1 Universe of Discourse: Road Traffic

Narration:

The domain is that of multi-modal transport **T**: land, sea and air,
of goods, **G**: passengers and merchandise,
by conveyors, **C**: bus, truck, train, ship and aircraft.
"K"ustomers, **K**, **inquire, order, deliver** and **receive** goods.
Firms, **F**, **offer, confirm order** and **convey** goods.
Conveyors **load** and **unload** merchandise at nodes, **N**,
travel along links, **L** of a transport net, **N**, and
keep firms and customers **inform**ed by messages, **M**.
Etcetera, etcetera.

Formalization:

type
  T, M, C, K, G, F, ..., N, L, N, M, ...
value
  inq, ordr, deliv, recv, offr, conf_ordr, convey, load, unload, travel, inf, ...
axiom
  ...

2 Endurants

2.1 External Qualities

2.1.1 The Parts

Narrative:

1. A road transport, rt:RT, is abstracted as a Cartesian of
2. a road net, RN and
3. an aggregate of automobiles, SA –
4. where the road net is a Cartesian of a set of hubs, AH,
5. and a set of links, AL.
6. An aggregate of automobiles is a set of automolbiles.
7. Automobiles are here considered atomic.

Formalization:

type
1. RT
2. RN
3. SA
4. AH = H-**set**
5. AL = L-**set**
6. AS = A-**set**
7. A

value
2. **obs**_RN: RT $\rightarrow$ RN
3. **obs**_SA: RT $\rightarrow$ SA
4. **obs**_AH: RN $\rightarrow$ AH
5. **obs**_AL: RN $\rightarrow$ AL
6. **obs**_AS: SA $\rightarrow$ AS

2.1.2 A Part State

8. There is the set of all hubs,
9. and the set of all links,
10. and the set of all automobiles.
11. The union of these form a state.

variable
8. hs:AH := **obs**_AH(**obs**_RN(rt))
9. ls:AL := **obs**_AL(**obs**_RN(rt))
10. as:SA := **obs**_AS(**obs**_SA(rt))
11. σ:(H|L|A)-**set** := $hs \cup ls \cup as$

2.2 Internal Qualities

2.2.1 Unique Identfication

The unique identifierss of a road transport, rt:RT, is here limited:

12. each hub has a unique identifier,
13. each link has a unique identifier, and
14. each automobile has a unique identifier.

type
12. HI
13. LI
14. AI

value
12. **uid**_H: H $\rightarrow$ HI
13. **uid**_H: L $\rightarrow$ LI
14. **uid**_H: A $\rightarrow$ AI

2.2.1.1 A Unique Identifier State

There are

15. the set of all hub identifiers,
16. the set of all link identifiers,
17. the set of all automobile identifiers.
18. Together they form a unique identifier state.
19. There are as many hubs, links and automobiles as there are hub, link and automobile identifiers.

variable
15. hs_{uids}:HI-**set** := { **uid**_H(h) | h:H•u $\in \sigma$ }
16. ls_{uids}:LI-**set** := { **uid**_L(l) | l:L•u $\in \sigma$ }
17. as_{uids}:AI-**set** := { **uid**_A(a) | a:A•u $\in \sigma$ }
18. σ_{uids}:(HI|LI|AI)-**set** := $hs_{uids} \cup ls_{uids} \cup as_{uids}$

axiom
19. $\Box$ **card** σ = **card** σ_{uids}

2.2.2 Mereology

We shall be concerned onlt with the mereology of some manifest parts.

20. The mereology of links is a 2 element set of hub identifiers of the road net[1].
21. The mereology of a hub is a possibly empty set of hub identifiers of the road net.
22. The mereology of an automobile is [some subset of] a set of hub and link identifiers[2]

type
20. ML = LI-**set** **axiom** $\forall$ ml:MK • **card** ml = 2 $\wedge$ ml $\subseteq ls_{uis}$
21. MH = HI-**set** **axiom** $\forall$ mh:MH • mh $\subseteq hs_{uis}$
22. MA = (HI|LI)-**set** **axiom** $\forall$ ma:MA • ma $\subseteq as_{uis}$

value
20. **mereo**_L: L $\rightarrow$ ML
21. **mereo**_H: H $\rightarrow$ MH
22. **mereo**_A: A $\rightarrow$ MA

[1] This is a simplified version: it allows for automoblie traffic in both directions of the link. We leave it to the reader to "cook" up othe such traffic possibilities.
[2] – a full set means that the specific automobile is allowed to travel all over the net.

2.2.3 Attributes

Example attributes are:

23. Hubs have states, hσ:HΣ: the set of pairs of link identifiers, ($f$li,$t$li), of the links $f$rom and to which automobiles may enter, respectively leave the hub.
24. Hubs have state spaces, hω:HΩ: the set of hub states "signaling" which states are open/closed, i.e., **green**/**red**.
25. Links that have lengths, LEN; and
26. Automobiles have road net positions, APos,
27. either *at a hub*, atH,
28. or *on a link*, onL, some fraction, f:**Real**, down a link, identified by li, from a hub, identified by fhi, towards a hub, identified by thi.
29. Links have states, lσ:LΣ: the set of pairs of link identifiers, ($f$li,$t$li), of the links $f$rom and to which automobiles may enter, respectively leave the hub.
30. Links have state spaces, lω:LΩ: the set of link states "signaling" which states are open/closed, i.e., **green**/**red**.
31. Hubs, links and automobiles have *histories:* time-stamped, chronologically ordered sequences of automobiles entering and leaving links and hubs, with automobile histories similarly recording hubs and links entered and left.
32. Link positions have well-defined identifiers and fractions.

type
23. HΣ = (LI$\times$LI)-**set**
24. HΩ = HΣ-**set**
25. LEN = **Nat**
26. APos = atH | onL
27. atH :: HI
28. onL :: LI $\times$ (fhi:HI $\times$ f:**Real** $\times$ thi:HI)
29. LΣ = (HI$\times$HI)-**set**
30. LΩ = LΣ-**set**
31. HHis,LHis = ($\mathbb{TIME}\times$AI)*
31. AHis = ($\mathbb{TIME}\times$(HI|LI))*

value
23. attr_HΣ: H $\rightarrow$ HΣ
24. attr_HΩ: H $\rightarrow$ HΩ
25. attr_LEN: L $\rightarrow$ LEN
26. attr_APos: A $\rightarrow$ APos
29. attr_LΣ: L $\rightarrow$ LΣ
30. attr_LΩ: L $\rightarrow$ LΩ
31. attr_HHis: H $\rightarrow$ HHis
31. attr_LHis: L $\rightarrow$ LHis
31. attr_AHis: A $\rightarrow$ AHis

axiom
32. $\forall$ mk_onL(li,(fhi,f,thi)):onL • 0<f<1 $\wedge$ li$\in ls_{uids}$ $\wedge$ {fhi,thi}$\subseteq hs_{uids}$ $\wedge$ ...

3 Perdurants

3.1 Channel

33. There is a set of channels between hubs, links and automobiles.
34. These channels communicate messages, `M`. `M` will "transpire" frm the behaviour definitions.

channel
33. { ch[{ui,uj}] | {ui,ij}:(HI|LI|AI)-**set** • ui$\neq$uj$\wedge${ui,uj}$\subseteq \sigma_{uids}$ } M

type
33. M

3.2 Behaviours

3.2.1 Definitions

35. We abstract automobile behaviour at a Hub (hi).
    (a) Either the automobile **remain**s at the hub,
    (b) or, **internally non-deterministically**,
    (c) **leaves** the hub entering a link,
    (d) or, **internally non-deterministically**,
    (e) **stops**.

35 automobile(ai)(ris)(...)(atH(hi),ahis,_) $\equiv$
35a   automobile_remain_at_hub(ai)(ris)(...)(atH(hi),ahis,_)
35b $\sqcap$
35c   automobile_leaving_hub(ai)(ris)(...)(atH(hi),ahis,_)
35d $\sqcap$
35e   automobile_stop(ai)(ris)(...)(atH(hi),ahis,_)

Automobile at Hub – Contd.

36. [35a] The automobile **remains** at a hub:
    (a) time is recorded,
    (b) informing the hub behaviour, whereupon
    (c) the automobile remains at that hub, "idling",

36 automobile_**remain**_at_hub(ai)(ris)(...)(atH(hi),ahis,_) $\equiv$
36a   **let** τ = **record**_$\mathbb{TIME}$ **in**
36b   ch[{ai,hi}] ! τ ;
36c   automobile(ai)(ris)(...)(atH(hi),$\langle(\tau$,hi)$\rangle^\frown$ahis,_) **end**

Automobile at Hub – Contd.

37. [35c] The automobile **leaves** the hub entering link li:
    (a) time is recorded;
    (b) hub is informed of automobile leaving and link that it is entering;
    (c) "whereupon" the vehicle resumes (i.e., "while at the same time" resuming) the vehicle behaviour positioned at the very beginning (0) of that link.

37 automobile_**leaving**_b(ai)({li}$\cup$ris)(...)(atH(hi),ahis,_) $\equiv$
37a   **let** τ = **record**_$\mathbb{TIME}$ **in**
37b   (ch[{ai,hi}] ! τ $\|$ ch[{ai,li}] ! τ) ;
37c   automobile(ai)(ris)(...)(onL(li,(hi,0,_)),$\langle(\tau$,li)$\rangle^\frown$ahis,_) **end**
37   **pre**: [hub is not isolated]

Automobile at Hub – Contd.

38. [35e] Or the automobile **stop**s, "disappears — off the radar" !

38 automobile_**stop**(ai)(ris),(...)(atH(hi),ahis,_) $\equiv$ **stop**