

DOMAIN ANALYSIS & DESCRIPTION

AN EXAMPLE

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Method & Methodology

Method

- By a **method** we shall understand
 - * a set of **principles** for selecting and applying
 - * a set of **procedures** – also for selecting and applying
 - * a set of **techniques** using
 - * a set of **tools**,for people to adhere to in the **construction** of an **artefact**.

Methodology

- * By **methodology** we shall understand the study of methods.

The Domain Modeling Method

- **Principles:**¹ abstraction.
- **Procedures:** the domain analysis & description ontology.
- **Techniques**²: Classical technique are that of establishing *invariants* and expressing *intentional pull*.
- **Tools**³: the analysis and description prompts and functions.

¹ **Principle:**

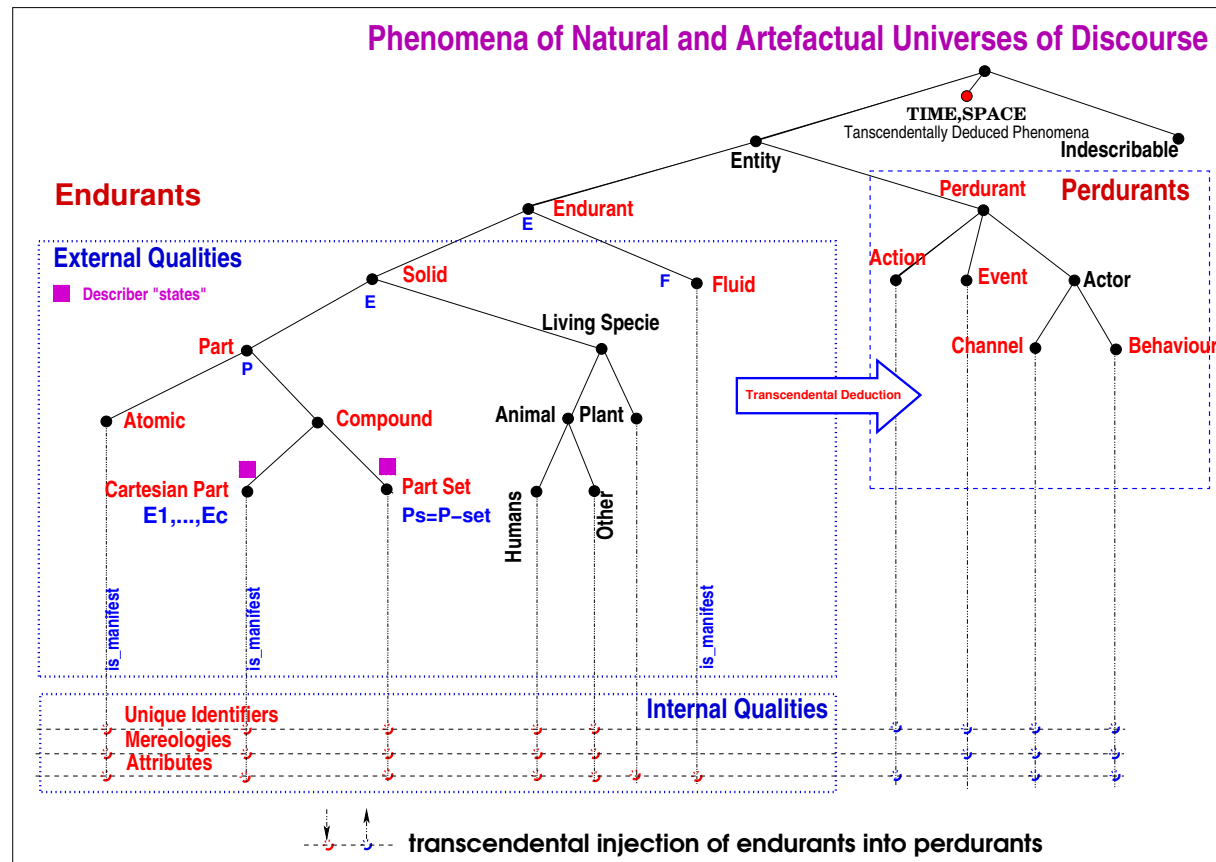
- (i) *elemental aspect of a craft or discipline,*
- (ii) *foundation,*
- (iii) *general law of nature, etc.*

² **Technique:**

- (i) *formal practical details in artistic, etc., expression,*
- (ii) *art, skill, craft in work".*

³ **Tool:**

- (i) *instrument, implement used by a craftsman or laborer, weapon,*
- (ii) *that with which one prepares something.*



The Domain Analysis & Description Ontology

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An Example

0. Universe of Discourse

Narration:

The domain is that of a road traffic system, **RT**
 of passengers, **P**,
 by automobiles, **A**, which **move** along a road net, **RN**.
 Passengers **embark** and **disembark** merchandise at hubs, **H**,
 and **travel** along links, **L** of the road net.
 Etcetera, etcetera.

Formalization:

type

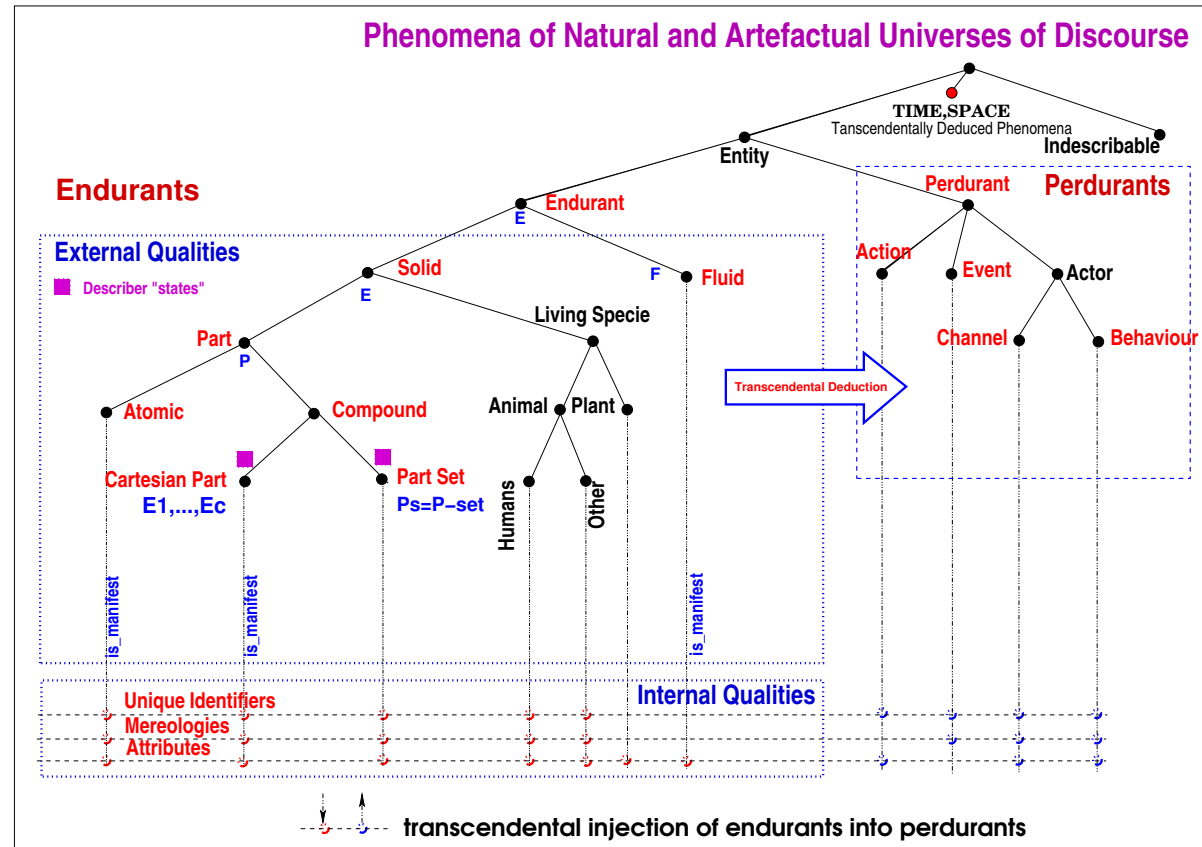
RT, P, A, RN, H, L, ...

value

move, embark, disembark, travel, ...

axiom

[The road net is connected, ...] ■

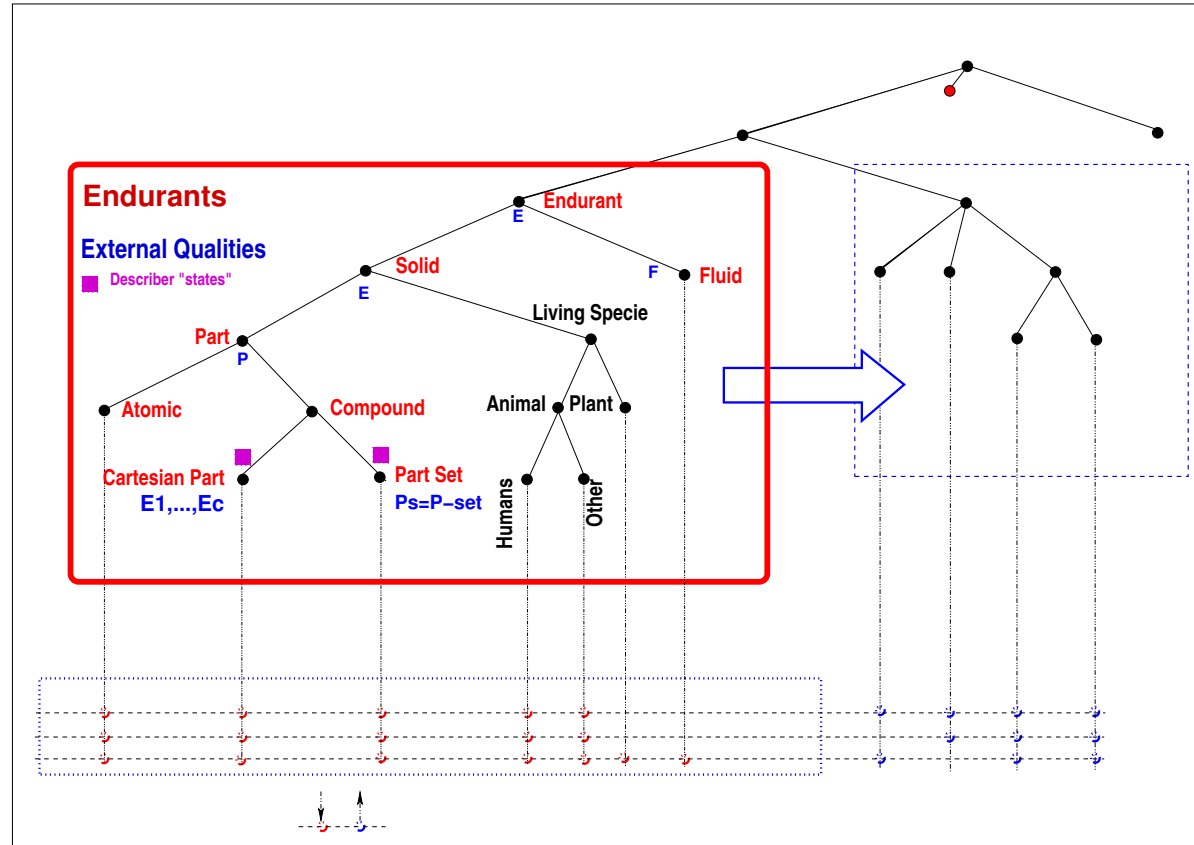


The Domain Analysis & Description Ontology

1. Endurants

1.1. External Qualities

1.1.1 Parts



Part Analysis & Description

Narrative:

1. A road transport, **rt:RT**, is abstracted as a Cartesian of
2. a road net, **RN** and
3. an aggregate of automobiles, **SA** –
4. where the road net is a Cartesian of a set of hubs, **AH**,
5. and a set of links, **AL**.
6. An aggregate of automobiles is a set of automolbiles.
7. Automobiles are here considered atomic.

Formalization:

type

1. RT
2. RN
3. SA
4. AH = H-set
5. AL = L-set
6. AS = A-set

7. A

value

2. **obs_RN**: RT \rightarrow RN
3. **obs_SA**: RT \rightarrow SA
4. **obs_AH**: RN \rightarrow AH
5. **obs_AL**: RN \rightarrow AL
6. **obs_AS**: SA \rightarrow AS ■

1.1.3 Part State

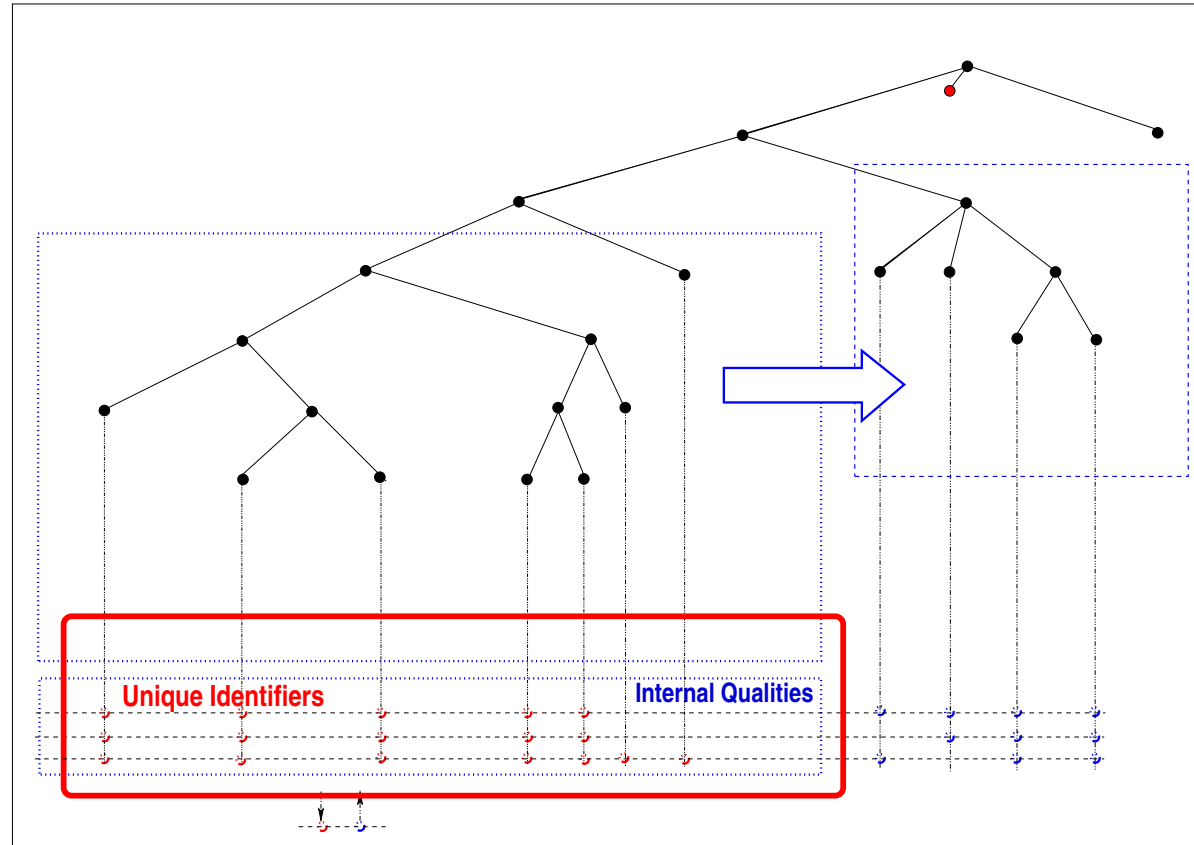
8. There is the set of all hubs,
9. and the set of all links,
10. and the set of all automobiles.
11. The union of these form a state.

variable

8. $hs:AH := \mathbf{obs_AH}(\mathbf{obs_RN}(rt))$
9. $ls:AL := \mathbf{obs_AL}(\mathbf{obs_RN}(rt))$
10. $as:SA := \mathbf{obs_AS}(\mathbf{obs_SA}(rt))$
11. $\sigma:(H|L|A)\text{-set} := hs \cup ls \cup as$ ■

1.2 Internal Qualities

1.2.1 Unique Identification



Unique Identification

- 12. Each hub has a unique identifier,
- 13. each link has a unique identifier, and
- 14. each automobile has a unique identifier.

type

- 12. H_I
- 13. L_I
- 14. A_I

value

- 12. **uid_H**: $H \rightarrow H_I$
- 13. **uid_H**: $L \rightarrow L_I$
- 14. **uid_H**: $A \rightarrow A_I$ ■

1.2.1.1 Unique Identifier State

There are

15. the set of all hub identifiers,
16. the set of all link identifiers,
17. the set of all automobile identifiers.
18. Together they form a unique identifier state.
19. There are as many hubs, links and automobiles as there are hub, link and automobile identifiers.

variable

15. $hs_{uids}:HI\text{-set} := \{ \mathbf{uid_H}(h) \mid h:H \cdot u \in \sigma \}$
16. $ls_{uids}:LI\text{-set} := \{ \mathbf{uid_L}(l) \mid l:L \cdot u \in \sigma \}$
17. $as_{uids}:AI\text{-set} := \{ \mathbf{uid_A}(a) \mid a:A \cdot u \in \sigma \}$
18. $\sigma_{uids}:(HI|LI|AI)\text{-set} := hs_{uids} \cup ls_{uids} \cup as_{uids}$

axiom

19. $\square \mathbf{card} \sigma = \mathbf{card} \sigma_{uids} \quad \blacksquare$

1.2.1.2 Part Uniqueness

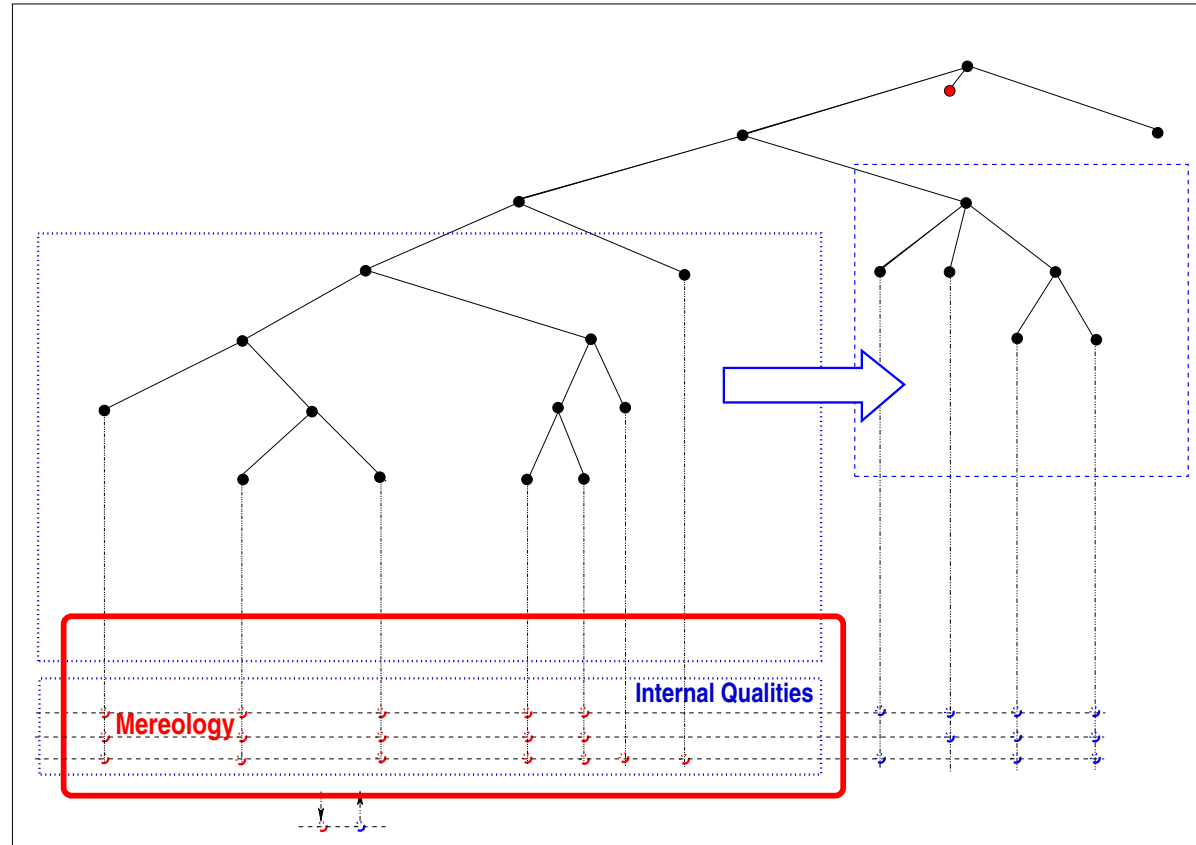
The unique identifiers of a road transport, **rt:RT**, consists of the unique identifiers of

20. the set of all hub identifiers,
21. the set of all link identifiers,
22. the set of all automobile identifiers.
23. Together they form a unique identifier state.
24. There are as many hubs, links and automobiles as there are hub, link and automobile identifiers.

variable

20. $hs_{uids}:\mathbf{Hl-set} := \{ \mathbf{uid_H}(h) \mid h:H \cdot h \in \sigma \}$
21. $ls_{uids}:\mathbf{LI-set} := \{ \mathbf{uid_L}(l) \mid l:L \cdot l \in \sigma \}$
22. $as_{uids}:\mathbf{AI-set} := \{ \mathbf{uid_A}(a) \mid a:A \cdot a \in \sigma \}$
23. $\sigma_{uids}:(\mathbf{HI|LI|AI-set}) := hs_{uids} \cup ls_{uids} \cup as_{uids}$
24. $\mathbf{card}\sigma = \mathbf{card}\sigma_{uids}$

1.2.2 Mereology



. Mereology

We shall be concerned only with the mereology of some manifest parts.

- 25. The mereology of links is a 2 element set of hub identifiers.
- 26. The mereology of a hub is a possibly empty set of hub identifiers.
- 27. The mereology of an automobile is a set of hub and link identifiers

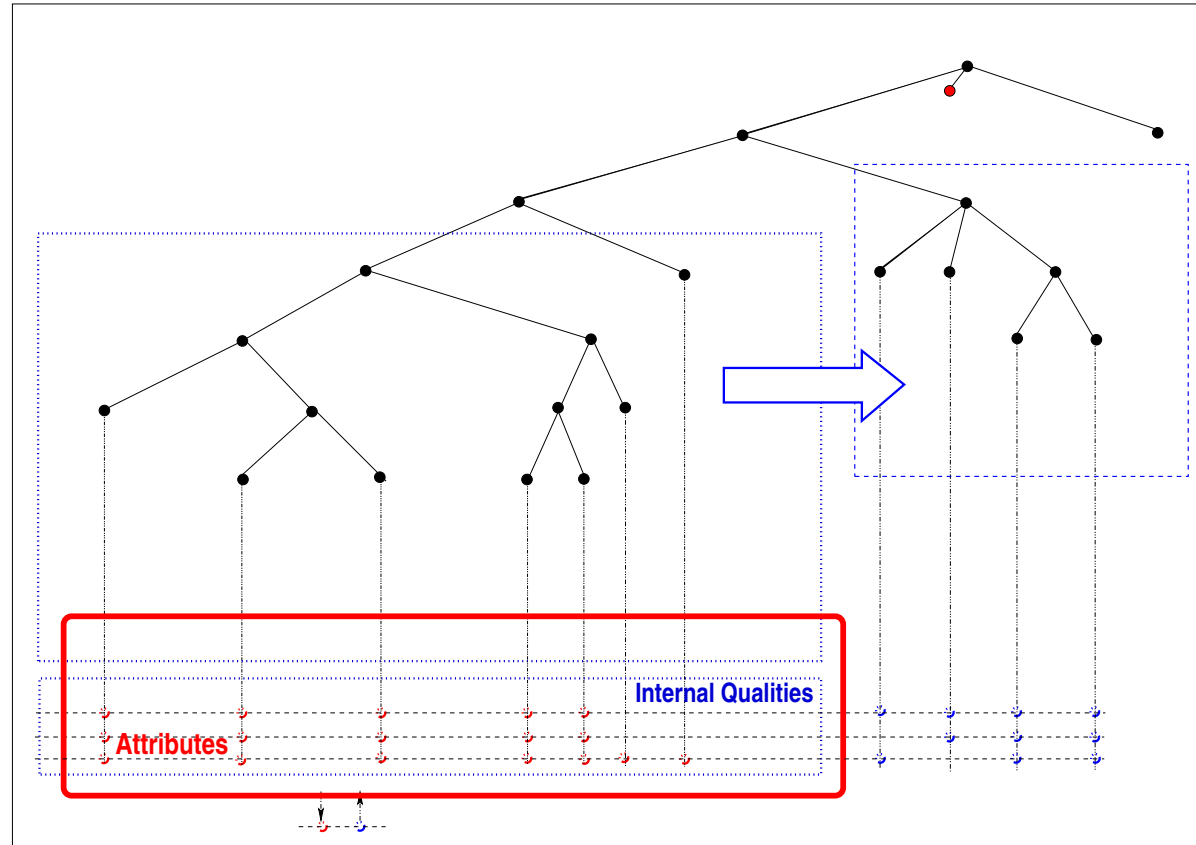
type

- 25. $ML = LI\text{-}set$ **axiom** $\forall ml:MK \cdot \text{card } ml = 2 \wedge ml \subseteq ls_{uis}$
- 26. $MH = HI\text{-}set$ **axiom** $\forall mh:MH \cdot mh \subseteq hs_{uis}$
- 27. $MA = (HI|LI)\text{-}set$ **axiom** $\forall ma:MA \cdot ma \subseteq as_{uis}$

value

- 25. **mereo_L**: $L \rightarrow ML$
- 26. **mereo_H**: $H \rightarrow MH$
- 27. **mereo_A**: $A \rightarrow MA$ ■

1.2.3 Attributes



.Attributes

Example attributes are:

28. Hubs have states, $\mathbf{h}\sigma:\mathbf{H}\Sigma$: the set of pairs of link identifiers, $(f\mathbf{li},t\mathbf{li})$, of the links *f*rom and *t*o which automobiles may enter, respectively leave the hub.
29. Hubs have state spaces, $\mathbf{h}\omega:\mathbf{H}\Omega$: the set of hub states “signaling” which states are open/closed, i.e., **green/red**.
30. Links that have lengths, **LEN**; and
31. Automobiles have road net positions, **APos**,
32. either *at a hub*, **atH**,
33. or *on a link*, **onL**, some fraction, **f:Real**, down a link, identified by **li**, from a hub, identified by **fhi**, towards a hub, identified by **thi**.
34. Links have states, $\mathbf{l}\sigma:\mathbf{L}\Sigma$: the set of pairs of link identifiers, $(f\mathbf{li},t\mathbf{li})$, of the links *f*rom and *t*o which automobiles may enter, respectively leave the hub.
35. Links have state spaces, $\mathbf{l}\omega:\mathbf{L}\Omega$: the set of link states “signaling” which states are open/closed, i.e., **green/red**.
36. Hubs, links and automobiles have *histories*: time-stamped, chronologically ordered sequences of automobiles entering and leaving links and hubs, with automobile histories similarly recording hubs and links entered and left.
37. Link positions have well-defined identifiers and fractions.

type

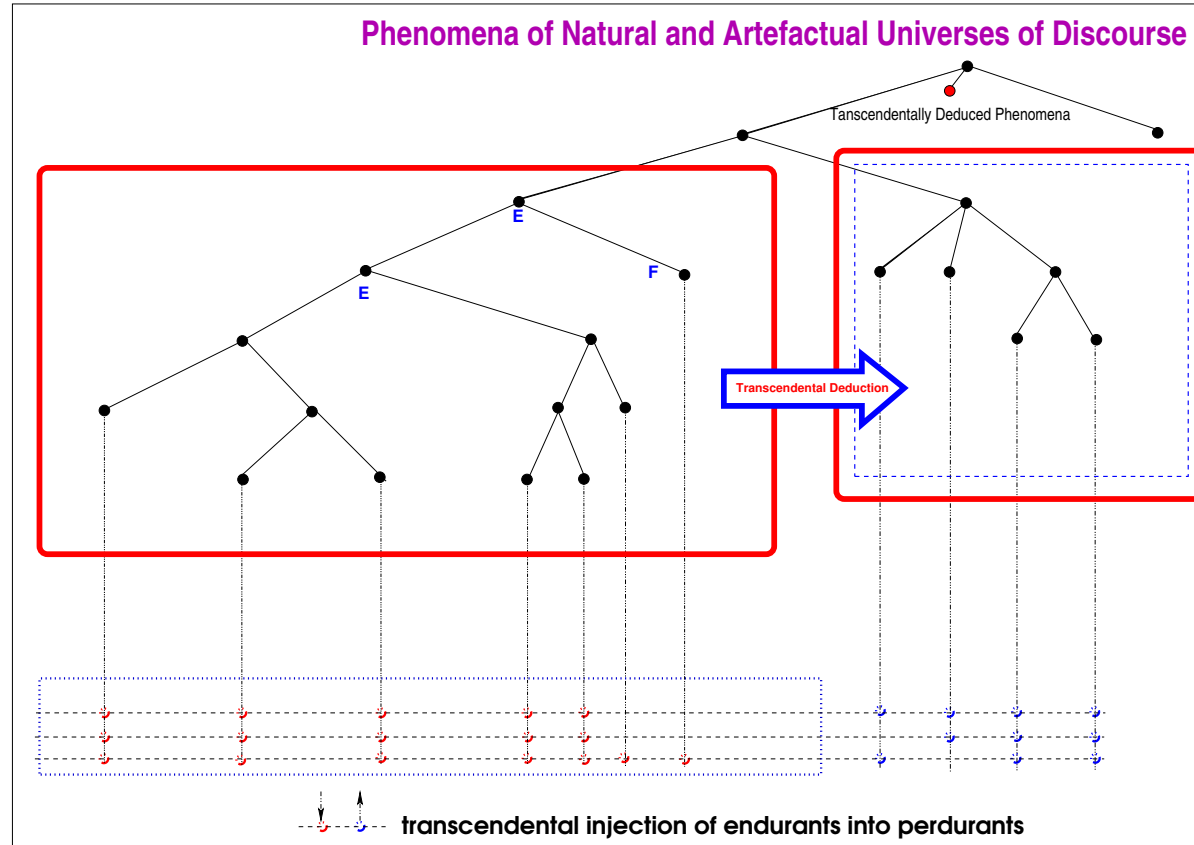
- 28. $H\Sigma = (LI \times LI)\text{-set}$
- 29. $H\Omega = H\Sigma\text{-set}$
- 30. $LEN = \mathbf{Nat}$
- 31. $APos = atH \mid onL$
- 32. $atH :: HI$
- 33. $onL :: LI \times (fhi:HI \times f:\mathbf{Real} \times thi:HI)$
- 34. $L\Sigma = (HI \times HI)\text{-set}$
- 35. $L\Omega = L\Sigma\text{-set}$
- 36. $HHis, LHis = (\mathbf{TIME} \times AI)^*$
- 36. $AHis = (\mathbf{TIME} \times (HI \mid LI))^*$

value

- 28. $attr_H\Sigma: H \rightarrow H\Sigma$
- 29. $attr_H\Omega: H \rightarrow H\Omega$
- 30. $attr_LEN: L \rightarrow LEN$
- 31. $attr_APos: A \rightarrow APos$
- 34. $attr_L\Sigma: L \rightarrow L\Sigma$
- 35. $attr_L\Omega: L \rightarrow L\Omega$
- 36. $attr_HHis: H \rightarrow HHis$
- 36. $attr_LHis: L \rightarrow LHis$
- 36. $attr_AHis: A \rightarrow AHis$

axiom

- 37. $\forall mk_onL(li, (fhi, f, thi)): onL \cdot 0 < f < 1 \wedge li \in ls_{uids} \wedge \{fhi, thi\} \subseteq hs_{uids} \wedge \dots \blacksquare$

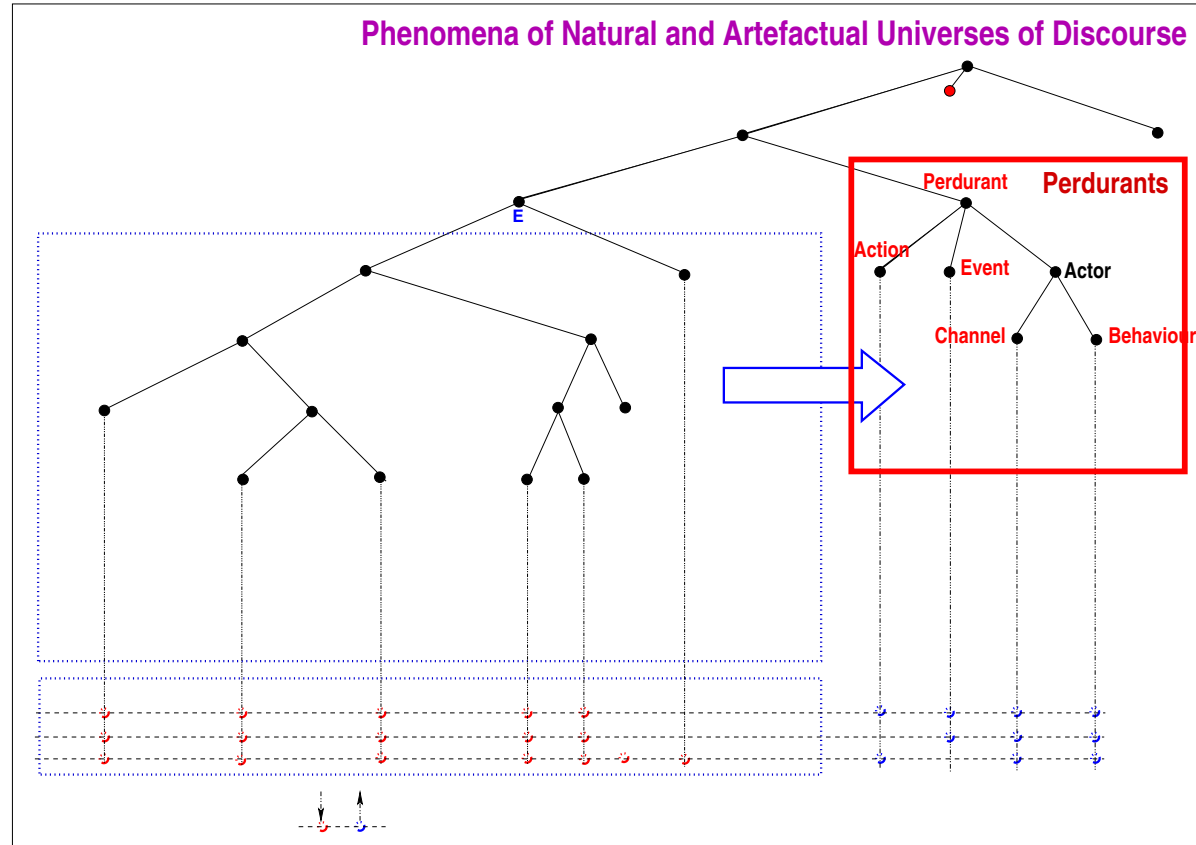


2. Transcendental Deduction

Transcendental Deductions:

- We decide to transcendental deduce the following manifest parts:
 - * stations,
 - * lines,
 - * junctions and
 - * passengers,
- into behaviours.

3. Perdurants



.

3.1 Channels

Channel

38. There is a set of channels between hubs, links and automobiles.
39. These channels communicate messages, M .
 M will “transpire” from the behaviour definitions.

channel

38. $\{ \text{ch}[\{u_i, u_j\}] \mid \{u_i, u_j\} : (\text{HI} \mid \text{LI} \mid \text{AI})\text{-set} \cdot u_i \neq u_j \wedge \{u_i, u_j\} \subseteq \sigma_{\text{wids}} \} \ M$

type

38. M ■

3.2 Behaviour Signatures

- The signature of behaviours follow the “Schönfinkel’ed pattern” of

names of behaviour: unique identifier

→ *mereology*

→ *static attributes*

[→ *inert and monitorable attributes*]

→ *programmable attributes*

→ *channel arrays and **Unit**.*

value

. hub: Hl

→ Mereoh

→ $(H\Omega \times \dots)$

→ $(H\Sigma \times \text{HHist} \times \dots)$

→ $\{\text{ch}[\{\mathbf{uid_H}(p), \text{ai}\}] \mid \text{ai:Al} \cdot \text{ai} \in as_{uid}\} \text{ Unit}$

?? link: Ll

→ Mereol→

→ $(L\Omega \times \text{LEN} \times \dots) \rightarrow$

→ $(L\Sigma \times \text{LHist} \times \dots)$

→ $\{\text{ch}[\{\mathbf{uid_L}(p), \text{ai}\}] \mid \text{ai:Al} \cdot \text{ai} \in as_{uid}\} \text{ Unit}$

?? automobile: Al

→ Mereoa

→ (...)

→ $(\text{AVel} \times \text{HAcc} \times \dots \times \text{APos} \times \text{AHist})$

→ $\{\text{ch}[\{\mathbf{uid_H}(p), \text{ri}\}] \mid \text{ri:}(Hl|Ll) \cdot \text{ri} \in h_{s_{uid}} \cup l_{s_{uid}}\} \text{ Unit}$

3.3 Behaviour Definitions

Automobile at Hub

40. We abstract automobile behaviour at a Hub (hi).

- (a) Either the automobile **remains** at the hub,
- (b) or, **internally non-deterministically**,
- (c) **leaves** the hub entering a link,
- (d) or, **internally non-deterministically**,
- (e) **stops**.

40 automobile(ai)(ris)(...)(atH(hi),ahis,_) \equiv

40a automobile_remain_at_hub(ai)(ris)(...)(atH(hi),ahis,_)

40b \sqcup

40c automobile_leaving_hub(ai)(ris)(...)(atH(hi),ahis,_)

40d \sqcup

40e automobile_stop(ai)(ris)(...)(atH(hi),ahis,_)

Automobile at Hub – Contd.

41. [40a] The automobile **remains** at a hub:

- (a) time is recorded,
- (b) informing the hub behaviour, whereupon
- (c) the automobile remains at that hub, “idling”,

```

41  automobile_remain_at_hub(ai)(ris)(...)(atH(hi),ahis,_) ≡
41a  let  $\tau$  = record_TIME in
41b  ch[ {ai,hi} ] !  $\tau$  ;
41c  automobile(ai)(ris)(...)(atH(hi), $\langle(\tau,hi)\rangle^{\wedge}$ ahis,_) end

```

Automobile at Hub – Contd.

42. [40c] The automobile **leaves** the hub entering link li:

- (a) time is recorded;
- (b) hub is informed of automobile leaving and link that it is entering;
- (c) “whereupon” the vehicle resumes (i.e., “while at the same time” resuming) the vehicle behaviour positioned at the very beginning (0) of that link.

42 automobile_**leaving**_b(ai)({li} ∪ ris)(...)(atH(hi),ahis,_) ≡

42a **let** τ = **record**_TIME **in**

42b (ch[{ai,hi}] ! τ || ch[{ai,li}] ! τ) ;

42c automobile(ai)(ris)(...)(onL(li,(hi,0,_)), $\langle(\tau,li)\rangle^{\wedge}$ ahis,_) **end**

42 **pre:** [hub is not isolated]

Automobile at Hub – Contd.

43. [40e] Or the automobile **stops**, “disappears — off the radar” !

43 automobile_**stop**(ai)(ris),(...)(atH(hi),ahis,_) \equiv stop ■

3.4 Behaviour Initialization

Initialization

44. Let us refer to the system initialization as a behaviour:

- (a) all hubs are initialized concurrently,
- (b) and, concurrently,
- (c) all links are initialized concurrently,
- (d) and, concurrently,
- (e) all automobiles are initialized concurrently.

value

44. `rts_initialisation`: **Unit** \rightarrow **Unit**

44. `rts_initialisation()` \equiv

44a. $\parallel \{ \text{hub}(\mathbf{uid_H}(l))(\mathbf{mereo_H}(l))(\mathbf{attr_H}\Omega(l),\dots)(\mathbf{attr_H}\Sigma(l),\dots) \mid h:H \cdot h \in h_s \}$

44b. \parallel

44c. $\parallel \{ \text{link}(\mathbf{uid_L}(l))(\mathbf{mereo_L}(l))(\mathbf{attr_LEN}(l),\dots)(\mathbf{attr_L}\Sigma(l),\dots) \mid l:L \cdot l \in l_s \}$

44d. \parallel

44e. $\parallel \{ \text{automobile}(\mathbf{uid_A}(a))(\mathbf{mereo_A}(a))(\mathbf{attr_APos}(a)\mathbf{attr_AHis}(a),\dots) \mid a:A \cdot a \in a_s \}$

Summing Up

- **This is NEITHER Computer NOR Computing Science:**

- * **This is Domain Science & Engineering**

- * It is, for example,

- a prerequisite for software requirements
 - and hence software design & coding!

- * We must abandon the “old” approach: just

- first software requirements.
 - then software design & coding!

- * **Now:**

- * First domain modeling \mathbb{D} .

- * Then “derive” requirements \mathbb{R} from \mathbb{D} .

- * Then “derive” software \mathbb{S} from \mathbb{R} .

- * Finally verify $\mathbb{D}, \mathbb{S} \models \mathbb{R}$

Review & Outlook

- **Four uses of Domain Models:**

- * **Understanding**

- * Theorems of Societal Infrastructures

- * **“Business Process Re-engineering”**

- * Redesigning Societal Infrastructures

- * **Software Development:** \mathbb{D} omains \rightarrow \mathbb{R} equirements \rightarrow \mathbb{S} oftware

- * **Basis for School Textbooks !**

- * Today we teach & learn about mathematics, physics, zoology, wt.

- * Tomorrow we could/should teach & learn about our own infrastructures:

· utilities,	· banking,	· judiciary system,	· retailing,
· health care,	· transport logistics,	· manufacturing,	· et cetera.

Review & Outlook – Continued:

● Exploratory Models – 1995–2025:

- * *A Retailer Market,*
- * *Documents,*
- * *Canals,*
- * *Container Terminals,*
- * *Credit Cards,*
- * *Double-entry Bookkeeping,*
- * *Graphs,*
- * *Rivers & Canals,*
- * *Railways,*
- * *Road Transport,*
- * *Shipping,*
- * *Stock Exchanges,*
- * *Swarms of Drones,*
- * *The “7 Seas”,*
- * *The “Blue Skies”,*
- * *Transport Logistics,*
- * *Urban Planning,*
- * *Weather Information,*
- * *Web Transactions,*
- * *Worldwide Banking.*

Review & Outlook – Continued:

● How to Do:

* Study the Domain:

- * visit/work in the domain
- * talk to domain stakeholders
- * read about the domain
- * etc., etc.

* **Exploratory Model:** one person⁴ 2 months! – then

* **Establish & Mantain Vocabulary:** throughout the project

* **Form a Team:** one or two per manifest endurant + perdurant⁵

* **Follow the Method:** Strictly, “no wavering!”

* Daily Work:

- * **Early am:** [Rotating/Shift] **Review** colleague’s work 8:30–10:00
- * **Mid am:** Team white board meeting: **Discuss** issues 10:00–10:45
- * **Late am + all pm:** **Modeling** 10:45–12:00, 13:00–16:30

⁴ – as for this example – or two for “larger” domain

⁵ – that is: 5 for this example!

Review & Outlook – Continued:

- **How Much – How Little ? :**

- * **For Understanding:**

- Any amount of a domain !

- * **For “Business Process Re-Engineering” :**

- Those aspects that appears to be re-oriented plus a little more !

- * **For Software Development:**

- A little more than what appears to be “computerized” !

- * **For School Textbooks:**

- That of a domain that he textbook would like to teach students.

Review & Outlook – Continued:

- **Commensurate Models:**

- * **“Families” of Domain Models:**

- * We assume that there are, or will be, two or more domain X models.

- * Then X_i, X_j, \dots, X_k , should/must satisfy some “Commensurateness” relations:

- * $\mathcal{C}_{ij}(X_i, X_j), \mathcal{C}_{ik}(X_i, X_k), \dots, \mathcal{C}_{jk}(X_j, X_k)$.

- * \mathcal{C} “being”: *extension, retraction, refinement, enlargement, ...*

That's it, Folks !

THANKS