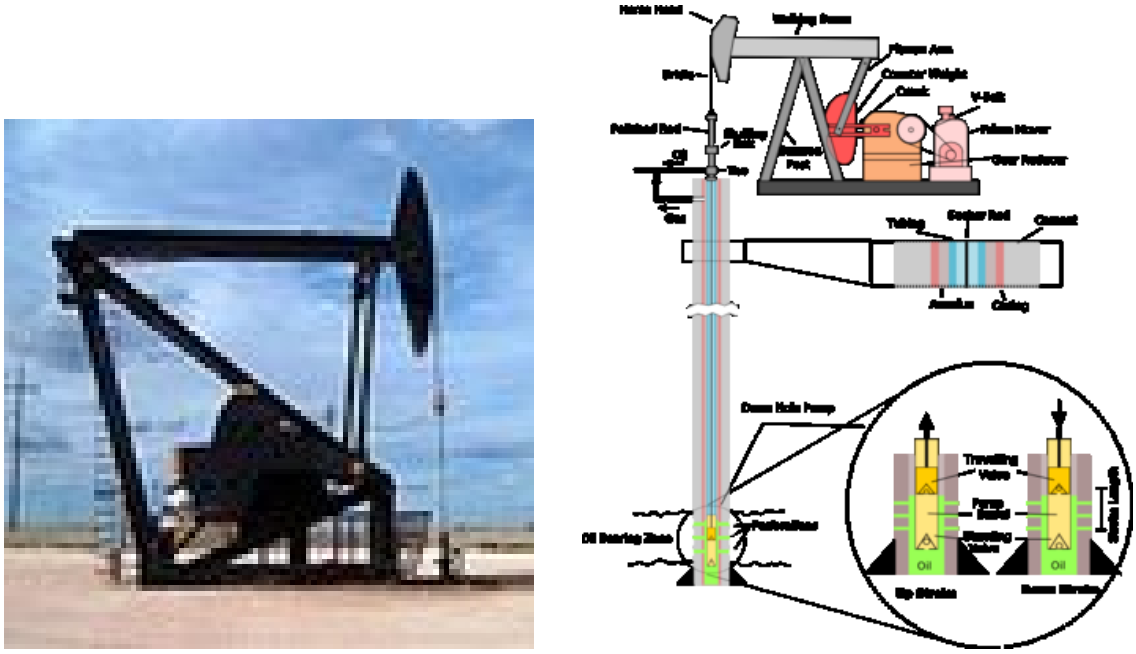

Fig. B.5 Oil Pumps

A pump allows for the "lifting" of, for example, oil, over hilly terrain. The concept of *pump head [height]* is relevant: The *head* is the height at which a pump can raise fluid up and is measured in meters or feet.

B.1.4 Compressors

Fig. B.6 Gas Compressors

A compressor is used for gaseous liquids. Compressors are similar to pumps: both increase the pressure on a fluid and both can transport the fluid through a pipe. As gases are compressible, the compressor also reduces the volume of a gas.

B.1.5 Pigs

Fig. B.7 New and Old Pigs

A "pig" is a tool that is sent down a pipeline and propelled by the pressure of the product flow in the pipeline itself. The primary purpose of pipeline pigs is to make sure that the pipe is clean and free from obstruction.

Fig. B.8 Pig Launcher, Receiver

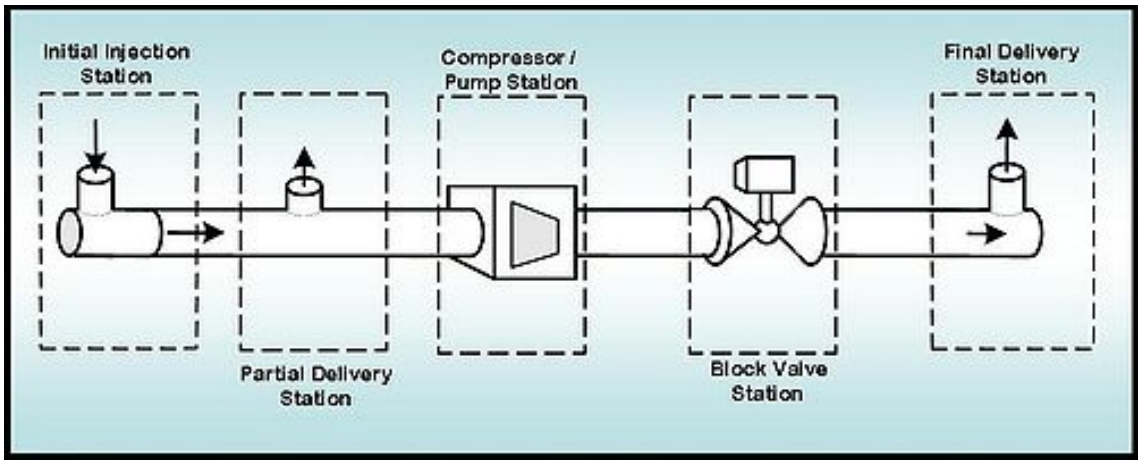

Fig. B.9 A Simple Pipeline:
Leftmost: A **Well**. 2nd from left: a **Fork**. Center: a **Pump**. 2nd from right a **Valve**. Rightmost: a **Sink**.

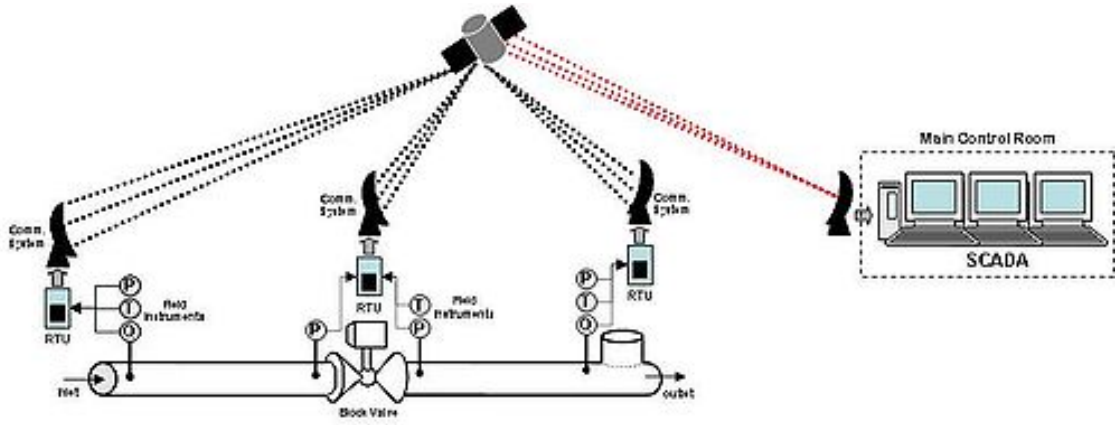

Fig. B.10 A Pipeline Monitoring & Control System Diagram A *SCADA [Supervisory Control And Data Acquisition]*

B.2 Endurants: External Qualities

We follow the ontology of Fig. 4.1 on page 38, the left hand dashed box labeled *External Qualities*.

B.2.1 Parts

288 A pipeline system contains a set of pipeline units and a pipeline system monitor.
289 The well-formedness of a pipeline system depends on its mereology (cf. Sect. B.3.2) and the routing of its pipes (cf. Sect. B.3.3.2).
290 A pipeline unit is either a well, a pipe, a pump, a valve, a fork, a join, a plate[4], or a sink unit.
291 We consider all these units to be distinguishable, i.e., the set of wells, the set pipe, etc., the set of sinks, to be disjoint.

type

[4] A *plate* unit is a usually circular, flat steel plate used to "begin" or "end" a pipe segment.

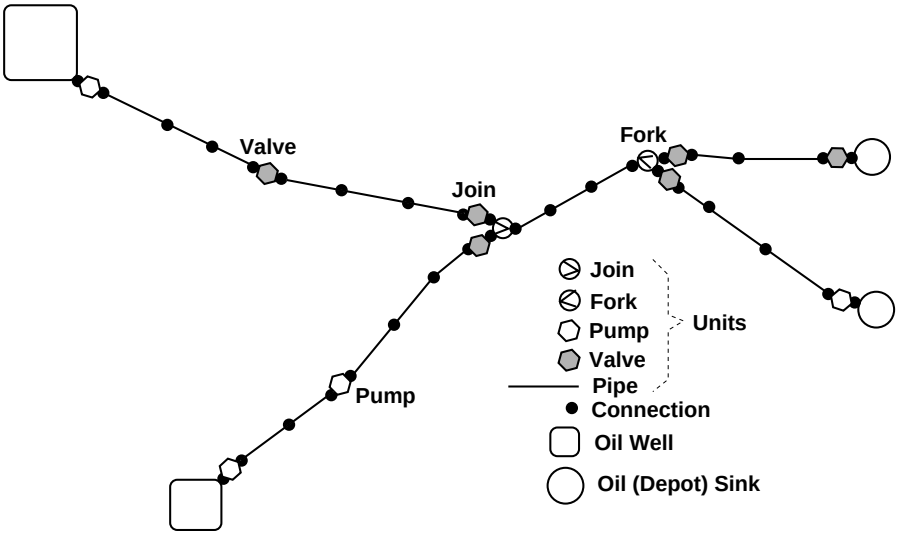

Fig. B.11 An example pipeline system

288. PLS′, U, M
289. PLS = {| pls:PLS′•wf_PLS(pls) |}

value

289. wf_PLS: PLS → **Bool**
289. wf_PLS(pls) ≡
289. wf_Mereology(pls)∧wf_Routes(pls)∧wf_Metrics(pls)[5]
288. obs_Us: PLS → U-**set**
288. obs_M: PLS → M

type

290. U = We | Pi | Pu | Va | Fo | Jo | Pl | Si
291. We :: Well
291. Pi :: Pipe
291. Pu :: Pump
291. Va :: Valv
291. Fo :: Fork
291. Jo :: Join
291. Pl :: Plate
291. Si :: Sink

B.2.2 An Endurant State

292 For a given pipeline system
293 we exemplify an endurant state σ
294 composed of the given pipeline system and all its manifest units, i.e., without plates.

value

292. pls:PLS

variable

[5] wf_Mereology, wf_Routes and wf_Metrics will be explained in Sects. B.3.2.2 on page 180, B.3.3.2 on page 181, and B.3.4.3 on page 185.

293. σ := collect_state(pls)
value
294. collect_state: PLS
294. collect_state(pls) ≡ {pls} ∪ obs_Us(pls) \ Pl

B.3 Endurants: Internal Qualities

We follow the ontology of Fig. 4.1 on page 38, the left hand vertical and horisontal lines.

B.3.1 Unique Identification

295 The pipeline system, as such,
296 has a unique identifier, distinct (different) from its pipeline unit identifiers.
297 Each pipeline unit is uniquely distinguished by its unit identifier.
298 There is a state of all unique identifiers.

type
296. PLSI
297. UI
value
295. pls:PLS
296. uid_PLS: PLS → PLSI
297. uid_U: U → UI
variable
298. σ_{uid} := { uid_PLS(pls) } ∪ xtr_UIs(pls)
axiom
297. ∀ u,u′:U•{u,u′}⊆obs_Us(pls)⇒(u≠u′⇒uid_UI(u)≠uid_UI(u′))
297. ∧ uid_PLS(pls) ∉ {uid_UI(u)|u:U•u ∈ obs_Us(pls)}

299 From a pipeline system one can observe the set of all unique unit identifiers.

value
299. xtr_UIs: PLS → UI-**set**
299. xtr_UIs(pls) ≡ {uid_UI(u)|u:U•u ∈ obs_Us(pls)}

300 We can prove that the number of unique unit identifiers of a pipeline system equals that of the units of that system.

theorem:
300. ∀ pls:PLS•**card** obs_Us(pl)=**card** xtr_UIs(pls)

B.3.2 Mereology

B.3.2.1 PLS Mereology

301 The mereology of a pipeline system is the set of unique identifiers of all the units of that system.

```
type
301.  PLS_Mer = UI-set
value
301.  mereo_PLS: PLS → PLS_Mer
axiom
301.  ∀ uis:PLS_Mer • uis = card xtr_UIs(pls)
```

B.3.2.2 Unit Mereologies

302 Each unit is connected to zero, one or two other existing input units and zero, one or two other existing output units as follows:

a A well unit is connected to exactly one output unit (and, hence, has no "input").
b A pipe unit is connected to exactly one input unit and one output unit.
c A pump unit is connected to exactly one input unit and one output unit.
d A valve is connected to exactly one input unit and one output unit.
e A fork is connected to exactly one input unit and two distinct output units.
f A join is connected to exactly two distinct input units and one output unit.
g A plate is connected to exactly one unit.
h A sink is connected to exactly one input unit (and, hence, has no "output").

```
type
302.    MER = UI-set × UI-set
value
302.    mereo_U: U → MER
axiom
302.    wf_Mereology: PLS → Bool
302.    wf_Mereology(pls) ≡
302.      ∀ u:U•u ∈ obs_Us(pls)⇒
302.        let (iuis,ouis) = mereo_U(u) in iuis ∪ ouis ⊆ xtr_UIs(pls) ∧
302.          case (u,(card iuis,card ouis)) of
302a.             (mk_We(we),(0,1)) → true,
302b.             (mk_Pi(pi),(1,1)) → true,
302c.             (mk_Pu(pu),(1,1)) → true,
302d.             (mk_Va(va),(1,1)) → true,
302e.             (mk_Fo(fo),(1,1)) → true,
302f.             (mk_Jo(jo),(1,1)) → true,
302f.             (mk_Pl(pl),(0,1)) → true ["begin"],
302f.             (mk_Pl(pl),(1,0)) → true ["end"],
302h.             (mk_Si(si),(1,0)) → true,
302.              _ → false end end
```

B.3.3 Pipeline Concepts, I

B.3.3.1 Pipe Routes

303 A route (of a pipeline system) is a sequence of connected units (of the pipeline system).
304 A route descriptor is a sequence of unit identifiers and the connected units of a route (of a pipeline system).

```
    type
303.    R′ = U^ω
303.    R = {| r:Route′•wf_Route(r) |}
304.    RD = UI^ω
    axiom
304.    ∀ rd:RD • ∃ r:R•rd=descriptor(r)
    value
304.    descriptor: R → RD
304.    descriptor(r) ≡ ⟨uid_UI(r[i])|i:Nat•1≤i≤len r⟩
```

305 Two units are adjacent if the output unit identifiers of one shares a unique unit identifier with the input identifiers of the other.

```
    value
305.    adjacent: U × U → Bool
305.    adjacent(u,u′) ≡ let (,ouis)=mereo_U(u),(iuis,)=mereo_U(u′) in ouis ∩ iuis ≠ {} end
```

306 Given a pipeline system, *pls*, one can identify the (possibly infinite) set of (possibly infinite) routes of that pipeline system.

a The empty sequence, ⟨⟩, is a route of *pls*.
b Let u,u' be any units of *pls*, such that an output unit identifier of u is the same as an input unit identifier of u', then $\langle u,u'\rangle$ is a route of *pls*.
c If r and r' are routes of *pls* such that the last element of r is the same as the first element of r', then $r^\frown\mathbf{tl}r'$ is a route of *pls*.
d No sequence of units is a route unless it follows from a finite (or an infinite) number of applications of the basis and induction clauses of Items 306a–306c.

```
    value
306.    Routes: PLS → RD-infset
306.    Routes(pls) ≡
306a.       let rs = ⟨⟩ ∪
306b.             {⟨uid_UI(u),uid_UI(u′)⟩|u,u′:U•{u,u′}⊆obs_Us(pls) ∧ adjacent(u,u′)}
306c.           ∪ {r^⌢tl r′|r,r′:R•{r,r′}⊆rs}
306d.       in rs end
```

B.3.3.2 Well-formed Routes

307 A route is acyclic if no two route positions reveal the same unique unit identifier.

```
    value
307.    is_acyclic_Route: R → Bool
307.    is_acyclic_Route(r) ≡ ~∃ i,j:Nat•{i,j}⊆inds r ∧ i≠j ∧ r[i]=r[j]
```

308 A pipeline system is well-formed if none of its routes are circular (and all of its routes embedded in well-to-sink routes).

```
    value
308.    wf_Routes: PLS → Bool
308.    wf_Routes(pls) ≡
308.        non_circular(pls) ∧ are_embedded_Routes(pls)
```

```
308. is_non_circular_PLS: PLS → Bool
308. is_non_circular_PLS(pls) ≡
308.     ∀ r:R•r ∈ routes(p)∧acyclic_Route(r)
```

309 We define well-formedness in terms of well-to-sink routes, i.e., routes which start with a well unit and end with a sink unit.

```
value
309. well_to_sink_Routes: PLS → R-set
309. well_to_sink_Routes(pls) ≡
309.   let rs = Routes(pls) in
309.   {r|r:R•r ∈ rs ∧ is_We(r[1]) ∧ is_Si(r[len r])} end
```

310 A pipeline system is well-formed if all of its routes are embedded in well-to-sink routes.

```
310. are_embedded_Routes: PLS → Bool
310. are_embedded_Routes(pls) ≡
310.     let wsrs = well_to_sink_Routes(pls) in
310.     ∀ r:R • r ∈ Routes(pls) ⇒
310.         ∃ r′:R,i,j:Nat •
310.             r′ ∈ wsrs ∧ {i,j}⊆inds r′∧i≤j ∧ r = ⟨r′[k]|k:Nat•i≤k≤j⟩ end
```

B.3.3.3 Embedded Routes

311 For every route we can define the set of all its embedded routes.

```
value
311. embedded_Routes: R → R-set
311. embedded_Routes(r) ≡ {⟨r[k]|k:Nat•i≤k≤j⟩ | i,j:Nat• i {i,j}⊆inds(r) ∧ i≤j}
```

B.3.3.4 A Theorem

312 The following theorem is conjectured:

a the set of all routes (of the pipeline system)
b is the set of all well-to-sink routes (of a pipeline system) and
c all their embedded routes

```
theorem:
312.  ∀ pls:PLS •
312.  let rs = Routes(pls),
312.      wsrs = well_to_sink_Routes(pls) in
312a.  rs =
312b.      wsrs ∪
312c.      ∪ {{r′|r′:R • r′ ∈ is_embedded_Routes(r″)} | r″:R • r″ ∈ wsrs}
311.  end
```

B.3.3.5 Fluids

313 The only fluid of concern to pipelines is the gas[6] or liquid[7] which the pipes transport[8].

type
313. GoL [= M]
value
313. obs_GoL: U → GoL

B.3.4 Attributes

B.3.4.1 Unit Flow Attributes

314 A number of attribute types characterise units:

a estimated current well capacity (barrels of oil, etc.),
b pump height (a static attribute),
c current pump status (not pumping, pumping; a programmable attribute),
d current valve status (closed, open; a programmable attribute) and
e flow (barrels/second, a biddable attribute).

type
314a. WellCap
314b. Pump_Height
314c. Pump_State == {|**not_pumping**,**pumping**|}
314d. Valve_State == {|**closed**,**open**|}
314e. Flow

315 Flows can be added and subtracted,
316 added distributively and
317 flows can be compared.

value
315. ⊕,⊖: Flow×Flow → Flow
316. ⊕: Flow-**set** → Flow
317. <,≤,=,≠,≥,>: Flow × Flow → **Bool**

318 Properties of pipeline units include

a estimated current well capacity (barrels of oil, etc.) [a biddable attribute],
b pipe length [a static attribute],
c current pump height [a biddable attribute],
d current valve open/close status [a programmable attribute],
e current [$\mathcal{L}$aminar] in-flow at unit input [a monitorable attribute],
f current $\mathcal{L}$aminar] in-flow leak at unit input [a monitorable attribute],
g maximum [$\mathcal{L}$aminar] guaranteed in-flow leak at unit input [a static attribute],
h current [$\mathcal{L}$aminar] leak unit interior [a monitorable attribute],
i current [$\mathcal{L}$aminar] flow in unit interior [a monitorable attribute],

[6] Gaseous materials include: air, gas, etc.
[7] Liquid materials include water, oil, etc.
[8] The description of this document is relevant only to gas or oil pipelines.

j maximum $\mathcal{L}$aminar] guaranteed flow in unit interior [a monitorable attribute],
k current [$\mathcal{L}$aminar] out-flow at unit output [a monitorable attribute],
l current [$\mathcal{L}$aminar] out-flow leak at unit output [a monitorable attribute] and
m maximum guaranteed $\mathcal{L}$aminar out-flow leak at unit output [a static attribute].

```
type
318e  In_Flow = Flow
318f  In_Leak = Flow
318g  Max_In_Leak = Flow
318h  Body_Flow = Flow
318i  Body_Leak = Flow
318j  Max_Flow = Flow
318k  Out_Flow = Flow
318l  Out_Leak = Flow
318m  Max_Out_Leak = Flow
value
318a  attr_WellCap: We → WellCap
318b  attr_LEN: Pi → LEN
318c  attr_Height: Pu → Height
318d  attr_ValSta: Va → VaSta
318e  attr_In_Flow: U → UI → Flow
318f  attr_In_Leak: U → UI → Flow
318g  attr_Max_In_Leak: U → UI → Flow
318h  attr_Body_Flow: U → Flow
318i  attr_Body_Leak: U → Flow
318j  attr_Max_Flow: U → Flow
318k  attr_Out_Flow: U → UI → Flow
318l  attr_Out_Leak: U → UI → Flow
318m  attr_Max_Out_Leak: U → UI → Flow
```

319 Summarising we can define a two notions of flow:

a static and
b monitorable.

```
type
319a  Sta_Flows = Max_In_Leak×In_Max_Flow>Max_Out_Leak
319b  Mon_Flows = In_Flow×In_Leak×Body_Flow×Body_Leak×Out_Flow×Out_Leak
```

B.3.4.2 Unit Metrics

Pipelines are laid out in the terrain. Units have length and diameters. Units are positioned in space: have altitude, longitude and latitude positions of its one, two or three connection PoinTs[9].

320 length (a static attribute),
321 diameter (a static attribute) and
322 position (a static attribute).

```
type
320.  LEN
321.  ◯
322.  POS == mk_One(pt:PT) | mk_Two(ipt:PT,opt:PT)
322.            | mk_OneTwo(ipt:PT,opts:(lpt:PT,rpt:PT))
322.            | mk_TwoOne(ipts:(lpt:PT,rpt:PT),opt:PT)
322.  PT = Alt × Lon × Lat
322.  Alt, Lon, Lat = ...
value
320.  attr_LEN: U → LEN
321.  attr_◯: U → ◯
322.  attr_POS: U → POS
```

We can summarise the metric attributes:

323 Units are subject to either of four (mutually exclusive) metrics:

[9] 1 for *wells*, *plates* and *sinks*; 2 for *pipes*, *pumps* and *valves*; 1+2 for *forks*, 2+1 for *joins*.

a Length, diameter and a one point position.
b Length, diameter and a two points position.
c Length, diameter and a one+two points position.
d Length, diameter and a two+one points position.

type
323. Unit_Sta = Sta1_Metric | Sta2_Metric | Sta12_Metric | Sta21_Metric
323a Sta1_Metric = LEN × Ø × mk_One(pt:PT)
323b Sta2_Metric = LEN × Ø × mk_Two(ipt:PT,opt:PT)
323c Sta12_Metric = LEN × Ø × mk_OneTwo(ipt:PT,opts:(lpt:PT,rpt:PT))
323d Sta21_Metric = LEN × Ø × mk_TwpOne(ipts:(lpt:PT,rpt:PT),opt:PT)

B.3.4.3 Wellformed Unit Metrics

The points positions of neighbouring units must "fit" one-another.

324 Without going into details we can define a predicate, wf_Metrics, that applies to a pipeline system and yields **true** iff neighbouring units must "fit" one-another.

value
324. wf_Metrics: PLS → **Bool**
324. wf_Metrics(pls) ≡ ...

B.3.4.4 Summary

We summarise the static, monitorable and programmable attributes for each manifest part of the pipeline system:

type
PLS_Sta = PLS_net×...
PLS_Mon = ...
PLS_Prg = PLS_Σ×...
Well_Sta = Sta1_Metric×Sta_Flows×Orig_Cap×...
Well_Mon = Mon_Flows×Well_Cap×...
Well_Prg = ...
Pipe_Sta = Sta2_Metric×Sta_Flows×LEN×...
Pipe_Mon = Mon_Flows×In_Temp×Out_Temp×...
Pipe_Prg = ...
Pump_Sta = Sta2_Metric×Sta_Flows×Pump_Height×...
Pump_Mon = Mon_Flows×...
Pump_Prg = Pump_State×...
Valve_Sta = Sta2_Metric×Sta_Flows×...
Valve_Mon = Mon_Flows×In_Temp×Out_Temp×...
Valve_Prg = Valve_State×...
Fork_Sta = Sta12_Metric×Sta_Flows×...
Fork_Mon = Mon_Flows×In_Temp×Out_Temp×...
Fork_Prg = ...
Join_Sta = Sta21_Metric×Sta_Flows×...
Join_Mon = Mon_Flows×In_Temp×Out_Temp×...
Join_Prg = ...
Sink_Sta = Sta1_Metric×Sta_Flows×Max_Vol×...
Sink_Mon = Mon_Flows×Curr_Vol×In_Temp×Out_Temp×...

Sink_Prg = ...

325 Corresponding to the above three attribute categories we can define "collective" attribute observers:

value
325. sta_A_We: We → Sta1_Metric×Sta_Flows×Orig_Cap×...
325. mon_A_We: We → ηMon_Flows×ηWell_Cap×ηIn_Temp×ηOut_Temp×...
325. prg_A_We: We → ...
325. sta_A_Pi: Pi → Sta2_Metric×Sta_Flows×LEN×...
325. mon_A_Pi: Pi → ℵMon_Flows×ηIn_Temp×ηOut_Temp×...
325. prg_A_Pi: Pi → ...
325. sta_A_Pu: Pu → Sta2_Metric×Sta_Flows×LEN×...
325. mon_A_Pu: Pu → ℵMon_Flows×ηIn_Temp×ηOut_Temp×...
325. prg_A_Pu: Pu → Pump_State×...
325. sta_A_Va: Va → Sta2_Metric×Sta_Flows×LEN×...
325. mon_A_Va: Va → ℵMon_Flows×ηIn_Temp×ηOut_Temp×...
325. prg_A_Va: Va → Valve_State×...
325. sta_A_Fo: Fo → Sta12_Metric×Sta_Flows×...
325. mon_A_Fo: Fo → ℵMon_Flows×ηIn_Temp×ηOut_Temp×...
325. prg_A_Fo: Fo → ...
325. sta_A_Jo: Jo → Sta21_Metric×Sta_Flows×...
325. mon_A_Jo: Jo → Mon_Flows×ηIn_Temp×ηOut_Temp×...
325. prg_A_Jo: Jo → ...
325. sta_A_Si: Si → Sta1_Metric×Sta_Flows×Max_Vol×...
325. mon_A_Si: Si → ℵMon_Flows×ηIn_Temp×ηOut_Temp×...
325. prg_A_Si: Si → ...

325. ℵMon_Flows ≡ (ηIn_Flow,ηIn_Leak,ηBody_Flow,ηBody_Leak,ηOut_Flow,ηOut_Leak)

Monitored flow attributes are [to be] passed as arguments to behaviours *by reference* so that their monitorable attribute values can be sampled.

B.3.4.5 Fluid Attributes

Fluids, we here assume, oil, as it appears in the pipeline units have no unique identity, have not mereology, but does have attributes: hydrocarbons consisting predominantly of aliphatic, alicyclic and aromatic hydrocarbons. It may also contain small amounts of nitrogen, oxygen, and sulfur compounds.

326 We shall simplify, just for illustration, crude oil fluid of units to have these attributes:

a volume,
b viscosity,
c temperature,
d paraffin content (%age),
e naphtenes content (%age),

type
326. Oil
326a. Vol
326b. Visc
326c. Temp
326d. Paraffin
326e. Naphtene

value
326b. obs_Oil: U → Oil
326a. attr_Vol: Oil → Vol
326b. attr_Visc: Oil → Visc
326c. attr_Temp: Oil → Temp
326d. attr_Paraffin: Oil → Paraffin
326e. attr_Naphtene: Oil → Naphtene

B.3.4.6 Pipeline System Attributes

The "root" pipeline system is a compound. In its transcendentally deduced behavioral form it is, amongst other "tasks", entrusted with the monitoring and control of all its units. To do so it must, as a basically static attribute possess awareness, say in the form of a net diagram of how these units are interconnected, together with all their internal qualities, by type and by value. Next we shall give a very simplified account of the possible pipeline system attribute.

327 We shall make use, in this example, of just a simple pipeline state, pls_ω.

The pipeline state, pls_ω, embodies all the information that is relevant to the monitoring and control of an entire pipeline system, whether static or dynamic.

type
327. PLS_Ω

B.3.5 Pipeline Concepts, II: Flow Laws

328 "What flows in, flows out !". For $\mathcal{L}$aminar flows: for any non-well and non-sink unit the sums of input leaks and in-flows equals the sums of unit and output leaks and out-flows.

Law:
328. ∀ u:U\We\Si •
328. sum_in_leaks(u) ⊕ sum_in_flows(u) =
328. attr_body_Leak$_{\mathcal{L}}$(u) ⊕
328. sum_out_leaks(u) ⊕ sum_out_flows(u)

value
sum_in_leaks: U → Flow
sum_in_leaks(u) ≡ **let** (iuis,) = mereo_U(u) **in** ⊕ {attr_In_Leak$_{\mathcal{L}}$(u)(ui)|ui:UI•ui ∈ iuis} **end**
sum_in_flows: U → Flow
sum_in_flows(u) ≡ **let** (iuis,) = mereo_U(u) **in** ⊕ {attr_In_Flow$_{\mathcal{L}}$(u)(ui)|ui:UI•ui ∈ iuis} **end**
sum_out_leaks: U → Flow
sum_out_leaks(u) ≡ **let** (,ouis) = mereo_U(u) **in** ⊕ {attr_Out_Leak$_{\mathcal{L}}$(u)(ui)|ui:UI•ui ∈ ouis} **end**
sum_out_flows: U → Flow
sum_out_flows(u) ≡ **let** (,ouis) = mereo_U(u) **in** ⊕ {attr_Out_Leak$_{\mathcal{L}}$(u)(ui)|ui:UI•ui ∈ ouis} **end**

329 "What flows out, flows in !". For $\mathcal{L}$aminar flows: for any adjacent pairs of units the output flow at one unit connection equals the sum of adjacent unit leak and in-flow at that connection.

Law:
329. ∀ u,u′:U•adjacent(u,u′) ⇒
329. **let** (,ouis)=mereo_U(u), (iuis′,)=mereo_U(u′) **in**
329. **assert**: uid_U(u′) ∈ ouis ∧ uid_U(u) ∈ iuis ′
329. attr_Out_Flow$_{\mathcal{L}}$(u)(uid_U(u′)) =
329. attr_In_Leak$_{\mathcal{L}}$(u)(uid_U(u))⊕attr_In_Flow$_{\mathcal{L}}$(u′)(uid_U(u)) **end**

These "laws" should hold for a pipeline system without plates.

B.4 Perdurants

We follow the ontology of Fig. 4.1 on page 38, the right-hand dashed box labeled *Perdurants* and the right-hand vertical and horisontal lines.

B.4.1 State

We introduce concepts of *manifest* and *structure* endurants. The former are such compound endurants (Cartesians of sets) to which we ascribe internal qualities; the latter are such compound endurants (Cartesians of sets) to which we **do not** ascribe internal qualities. The distinction is pragmatic.

330 For any given pipeline system we suggest the state to consist of the manifest endurants of all its non-plate units.

value
330. σ = obs_Us(pls)

B.4.2 Channel

331 There is a [global] array channel indexed by a "set pair" of distinct manifest endurant part identifiers – signifying the possibility of the syncharonisation and communication between any pair of pipeline units and between these and the pipeline system, cf. last, i.e., bottom-most diagram of Fig. B.10 on page 177.

channel
331. { ch[{i,j}] | {i,j}:(PLSI|UI) • {i,j}⊆σ_{id} }

B.4.3 Actions

These are, informally, some of the actions of a pipeline system:

332 **start pumping**: from a state of not pumping to a state of pumping "at full blast !".[10]
333 **stop pumping**: from a state of (full) pumping to a state of no pumping at all.
334 **open valve**: from a state of a fully closed valve to a state of fully open valve.[11]
335 **close valve**: from a state of a fully opened valve to a state of fully closed valve.

We shall not define these actions in this paper. But they will be referred to in the *pipeline_system* (Items 354a, 354b, 354c), the *pump* (Items 357a, 357b) and the *valve* (Items 360a, 360b) behaviours.

B.4.4 Behaviours

B.4.4.1 Behaviour Kinds

There are eight kinds of behaviours:

[10] – that is, we simplify, just for the sake of illustration, and do not consider "intermediate" states of pumping.
[11] – cf. Footnote 10.

336 the pipeline system behaviour;[12]
337 the [generic] well behaviour,
338 the [generic] pipe behaviour,
339 the [generic] pump behaviour,
340 the [generic] valve behaviour,
341 the [generic] fork behaviour,
342 the [generic] join behaviour,
343 the [generic] sink behaviour.

B.4.4.2 Behaviour Signatures

344 The *pipeline_system* behaviour, *pls*,
345 The *well* behaviour signature lists the unique well identifier, the well mereology, the static well attributes, the monitorable well attributes, the programmable well attributes and the channels over which the well [may] interact with the pipeline system and a pipeline unit.
346 The *pipe* behaviour signature lists the unique pipe identifier, the pipe mereology, the static pipe attributes, the monitorable pipe attributes, the programmable pipe attributes and the channels over which the pipe [may] interact with the pipeline system and its two neighbouring pipeline units.
347 The *pump* behaviour signature lists the unique pump identifier, the pump mereology, the static pump attributes, the monitorable pump attributes, the programmable pump attributes and the channels over which the pump [may] interact with the pipeline system and its two neighbouring pipeline units.
348 The *valve* behaviour signature lists the unique valve identifier, the valve mereology, the static valve attributes, the monitorable valve attributes, the programmable valve attributes and the channels over which the valve [may] interact with the pipeline system and its two neighbouring pipeline units.
349 The *fork* behaviour signature lists the unique fork identifier, the fork mereology, the static fork attributes, the monitorable fork attributes, the programmable fork attributes and the channels over which the fork [may] interact with the pipeline system and its three neighbouring pipeline units.
350 The *join* behaviour signature lists the unique join identifier, the join mereology, the static join attributes, the monitorable join attributes, the programmable join attributes and the channels over which the join [may] interact with the pipeline system and its three neighbouring pipeline units.
351 The *sink* behaviour signature lists the unique sink identifier, the sink mereology, the static sing attributes, the monitorable sing attributes, the programmable sink attributes and the channels over which the sink [may] interact with the pipeline system and its one or more pipeline units.

value
344. pls: plso:PLSI → pls_mer:PLS_Mer → PLS_Sta → PLS_Mon →
344. PLS_Prg → { ch[{plsi,ui}] | ui:UI • ui ∈ σ_{ui} } **Unit**
345. well: wid:WI → well_mer:MER → Well_Sta → Well_mon →
345. Well_Prgr → { ch[{plsi,ui}] | wi:WI • ui ∈ σ_{ui} } **Unit**
346. πipe: UI → pipe_mer:MER → Pipe_Sta → Pipe_mon →
346. Pipe_Prgr → { ch[{plsi,ui}] | ui:UI • ui ∈ σ_{ui} } **Unit**
347. pump: pi:UI → pump_mer:MER → Pump_Sta → Pump_Mon →
347. Pump_Prgr → { ch[{plsi,ui}] | ui:UI • ui ∈ σ_{ui} } **Unit**
348. valve: vi:UI → valve_mer:MER → Valve_Sta → Valve_Mon →
348. Valve_Prgr → { ch[{plsi,ui}] | ui:UI • ui ∈ σ_{ui} } **Unit**
349. fork: fi:FI → fork_mer:MER → Fork_Sta → Fork_Mon →
349. Fork_Prgr → { ch[{plsi,ui}] | ui:UI • ui ∈ σ_{ui} } **Unit**
350. join: ji:JI → join_mer:MER → Join_Sta → Join_Mon →
350. Join_Prgr → { ch[{plsi,ui}] | ui:UI • ui ∈ σ_{ui} } **Unit**
351. sink: si:SI → sink_mer:MER → Sink_Sta → Sink_Mon →
351. Sink_Prgr → { ch[{plsi,ui}] | ui:UI • ui ∈ σ_{ui} } **Unit**

[12] This "PLS" behaviour summarises the either global, i.e., *SCADA*[13]-like behaviour, or the fully distributed, for example, manual, human-operated behaviour of the monitoring and control of the entire pipeline system.
[13] Supervisory Control And Data Acquisition

B.4.4.2.1 Behaviour Definitions

We show the definition of only three behaviours:

- the **pipe_line_system** behaviour,
- the **pump** behaviour and
- the **valve** behaviour.

B.4.4.2.2 The Pipeline System Behaviour

352 The pipeline system behaviour
353 calculates, based on its programmable state, its next move;
354 if that move is [to be] an action on a named

a pump, whether to start or stop pumping, then the named pump is so informed, whereupon the pipeline system behaviour resumes in the new pipeline state; or
b valve, whether to open or close the valve, then the named valve is so informed, whereupon the pipeline system behaviour resumes in the new pipeline state; or
c unit, to collect its monitorable attribute values for monitoring, whereupon the pipeline system behaviour resumes in the further updated pipeline state;
d et cetera;

```
value
352.  pls(plsi)(uis)(pls_msta)(pls_mon)(pls_ω) ≡
353.    let (to_do,pls_ω′) = calculate_next_move(plsi,pls_mer,pls_msta,pls_mon,pls_prgr) in
354.    case to_do of
354a      mk_Pump(pi,α) →
354a            ch[{plsi,pi}] ! α assert: α ∈ {stop_pumping,pump};
354a            pls(plsi)(pls_mer)(pls_msta)(pls_mon)(pls_ω′),
354b      mk_Valve(vi,α) →
354b            ch[{plsi,vi}] ! α assert: α ∈ {open_valve,close_valve};
354b            pls(plsi)(pls_mer)(pls_msta)(pls_mon)(pls_ω′),
354c      mk_Unit(ui,monitor) →
354c           ch[{plsi,ui}] ! monitor;
354c           pls(plsi)(pls_mer)(pls_msta)(pls_mon)(update_pls_ω(ch[{plsi,ui}] ?,ui)(pls_ω′)),
354d      ... end
352     end
```

We leave it to the reader to define the calculate_next_move function!

B.4.4.2.3 The Pump Behaviours

355 The [generic] pump behaviour internal non-deterministically alternates between
356 doing own work (...), or
357 accepting pump directives from the pipeline behaviour.

a If the directive is either to start or stop pumping, then that is what happens – whereupon the pump behaviour resumes in the new pumping state.
b If the directive requests the values of all monitorable attributes, then these are *gathered*, communicated to the pipeline system behaviour – whereupon the pump behaviour resumes in the "old" state.

```
value
355.  pump(π)(pump_mer)(pump_sta)(pump_mon)(pump_prgr) ≡
```

```
356.     ...
357.   ∏ let α = ch[{plsi,π}] ? in
357.       case α of
357a.           stop_pumping ∨ pump
357a.               → pump(π)(pump_mer)(pump_sta)(pump_mon)(α)[14]end,
357b.           monitor
357b.               → let mvs = gather_monitorable_values(π,pump_mon) in
357b.                 ch[{plsi,π}] ! mvs;
357b.                 pump(π)(pump_mer)(pump_sta)(pump_mon)(pump_prgr) end
357.       end
```

We leave it to the reader to define the gather_monitorable_values function.

B.4.4.2.4 The Valve Behaviours

358 The [generic] valve behaviour internal non-deterministically alternates between
359 doing own work (...), or
360 accepting valve directives from the pipeline system.

a If the directive is either to open or close the valve, then that is what happens – whereupon the pump behaviour resumes in the new valve state.
b If the directive requests the values of all monitorable attributes, then these are *gathered*, communicated to the pipeline system behaviour – whereupon the valve behaviour resumes in the new valve state.

```
value
358.  valve(vi)(valv_mer)(valv_sta)(valv_mon)(valv_prgr) ≡
359.    ...
360.  ∏ let α = ch[{plsi,π}] ? in
360.      case α of
360a.         open_valve ∨ close_valve
360a.             → valve(vi)(val_mer)(val_sta)(val_mon)(α)[15]end,
360b.         monitor
360b.             → let mvs = gather_monitorable_values(vi,val_mon) in
360b.               ch[{plsi,π}] ! (vi,mvs);
360b.               valve(vi)(val_mer)(val_sta)(val_mon)(val_prgr) end
360.      end
```

B.4.4.3 Sampling Monitorable Attribute Values

Static and programmable attributes are, as we have seen, *passed by value* to behaviours. Monitorable attributes "surreptitiously" change their values so, as a technical point, these are *passed by reference* – by *passing attribute type names*.

361 From the name, ηA, of a monitorable attribute and the unique identifier, u_i, of the part having the named monitorable attribute one can then, "dynamically", "on-the-fly", as the part behaviour "moves-on", retrieve the value of the monitorable attribute. This can be illustrated as follows:

[14] Updating the programmable pump state to either **stop_pumping** or **pump** shall here be understood to mean that the pump is set to not pump, respectively to pump.

[15] Updating the programmable valve state to either **open_valve** or **close_valve** shall here be understood to mean that the valve is set to open, respectively to closed position.

362 The unique identifier u_i is used in order to retrieve, from the global parts state, σ, that identified part, p.
363 Then attr_A is applied to p.

value
361. retr_U: UI → Σ → U
361. retr_U(ui)(σ) ≡ **let** u:U • u ∈ σ∧uid_U(u)=ui **in** u **end**
362. retr_AttrVal: UI × ηA → Σ → A
363. retr_AttrVal(ui)(ηA)(σ) ≡ attr_A(retr_U(ui)(σ))

retr_AttrVal(...)(...)(...) can now be applied in the body of the behaviour definitions, for example in gather_monitorable_values.

B.4.4.4 System Initialisation

System initialisation means to "morph" all manifest parts into their respective behaviours, initialising them with their respective attribute values.

364 The *pipeline system* behaviour is initialised and "put" in parallel with the parallel compositions of
365 all initialised *well*,
366 all initialised *pipe*,
367 all initialised *pump*,
368 all initialised *valve*,
369 all initialised *fork*,
370 all initialised *join* and
371 all initialised *sink* behaviours.[16]

value
364. pls(uid_PLS(pls))(mereo_PLS(pls))((pls))((pls))((pls))
365. ‖ ‖ { well(uid_U(we))(mereo_U(we))(sta_A_We(we))(mon_A_We(we))(prg_A_We(we)) | we:Well • w ∈ σ }
366. ‖ ‖ { pipe(uid_U(pi))(mereo_U(pi))(sta_A_Pi(pi))(mon_A_Pi(pi))(prg_A_Pi(pi)) | pi:Pi • pi ∈ σ }
367. ‖ ‖ { pump(uid_U(pu))(mereo_U(pu))(sta_A_Pu(pu))(mon_A_Pu(pu))(prg_A_Pu(pu)) | pu:Pump • pu ∈ σ }
368. ‖ ‖ { valv(uid_U(va))(mereo_U(va))(sta_A_Va(va))(mon_A_Va(va))(prg_A_Va(va)) | va:Well • va ∈ σ }
369. ‖ ‖ { fork(uid_U(fo))(mereo_U(fo))(sta_A_Fo(fo))(mon_A_Fo(fo))(prg_A_Fo(fo)) | fo:Fork • fo ∈ σ }
370. ‖ ‖ { join(uid_U(jo))(mereo_U(jo))(sta_A_Jo(jo))(mon_A_J(jo))(prg_A_J(jo)) | jo:Join • jo ∈ σ }
371. ‖ ‖ { sink(uid_U(si))(mereo_U(si))(sta_A_Si(si))(mon_A_Si(si))(prg_A_Si(si)) | si:Sink • si ∈ σ }

The sta_..., mon_..., and prg_A... functions are defined in Items 325 on page 186.
Note: ‖ { f(u)(...) | u:U • u ∈ {} } ≡ ().

B.5 Index

[16] Plates are treated as are structures, i.e., not "behaviourised"!

Appendix C
A RAISE Specification Language Primer

Contents

We present an RSL Primer. Indented text, in slanted font, such as this, presents informal material and examples. Non-indented text, in roman font, presents narrative and formal explanation of RSL constructs.

This RSL Primer omits treatment of a number of language constructs, notably the RSL module concepts of *schemes, classes* and *objects*. Although we do cover the imperative language construct of [declaration of] variables and, hence, assignment, we shall omit treatment of structured imperative constructs like **for** ..., **do** *s* **while** *b*, **while** *b* **do** *s* loops.

Section C.13 on page 222 introduces additional language constructs, thereby motivation the $^+$ in the RSL$^+$ name.

C.1 Specification Units

An RSL specification consists of a set (textually ordered in any linear sequence) of *specification units*. We shall only treat five kinds of such units:

- *type:* Prefixed by the keyword (literal) **type**, type specification units introduce distinct type names. The general form of a type specification unit is:

  type T = Type_Expression

  where T is a distinct (type) identifier. Type specification units may introduce several (new, distinct) types:

  type T1 = Type_Expression_1, T2 = Type_Expression_2, ..., Tn = Type_Expression_n

For more on types, see Sect. C.2.

- *value:* Prefixed by the keyword (literal) **value** value specification units introduce distinct value names. The general form of a value specification unit is:

  value a:A = $\mathcal{E}$(...)

  where $\mathcal{E}$(...) is a value expression. Value specification units may introduce several (new, distinctly named) values:

  value a_1:A_1 = $\mathcal{E}_1$(...), a_2:A_2 = $\mathcal{E}_2$(...), ..., a_n:A_n = $\mathcal{E}_n$(...)

  Quite often the value specification is of the form:

  value f: A → B, f(arg) ≡ $\mathcal{E}$(...[f]...)

  that is: the value is a function, f, whose *signature* gives the name, f, and the type of the functionality A → B (or A $\xrightarrow{\sim}$ B). Most of this chapter will cover aspects of value expressions.
- *axiom:* Prefixed by the keyword (literal) **axiom**, axiom specification units serve to limit values. The general form of an axiom specification unit is:

  axiom $\mathcal{A}$(...)

  where $\mathcal{A}$(...) is some predicate expression over (specification unit) defined quantities. For more on axiomatic expressions, see Sect. C.3.
- *variable:* Prefixed by the keyword (literal) **variable**, variable specification units introduce distinct variable names. The general form of a variable specification unit is:

  variable v:T := expression

  where v is ..., T is ..., and expression is a value expression. For variables, see Sect. C.9.2.
- *channel:* Prefixed by the keyword (literal) **channel**, channel specification units introduce distinct channel names.
  The general form of a channel specification unit is:

  channel { ch[{i,j}] | i,j:UI • ... }: M

  where ch is a distinct, here channel, name,we are declaring an array of channels. ch[{i, j}] expresses that , {i, j} ranges of so-called unique identifier indices of type UI, and M is a type expression. For more on channels, see Sect. C.10.1.

This chapter will otherwise be conventionally structured.

C.2 Types and Values

$\mathcal{I}$: Types are, in general, set-like structures[17] *of things, i.e., values, having common characteristics.*

A bunch of zero, one or more apples (type apples) may thus form a [sub]set of type Belle de Boskoop apples. A bunch of zero, one or more pears (type pears) may thus form a [sub]set of type Concorde pears. A union of zero, one or more of these apples and pears then form a [sub]set of entities of type fruits. ▪

[17] We shall not, in this primer, go into details as to the mathematics of types.

C.2.1 Sort and Type Expressions

Sort and type expressions are expressions whose values are types, that is, possibly infinite set-like structures of values (of "that" type).

C.2.1.1 Atomic Types: Identifier Expressions and Type Values

Atomic types have (atomic) values. That is, values which we consider to have no proper constituent (sub-)values, i.e., cannot, to us, be meaningfully "taken apart".

RSL has a number of [so-called] built-in atomic types. They are expressed in terms of literal identifiers. These are the **Boolean**s, **int**egers, **Nat**ural numbers, **Real**s, **Char**acters, and **Text**s. **Text**s are free-form texts and are more general than just texts of RSL-like formulas. RSL-**Text**'s will be introduced in Sect. C.13 on page 222.

We shall not need the base types **Char**acters, nor the general type **Text**s for domain modeling in this primer. They will be listed below, but not mentioned further.

The base types are:

Basic Types

type
[1] **Bool**
[2] **Int**
[3] **Nat**
[4] **Real**
[5] **Char**
[6] **Text**

1 The Boolean type of truth values **false** and **true**. indexsymbindaaab@**true**
2 The integer type on integer values ..., −2, −1, 0, 1, 2, ...
3 The natural number type of positive integer values 0, 1, 2, ...
4 The real number type of real values, i.e., values whose numerals can be written as an integer, followed by a period ("."), followed by a natural number (the fraction).
5 The character type of character values ″a″, ″bbb″, ...
6 The text type of character string values ″aa″, ″aaa″, ..., ″abc″, ...

C.2.1.2 Composite Types: Expressions and Type Values

Composite types have composite values. That is, values which we consider to have proper constituent (sub-)values, i.e., can, to us, be meaningfully "taken apart".

From these one can form type expressions: finite sets, infinite sets, Cartesian products, lists, maps, etc.

Let A, B and C be any type names or type expressions, then these are the composite types, hence, type expressions:

Composite Type Expressions

[7] A-**set**
[8] A-**infset**
[9] A × B × ... × C
[10] A^*
[11] A^ω
[12] A $\overrightarrow{m}$ B

[13] A → B
[14] A ~→ B
[15] A | B | ... | C
[16] mk_id(sel_a:A,...,sel_b:B)
[17] sel_a:A ... sel_b:B

The following are generic type expressions:

7 The set type of finite cardinality set values.
8 The set type of infinite and finite cardinality set values.
9 The Cartesian type of Cartesian values.
10 The list type of finite length list values.
11 The list type of infinite and finite length list values.
12 The map type of finite definition set map values.
13 The function type of total function values.
14 The function type of partial function values.
15 The postulated disjoint union of types A, B, ..., and C.
16 The record type of mk_id-named record values mk_id(av,...,bv), where av, ..., bv, are values of respective types. The distinct identifiers sel_a, etc., designate selector functions.
17 The record type of unnamed record values (av,...,bv), where av, ..., bv, are values of respective types. The distinct identifiers sel_a, etc., designate selector functions.

Section C.13 on page 222 introduces the extended RSL concepts of type name values and the type, 𝕋, of type names.

C.2.2 Type Definitions

C.2.2.1 Sorts — Abstract Types

Types can be (abstract) sorts in which case their structure is not specified:

Sorts

type
A, B, ..., C

C.2.2.2 Concrete Types

Types can be concrete in which case the structure of the type is specified by type expressions:

Type Definition

type
A = Type_expr

RSL Example: Sets. Narrative: *H* stand for the domain type of street intersections – we shall call then hubs, and let *L* stand for the domain type of segments of streets between immediately neighboring hubs – we shall call then links. Then *Hs* and *Ls* are to designate the types of finite sets of zero, one or more hubs, respectively links. Formalisation:

type H, L, Hs=H-**set**, Ls=L-**set** •

RSL Example: Cartesians. Narrative: Let *RN* stand for the domain type of road nets consisting of hub aggregates, *HA*, and link aggregates, *LA*. Hub and link aggregates can be observed from road nets, and hub sets and link sets can be observed from hub, respectively link aggregates. Formalisation:

type RN = HA×LA, Hs, Ls
value obs_HA: RN→HA, obs_LA: RN− LA, obs_Hs: HA→Hs, obs_Ls: LA→Ls

Observer functions, obs_... are not further defined – beyond their signatures. They will (subsequently) be defined through axioms over their results •
Some schematic type definitions are:

Variety of Type Definitions

```
[18] Type_name = Type_expr /* without |s or subtypes */
[19] Type_name = Type_expr_1 | Type_expr_2 | ... | Type_expr_n
[20] Type_name ==
         mk_id_1(s_a1:Type_name_a1,...,s_ai:Type_name_ai) |
         ... |
         mk_id_n(s_z1:Type_name_z1,...,s_zk:Type_name_zk)
[21] Type_name :: sel_a:Type_name_a  ...  sel_z:Type_name_z
[22] Type_name = {| v:Type_name′ • 𝒫(v) |}
```

where a form of [19–20] is provided by combining the types:

Record Types

```
[23] Type_name = A | B | ... | Z
[24] A == mk_id_1(s_a1:A_1,...,s_ai:A_i)
[25] B == mk_id_2(s_b1:B_1,...,s_bj:B_j)
[26] ...
[27] Z == mk_id_n(s_z1:Z_1,...,s_zk:Z_k)
```

Of these we shall almost exclusively make use of [23–27].

Disjoint Types. Narrative: A pipeline consists of a finite set of zero, one or more [interconnected][18] pipe units. Pipe units are either wells, or are pumps, or are valves, or are joins, or are forks, or are sinks. Formalisation:

type PL = P-**set**, P == WU|PU|VA|JO|FO|SI, Wu,Pu,Vu,Ju,Fu,Su
WU::mkWU(swu:Wu), PU::mkPU(spu:Pu), VA::mkVU(svu:Vu),
JO::mkJu(sju:Ju), FO::mkFu(sfu:Fu), SI::mkSi(ssu:Su)

where we leave types Wu, Pu, Vu, Ju, Fu and Su further undefined •
Types A, B, ..., Z are disjoint, i.e., shares no values, provided all mk_id_k are distinct and due to the use of the disjoint record type constructor ==.

axiom
∀ a1:A_1, a2:A_2, ..., ai:Ai •
s_a1(mk_id_1(a1,a2,...,ai))=a1 ∧ s_a2(mk_id_1(a1,a2,...,ai))=a2 ∧
... ∧ s_ai(mk_id_1(a1,a2,...,ai))=ai ∧
∀ a:A • **let** mk_id_1(a1′,a2′,...,ai′) = a **in**
a1′ = s_a1(a) ∧ a2′ = s_a2(a) ∧ ... ∧ ai′ = s_ai(a) **end**

[18] Although interconnected we shall not model pipelines as lists.

Note: Values of type A, where that type is defined by A::B×C×D, can be expressed A(b,c,d) for b:B, c:D, d:D.

C.2.2.3 Subtypes

In RSL, each type represents a set of values. Such a set can be delimited by means of predicates. The set of values b which have type B and which satisfy the predicate $\mathcal{P}$, constitute the subtype A:

Subtypes

type
A = {| b:B • $\mathcal{P}$(b) |}

Subtype. Narrative: The subtype of even natural numbers.
Formalisation: **type** ENat = {| en | en:**Nat** • is_even_natural_number(en) |} •

C.3 The Propositional and Predicate Calculi

C.3.1 Propositions

ℐ: In logic, a proposition is the meaning of a declarative sentence. [A declarative sentence is a type of sentence that makes a statement] ▪

C.3.1.1 Propositional Expressions

ℐ: Propositional expressions, informally speaking, are quantifier-free expressions having truth (or **chaos***) values.* ∀, ∃ *and* ∃! *are quantifiers, see below.*

Below, we will first treat propositional expressions all of whose identifiers denote truth values. As we progress, in sections on arithmetic, sets, list, maps, *etc., we shall extend the range of propositional expressions* ▪

Let identifiers (or propositional expressions) a, b, ..., c designate Boolean values (**true** or **false** [or **chaos**]). Then:

Propositional Expressions

false, **true**
a, b, ..., c ~a, a∧b, a∨b, a⇒b, a=b, a≠b

are propositional expressions having Boolean values. ~, ∧, ∨, ⇒, =, ≠ and □ are Boolean connectives (i.e., operators). They can be read as: **not**, **and**, **or**, **if then** (or **implies**), **equal**, **not equal** and **always**.

C.3.1.2 Propositional Calculus

ℐ: Propositional calculus is a branch of logic. It is also called propositional logic, statement logic, sentential calculus, sentential logic, or sometimes zeroth-order logic. It deals with propositions (which can be true or false) and relations between propositions, including the construction of arguments based on

them. Compound propositions are formed by connecting propositions by logical connectives. Propositions that contain no logical connectives are called atomic propositions [Wikipedia] ▪

A simple two-value Boolean logic can be defined as follows:

type
  Bool
value
  true, **false**
  ~: **Bool** → **Bool**
  ∧, ∨, ⇒, =, ≠, ≡: **Bool** × **Bool** → **Bool**
axiom
  ∀ b,b′:**Bool** •
    ~b ≡ **if** b **then false else true end**
    b ∧ b′ ≡ **if** b **then** b′ **else false end**
    b ∨ b′ ≡ **if** b **then true else** b′ **end**
    b ⇒ b′ ≡ **if** b **then** b′ **else true end**
    b = b′ ≡ **if** (b∧b′)∨(~b∧~b′) **then true else false end**
    (b ≠ b′) ≡ ~(b = b′)
    (b ≡ b′) ≡ (b = b′)

We shall, however, make use of a three-value Boolean logic. The model-theory explanation of the meaning of propositional expressions is now given in terms of the *truth tables* for the logic connectives:

∨, ∧, **and** ⇒ **Syntactic Truth Tables**

∨	true	false	chaos
true	true	true	true
false	true	false	chaos
chaos	chaos	chaos	chaos

∧	true	false	chaos
true	true	false	chaos
false	false	false	false
chaos	chaos	chaos	chaos

⇒	true	false	chaos
true	true	false	chaos
false	true	true	true
chaos	chaos	chaos	chaos

The two-value logic defined earlier 'transpires' from the **true**,**false** columns and rows of the above truth tables.

C.3.2 Predicates

ℐ: Predicates are mathematical assertions that contains variables, sometimes referred to as predicate variables, and may be true or false depending on those variables' value or values[19] ▪

C.3.2.1 Predicate Expressions

Let x, y, ..., z (or term expressions) designate non-Boolean values, and let $\mathcal{P}(x)$, $\mathcal{Q}(y)$ and $\mathcal{R}(z)$ be propositional or predicate expressions, then:

Simple Predicate Expressions

[28] ∀x:X • $\mathcal{P}(x)$
[29] ∃y:Y • $\mathcal{Q}(y)$
[30] ∃!z:Z • $\mathcal{R}(z)$

[19] https://calcworkshop.com/logic/predicate-logic/, and: predicate logic, first-order logic or quantified logic is a formal language in which propositions are expressed in terms of predicates, variables and quantifiers. It is different from propositional logic which lacks quantifiers https://brilliant.org/wiki/predicate-logic/.

are quantified, i.e., predicate expressions. ∀, ∃ and ∃! are the quantifiers.

C.3.2.2 Predicate Calculus

They are "read" as:

[28] For all x (values in type X) the predicate $\mathcal{P}(x)$ holds – if that is not the case the expression yields truth value **false**.

[29] There exists (at least) one y (value in type Y) such that the predicate $\mathcal{Q}(y)$ holds – if that is not the case the expression yields truth value **false**.

[30] There exists a unique z (value in type Z) such that the predicate $\mathcal{R}(z)$ holds – if that is not the case the expression yields truth value **false**.

[28–30] The predicates $\mathcal{P}(x)$, $\mathcal{Q}(y)$ or $\mathcal{R}(z)$ may yield **chaos** in which case the whole expression yields **chaos**.

C.4 Arithmetics

ℐ: RSL offers the usual set of arithmetic operators. From these the usual kind of arithmetic expressions can be formed. ▪

Arithmetic

```
type
   Nat, Int, Real
value
   +,−,*: Nat×Nat→Nat | Int×Int→Int | Real×Real→Real
   /: Nat×Nat~→Nat | Int×Int~→Int | Real×Real~→Real
   <,≤,=,≠,≥,> (Nat|Int|Real) → (Nat|Int|Real)
```

C.5 Comprehensive Expressions

ℐ: Comprehensive expressions are common in mathematics texts. They capture properties conveniently abstractly ▪

C.5.1 Set Enumeration and Comprehension

C.5.1.1 Set Enumeration

Let the below a's denote values of type A:

Set Enumerations

{{}, {a}, {e$_1$,e$_2$,...,e$_n$}, ...} ∈ A-**set**
{{}, {a}, {e$_1$,e$_2$,...,e$_n$}, ..., {e$_1$,e$_2$,...}} ∈ A-**infset**

C.5.1.2 Set Comprehension

The expression, last line below, to the right of the ≡, expresses set comprehension. The expression "builds" the set of values satisfying the given predicate. It is abstract in the sense that it does not do so by following a concrete algorithm.

Set Comprehension

```
type
    A, B
    P = A → Bool
    Q = A ~→ B
value
    comprehend: A-infset × P × Q → B-infset
    comprehend(s,P,Q) ≡ { Q(a) | a:A • a ∈ s ∧ P(a)}
```

C.5.1.3 Cartesian Enumeration

Let e range over values of Cartesian types involving A, B, ..., C, then the below expressions are simple Cartesian enumerations:

Cartesian Enumerations

```
type
    A, B, ..., C
    A × B × ... × C
value
    (e1,e2,...,en)
```

C.5.2 List Enumeration and Comprehension

C.5.2.1 List Enumeration

Let a range over values of type A, then the below expressions are simple list enumerations:

List Enumerations

```
{⟨⟩, ⟨e⟩, ..., ⟨e1,e2,...,en⟩, ...} ∈ A*
{⟨⟩, ⟨e⟩, ..., ⟨e1,e2,...,en⟩, ..., ⟨e1,e2,...,en,... ⟩, ...} ∈ Aω

⟨ a_i .. a_j ⟩
```

The last line above assumes a_i and a_j to be integer-valued expressions. It then expresses the set of integers from the value of e_i to and including the value of e_j. If the latter is smaller than the former, then the list is empty.

C.5.2.2 List Comprehension

The last line below expresses list comprehension.

List Comprehension

```
type
  A, B, P = A → Bool, Q = A ~→ B
value
  comprehend: A^ω × P × Q ~→ B^ω
  comprehend(l,P,Q) ≡ ⟨ Q(l(i)) | i in ⟨1..len l⟩ • P(l(i))⟩
```

C.5.3 Map Enumeration and Comprehension

C.5.3.1 Map Enumeration

Let (possibly indexed) u and v range over values of type $T1$ and $T2$, respectively, then the below expressions are simple map enumerations:

Map Enumerations

```
type
  T1, T2
  M = T1 ~m~→ T2
value
  u,u1,u2,...,un:T1, v,v1,v2,...,vn:T2
  [], [u↦v], ..., [u1↦v1,u2↦v2,...,un↦vn] ∀ ∈ M
```

C.5.3.2 Map Comprehension

The last line below expresses map comprehension:

Map Comprehension

```
type
  U, V, X, Y
  M = U ~m~→ V
  F = U ~→ X
  G = V ~→ Y
  P = U → Bool
value
  comprehend: M×F×G×P → (X ~m~→ Y)
  comprehend(m,F,G,P) ≡ [ F(u) ↦ G(m(u)) | u:U • u ∈ dom m ∧ P(u) ]
```

C.6 Operations

C.6.1 Set Operations

C.6.1.1 Set Operator Signatures

Set Operator Signatures

```
value
  18 ∈: A × A-infset → Bool
  19 ∉: A × A-infset → Bool
  20 ∪: A-infset × A-infset → A-infset
  21 ∪: (A-infset)-infset → A-infset
  22 ∩: A-infset × A-infset → A-infset
  23 ∩: (A-infset)-infset → A-infset
  24 \: A-infset × A-infset → A-infset
  25 ⊂: A-infset × A-infset → Bool
  26 ⊆: A-infset × A-infset → Bool
  27 =: A-infset × A-infset → Bool
  28 ≠: A-infset × A-infset → Bool
  29 card: A-infset ~→ Nat
```

C.6.1.2 Set Operation Examples

Set Operation Examples

```
examples
  a ∈ {a,b,c}
  a ∉ {}, a ∉ {b,c}
  {a,b,c} ∪ {a,b,d,e} = {a,b,c,d,e}
  ∪{{a},{a,b},{a,d}} = {a,b,d}
  {a,b,c} ∩ {c,d,e} = {c}
  ∩{{a},{a,b},{a,d}} = {a}
  {a,b,c} \ {c,d} = {a,b}
  {a,b} ⊂ {a,b,c}
  {a,b,c} ⊆ {a,b,c}
  {a,b,c} = {a,b,c}
  {a,b,c} ≠ {a,b}
  card {} = 0, card {a,b,c} = 3
```

C.6.1.3 Informal Set Operator Explication

The following is **not** a definition of RSL semantics. In RSL formulas we present an explication of RSL operators. Read, what appears as definitions, ≡, as [a kind of] identities.

18 ∈: The membership operator expresses that an element is a member of a set.
19 ∉: The nonmembership operator expresses that an element is not a member of a set.

20 ∪: The infix union operator. When applied to two sets, the operator gives the set whose members are in either or both of the two operand sets.
21 ∪: The distributed prefix union operator. When applied to a set of sets, the operator gives the set whose members are in all of the operand sets.
22 ∩: The infix intersection operator. When applied to two sets, the operator gives the set whose members are in both of the two operand sets.
23 ∩: The prefix distributed intersection operator. When applied to a set of sets, the operator gives the set whose members are in some of the operand sets.
24 \: The set complement (or set subtraction) operator. When applied to two sets, the operator gives the set whose members are those of the left operand set which are not in the right operand set.
25 ⊆: The proper subset operator expresses that all members of the left operand set are also in the right operand set.
26 ⊂: The proper subset operator expresses that all members of the left operand set are also in the right operand set, and that the two sets are not identical.
27 =: The equal operator expresses that the two operand sets are identical.
28 ≠: The nonequal operator expresses that the two operand sets are not identical.
29 **card**: The cardinality operator gives the number of elements in a finite set.

C.6.1.4 Set Operator Explications

The set operations can be "equated" as follows:

Set Operator Explications

```
value
  s′ ∪ s″ ≡ { a | a:A • a ∈ s′ ∨ a ∈ s″ }
  s′ ∩ s″ ≡ { a | a:A • a ∈ s′ ∧ a ∈ s″ }
  s′ \ s″ ≡ { a | a:A • a ∈ s′ ∧ a ∉ s″ }
  s′ ⊆ s″ ≡ ∀ a:A • a ∈ s′ ⇒ a ∈ s″
  s′ ⊂ s″ ≡ s′ ⊆ s″ ∧ ∃ a:A • a ∈ s″ ∧ a ∉ s′
  s′ = s″ ≡ ∀ a:A • a ∈ s′ ≡ a ∈ s″ ≡ s⊆s′ ∧ s′⊆s
  s′ ≠ s″ ≡ s′ ∩ s″ ≠ {}
  card s ≡
    if s = {} then 0 else
    let a:A • a ∈ s in 1 + card (s \ {a}) end end
    pre s /* is a finite set */
  card s ≡ chaos /* tests for infinity of s */
```

C.6.2 Cartesian Operations

Cartesian Operations

```
type
    A, B, C
    g0: G0 = A × B × C
    g1: G1 = ( A × B × C )
    g2: G2 = ( A × B ) × C
    g3: G3 = A × ( B × C )

value
    va:A, vb:B, vc:C, vd:D
    (va,vb,vc):G0,
    (va,vb,vc):G1
    ((va,vb),vc):G2
    (va3,(vb3,vc3)):G3

decomposition expressions
    let (a1,b1,c1) = g0,
        (a1′,b1′,c1′) = g1 in .. end
    let ((a2,b2),c2) = g2 in .. end
    let (a3,(b3,c3)) = g3 in .. end
```

C.6.3 List Operations

C.6.3.1 List Operator Signatures

List Operator Signatures

```
value
    hd: A^ω ~→ A
    tl: A^ω ~→ A^ω
    len: A^ω ~→ Nat
    inds: A^ω → Nat-infset
    elems: A^ω → A-infset
    .(.): A^ω × Nat ~→ A
    ⌢: A* × A^ω → A^ω
    =: A^ω × A^ω → Bool
    ≠: A^ω × A^ω → Bool
```

C.6.3.2 List Operation Examples

List Operation Examples

```
examples
    hd⟨a1,a2,...,am⟩=a1
    tl⟨a1,a2,...,am⟩=⟨a2,...,am⟩
    len⟨a1,a2,...,am⟩=m
    inds⟨a1,a2,...,am⟩={1,2,...,m}
    elems⟨a1,a2,...,am⟩={a1,a2,...,am}
    ⟨a1,a2,...,am⟩(i)=ai
    ⟨a,b,c⟩⌢⟨a,b,d⟩ = ⟨a,b,c,a,b,d⟩
```

```
⟨a,b,c⟩=⟨a,b,c⟩
⟨a,b,c⟩ ≠ ⟨a,b,d⟩
```

C.6.3.3 Informal List Operator Explication

The following is **not** a definition of RSL semantics. In RSL formulas we present an explication of RSL operators. Read, what appears as definitions, ≡, as [a kind of] identities.

- **hd**: Head gives the first element in a nonempty list.
- **tl**: Tail gives the remaining list of a nonempty list when Head is removed.
- **len**: Length gives the number of elements in a finite list.
- **inds**: Indices give the set of indices from 1 to the length of a nonempty list. For empty lists, this set is the empty set as well.
- **elems**: Elements gives the possibly infinite set of all distinct elements in a list.
- $\ell(i)$: Indexing with a natural number, i larger than 0, into a list ℓ having a number of elements larger than or equal to i, gives the ith element of the list.
- $\frown$: Concatenates two operand lists into one. The elements of the left operand list are followed by the elements of the right. The order with respect to each list is maintained.
- =: The equal operator expresses that the two operand lists are identical.
- ≠: The nonequal operator expresses that the two operand lists are not identical.

The operations can also be defined as follows:

C.6.3.4 List Operator Explications

The following is **not** a definition of RSL semantics. In RSL formulas we present an explication of RSL operators. Read, what appears as definitions, ≡, as [a kind of] identities.

List Operator Explications

```
value
  is_finite_list: A^ω → Bool

  len q ≡
    case is_finite_list(q) of
      true → if q = ⟨⟩ then 0 else 1 + len tl q end,
      false → chaos end

  inds q ≡
    case is_finite_list(q) of
      true → { i | i:Nat • 1 ≤ i ≤ len q },
      false → { i | i:Nat • i≠0 } end

  elems q ≡ { q(i) | i:Nat • i ∈ inds q }

  q(i) ≡
    if i=1
      then
        if q≠⟨⟩
          then let a:A,q′:Q • q=⟨a⟩⌢q′ in a end
          else chaos end
```

```
        else q(i−1) end

  fq ⌢ iq ≡
        ⟨ if 1 ≤ i ≤ len fq then fq(i) else iq(i − len fq) end
        | i:Nat • if len iq≠chaos then i ≤ len fq+len end ⟩
    pre is_finite_list(fq)

  iq′ = iq″ ≡
    inds iq′ = inds iq″ ∧ ∀ i:Nat • i ∈ inds iq′ ⇒ iq′(i) = iq″(i)

  iq′ ≠ iq″ ≡ ~(iq′ = iq″)
```

C.6.4 Map Operations

C.6.4.1 Map Operator Signatures

Map Operator Signatures

```
value
[30] ·(·): M → A ~→ B
[31] dom: M → A-infset [domain of map]
[32] rng: M → B-infset [range of map]
[33] †: M × M → M [override extension]
[34] ∪: M × M → M [merge ∪]
[35] \: M × A-infset → M [restriction by]
[36] /: M × A-infset → M [restriction to]
[37] =,≠: M × M → Bool
[38] °: (A ~m→ B) × (B ~m→ C) → (A ~m→ C) [composition]
```

C.6.4.2 Map Operation Examples

Map Operation Examples

```
value
[30] m(a) = b
[31] dom [a1↦b1,a2↦b2,...,an↦bn] = {a1,a2,...,an}
[32] rng [a1↦b1,a2↦b2,...,an↦bn] = {b1,b2,...,bn}
[33] [a↦b,a′↦b′,a″↦b″] † [a′↦b″,a″↦b′] = [a↦b,a′↦b″,a″↦b′]
[34] [a↦b,a′↦b′,a″↦b″] ∪ [a‴↦b‴] = [a↦b,a′↦b′,a″↦b″,a‴↦b‴]
[35] [a↦b,a′↦b′,a″↦b″]\{a} = [a′↦b′,a″↦b″]
[37] [a↦b,a′↦b′,a″↦b″]/{a′,a″} = [a′↦b′,a″↦b″]
[38] [a↦b,a′↦b′] ° [b↦c,b′↦c′,b″↦c″] = [a↦c,a′↦c′]
```

C.6.4.3 Informal Map Operation Explication

- $m(a)$: Application gives the element that a maps to in the map m.
- **dom**: Domain/Definition Set gives the set of values which maps to in a map.
- **rng**: Range/Image Set gives the set of values which are mapped to in a map.
- †: Override/Extend. When applied to two operand maps, it gives the map which is like an override of the left operand map by all or some "pairings" of the right operand map.
- ∪: Merge. When applied to two operand maps, it gives a merge of these maps.
- \: Restriction. When applied to two operand maps, it gives the map which is a restriction of the left operand map to the elements that are not in the right operand set.
- /: Restriction. When applied to two operand maps, it gives the map which is a restriction of the left operand map to the elements of the right operand set.
- =: The equal operator expresses that the two operand maps are identical.
- ≠: The nonequal operator expresses that the two operand maps are not identical.
- °: Composition. When applied to two operand maps, it gives the map from definition set elements of the left operand map, m_1, to the range elements of the right operand map, m_2, such that if a is in the definition set of m_1 and maps into b, and if b is in the definition set of m_2 and maps into c, then a, in the composition, maps into c.

C.6.4.4 Map Operator Explication

The following is **not** a definition of RSL semantics. In RSL formulas we present an explication of RSL operators. Read, what appears as definitions, ≡, as [a kind of] identities.
The map operations can also be defined as follows:

Map Operator Explications

```
value
  rng m ≡ { m(a) | a:A • a ∈ dom m }

  m1 † m2 ≡
    [ a↦b | a:A,b:B •
      a ∈ dom m1 \ dom m2 ∧ b=m1(a) ∨ a ∈ dom m2 ∧ b=m2(a) ]

  m1 ∪ m2 ≡ [ a↦b | a:A,b:B •
              a ∈ dom m1 ∧ b=m1(a) ∨ a ∈ dom m2 ∧ b=m2(a) ]

  m \ s ≡ [ a↦m(a) | a:A • a ∈ dom m \ s ]
  m / s ≡ [ a↦m(a) | a:A • a ∈ dom m ∩ s ]

  m1 = m2 ≡
    dom m1 = dom m2 ∧ ∀ a:A • a ∈ dom m1 ⇒ m1(a) = m2(a)
  m1 ≠ m2 ≡ ~(m1 = m2)

  m°n ≡
    [ a↦c | a:A,c:C • a ∈ dom m ∧ c = n(m(a)) ]
    pre rng m ⊆ dom n
```

C.7 λ-Calculus + Functions

I: The λ-Calculus is a foundation for the abstract specification language that RSL *is* ▪

C.7.1 The λ-Calculus Syntax

λ-Calculus Syntax

```
type /* A BNF Syntax: */
    ⟨L⟩ ::= ⟨V⟩ | ⟨F⟩ | ⟨A⟩ | ( ⟨A⟩ )
    ⟨V⟩ ::= /* variables, i.e. identifiers */
    ⟨F⟩ ::= λ⟨V⟩ • ⟨L⟩
    ⟨A⟩ ::= ( ⟨L⟩⟨L⟩ )
value /* Examples */
    ⟨L⟩: e, f, a, ...
    ⟨V⟩: x, ...
    ⟨F⟩: λ x • e, ...
    ⟨A⟩: f a, (f a), f(a), (f)(a), ...
```

C.7.2 Free and Bound Variables

Free and Bound Variables

Let x, y be variable names and e, f be λ-expressions.

- ⟨V⟩: Variable x is free in e.
- ⟨F⟩: x is free in $\lambda y \cdot e$ if $x \neq y$ and x is free in e.
- ⟨A⟩: x is free in $f(e)$ if it is free in either f or e (i.e., also in both).

C.7.3 Substitution

In RSL, the following rules for substitution apply:

Substitution

- subst([N/x]x) ≡ N;
- subst([N/x]a) ≡ a,

  for all variables a≠ x;
- subst([N/x](P Q)) ≡ (subst([N/x]P) subst([N/x]Q));
- subst([N/x](λx•P)) ≡ λ y•P;
- subst([N/x](λ y•P)) ≡ λy• subst([N/x]P),

  if x≠y and y is not free in N or x is not free in P;
- subst([N/x](λy•P)) ≡ λz•subst([N/z]subst([z/y]P)),

if y≠x and y is free in N and x is free in P
(where z is not free in (N P)).

C.7.4 α-Renaming and β-Reduction

α and β Conversions

- α-renaming: λx•M
  If x, y are distinct variables then replacing x by y in λx•M results in λy•subst([y/x]M). We can rename the formal parameter of a λ-function expression provided that no free variables of its body M thereby become bound.
- β-reduction: (λx•M)(N)
  All free occurrences of x in M are replaced by the expression N provided that no free variables of N thereby become bound in the result. (λx•M)(N) ≡ subst([N/x]M)

C.7.5 Function Signatures

For sorts we may want to postulate some functions:

Sorts and Function Signatures

```
type
    A, B, C
value
    obs_B: A → B,
    obs_C: A → C,
    gen_A: B×C → A
```

C.7.6 Function Definitions

Functions can be defined explicitly:

Explicit Function Definitions

```
value
    f: Arguments → Result
    f(args) ≡ DValueExpr

    g: Arguments ~→ Result
    g(args) ≡ ValueAndStateChangeClause
    pre P(args)
```

Or functions can be defined implicitly:

Implicit Function Definitions

```
value
  f: Arguments → Result
  f(args) as result
  post P1(args,result)

  g: Arguments ~→ Result
  g(args) as result
  pre P2(args)
  post P3(args,result)
```

The symbol $\tilde{\rightarrow}$ indicates that the function is partial and thus not defined for all arguments. Partial functions should be assisted by preconditions stating the criteria for arguments to be meaningful to the function.

C.8 Other Applicative Expressions

I: RSL *offers the usual collection of applicative constructs that functional programming languages (*Standard ML *[138, 138] or* F# *[106]) offer* ▪

C.8.1 Simple let Expressions

Simple (i.e., nonrecursive) **let** expressions:

Let Expressions

let a = $\mathcal{E}_d$ **in** $\mathcal{E}_b$(a) **end**

is an "expanded" form of:

$(\lambda a.\mathcal{E}_b(a))(\mathcal{E}_d)$

C.8.2 Recursive let Expressions

Recursive **let** expressions are written as:

Recursive let Expressions

let f = λa:A • E(f) **in** B(f,a) **end**

is "the same" as:

let f = YF **in** B(f,a) **end**

where:

F ≡ λg•λa•(E(g)) and YF = F(YF)

C.8.3 Predicative let Expressions

Predicative **let** expressions:

Predicative **let** Expressions

let a:A • $\mathcal{P}$(a) **in** $\mathcal{B}$(a) **end**

express the selection of a value a of type A which satisfies a predicate $\mathcal{P}$(a) for evaluation in the body $\mathcal{B}$(a).

C.8.4 Pattern and "Wild Card" let Expressions

Patterns and wild cards can be used:

Patterns

```
let {a} ∪ s = set in ... end
let {a,_} ∪ s = set in ... end

let (a,b,...,c) = cart in ... end
let (a,_,...,c) = cart in ... end

let ⟨a⌢ℓ = list in ... end
let ⟨a,_,b⟩⌢ℓ = list in ... end

let [a↦b] ∪ m = map in ... end
let [a↦b,_] ∪ m = map in ... end
```

C.8.4.1 Conditionals

Various kinds of conditional expressions are offered by RSL:

Conditionals

```
if b_expr then c_expr else a_expr
end

if b_expr then c_expr end ≡ /* same as: */
   if b_expr then c_expr else skip end

if b_expr_1 then c_expr_1
elsif b_expr_2 then c_expr_2
```

```
    elsif b_expr_3 then c_expr_3
    ...
    elsif b_expr_n then c_expr_n end

    case expr of
      choice_pattern_1 → expr_1,
      choice_pattern_2 → expr_2,
      ...
      choice_pattern_n_or_wild_card → expr_n
    end
```

C.8.5 Operator/Operand Expressions

Operator/Operand Expressions

⟨Expr⟩ ::=
⟨Prefix_Op⟩ ⟨Expr⟩
| ⟨Expr⟩ ⟨Infix_Op⟩ ⟨Expr⟩
| ⟨Expr⟩ ⟨Suffix_Op⟩
| ...
⟨Prefix_Op⟩ ::=
− | ∼ | ∪ | ∩ | **card** | **len** | **inds** | **elems** | **hd** | **tl** | **dom** | **rng**
⟨Infix_Op⟩ ::=
= | ≠ | ≡ | + | − | ∗ | ↑ | / | < | ≤ | ≥ | > | ∧ | ∨ | ⇒
| ∈ | ∉ | ∪ | ∩ | \ | ⊂ | ⊆ | ⊇ | ⊃ | ⌢ | † | °
⟨Suffix_Op⟩ ::= !

C.9 Imperative Constructs

I: RSL *offers the usual collection of imperative constructs that imperative programming languages (*Java *[100, 160] or* Oberon *(!) [172]) offer* ▪

C.9.1 Statements and State Changes

Often, following the RAISE method, software development starts with highly abstract-applicative constructs which, through stages of refinements, are turned into concrete and imperative constructs. Imperative constructs are thus inevitable in RSL.

Statements and State Change

```
  Unit
value
  stmt: Unit → Unit
```

stmt()

- Statements accept no arguments.
- Statement execution changes the state (of declared variables).
- **Unit → Unit** designates a function from states to states.
- Statements, stmt, denote state-to-state changing functions.
- Writing () as "only" arguments to a function "means" that () is an argument of type **Unit**.

C.9.2 Variables and Assignment

Variables and Assignment

0. **variable** v:Type := expression
1. v := expr

C.9.3 Statement Sequences and skip

Sequencing is expressed using the ';' operator. **skip** is the empty statement having no value or side-effect.

Statement Sequences and **skip**

2. **skip**
3. stm_1;stm_2;...;stm_n

C.9.4 Imperative Conditionals

Imperative Conditionals

4. **if** expr **then** stm_c **else** stm_a **end**
5. **case** e **of**: p_1→S_1(p_1),...,p_n→S_n(p_n) **end**

C.9.5 Iterative Conditionals

Iterative Conditionals

6. **while** expr **do** stm **end**
7. **do** stmt **until** expr **end**

C.9.6 Iterative Sequencing

Iterative Sequencing

8. **for** e **in** list_expr • P(b) **do** S(b) **end**

C.10 Process Constructs

I: RSL *offers several of the constructs that* CSP *[114] offers* ▪

C.10.1 Process Channels

As for channels we deviate from common RSL [98] in that we directly *declare* channels – and not via common RSL *objects* etc.

Let A and B stand for two types of (channel) messages and i:KIdx for channel array indexes, then:

Process Channels

channel c:A
channel { k[i]:B • i:Idx }
channel { k[i,j,...,k]:B • i:Idx,j:Jdx,...,k:Kdx }

declare a channel, c, and a set (an array) of channels, k[i], capable of communicating values of the designated types (A and B).

C.10.2 Process Composition

Let P and Q stand for names of process functions, i.e., of functions which express willingness to engage in input and/or output events, thereby communicating over declared channels. Let P() and Q stand for process expressions, then:

Process Composition

P ‖ Q Parallel composition
P □ Q Nondeterministic external choice (either/or)
P ⨅ Q Nondeterministic internal choice (either/or)
P ⫲ Q Interlock parallel composition

express the parallel (‖) of two processes, or the nondeterministic choice between two processes: either external (□) or internal (⨅). The interlock (⫲) composition expresses that the two processes are forced to communicate only with one another, until one of them terminates.

C.10.3 Input/Output Events

Let c, k[i] and e designate channels of type A and B, then:

Input/Output Events

```
c ?, k[i] ?      Input
c ! e, k[i] ! e  Output
```

expresses the willingness of a process to engage in an event that "reads" an input, respectively "writes" an output.

C.10.4 Process Definitions

The below signatures are just examples. They emphasise that process functions must somehow express, in their signature, via which channels they wish to engage in input and output events.

Process Definitions

```
value
  P: Unit → in c out k[i]
  Unit
  Q: i:KIdx →  out c in k[i] Unit

  P() ≡ ... c ? ... k[i] ! e ...
  Q(i) ≡ ... k[i] ? ... c ! e ...
```

The process function definitions (i.e., their bodies) express possible events.

C.11 RSL Module Specifications

We shall not include coverage nor use of the RSL module concepts of *schemes, classes* and *objects*.

C.12 Simple RSL Specifications

Often, we do not want to encapsulate small specifications in schemas, classes, and objects, as is often done in RSL. An RSL specification is simply a sequence of one or more types, values (including functions), variables, channels and axioms:

Simple RSL Specifications

```
type
  ...
variable
  ...
channel
  ...
value
```

```
    ...
axiom
    ...
```

C.13 RSL$^+$: Extended RSL

Section C.2 on page 199 covered standard RSL types. To them we now add two new types: Type names and RSL-Text.

We refer to Sect. 4.5.1.2.1.1 (the *An RSL Extension* box) Page 44 for a first introduction to extended RSL.

C.13.1 Type Names and Type Name Values

C.13.1.1 Type Names

- Let T be a type name.
- Then ηT is a type name value.
- And $\eta\mathbb{T}$ is the type of type names.

C.13.1.2 Type Name Operations

- η can be considered an operator.
  - It (prefix) applies, then, to type (T) identifiers and yields the name of that type.
  - Two type names, nT_i, nT_j, can be compared for equality: $\mathrm{nT}_i = \mathrm{nT}_j$ iff $i = j$.
- It, vice-versa, suffix applies to type name (nT) identifiers and yields the name, T, of that type: $\mathrm{nT}\eta = \mathrm{T}$.

C.13.2 RSL-Text

C.13.2.1 The RSL-Text Type and Values

- RSL-Text is the type name for ordinary, non-extended RSL texts.

We shall not here give a syntax for ordinary, non-extended RSL texts – but refer to [98].

C.13.2.2 RSL-Text Operations

- RSL-Texts can be compared and concatenated:
  - rsl-text_a=rsl-text_b
  - $\text{rsl-text}_a \frown \text{rsl-text}_b$

The $\frown$ operator thus also applies, besides, lists (tuples), to RSL texts – treating RSL texts as (if they were) lists of characters.

Appendix D
Indexes

Contents

Some readers may use an index, such as these presented in this chapter, as an introduction to the ideas [etc.] of a book; Please, that is not the purpose of this chapter. Other readers may use an index, such as meant here, while studying a chapter, to remind themselves of the definition of a term, or where a term is [otherwise] used, etc. That is the purpose of this chapter. There are either far too many index references: with no obvious "structure"; or there are missing index references. or both ! I should have liked to work on these issues: to weed out "superfluous" references, to insert "missing" references, to "boil" them all into the essential ones, etcetera. Alas ! At the time I am writing this, trying to wrap up this book, I am 88. No real chance to get that – further 4–5 year work done !

D.1 Definitions

D.2 Concepts

D.3 Examples

D.4 Method

Note: We have yet to index all method principles, procedures, techniques and tools.

D.5 Analysis Predicate Prompts

D.6 Analysis Function Prompts

D.7 Description Prompts

D.8 Attribute Categories

D.9 Symbols

D.10 RSL Symbols