RESTORATION OF GERIS DATA USING THE MAXIMUM NOISE FRACTIONS TRANSFORM[1]

Allan Aasbjerg Nielsen
Rasmus Larsen

IMSOR Image Analysis Group
Institute of Mathematical Modelling
Technical University of Denmark, Building 321
DK–2800 Lyngby, Denmark

ABSTRACT

The Maximum Noise Fractions (MNF) transformation is used as a restoration tool in a 512×512 subscene of a 63 channel spectral dataset recorded over the Pyrite Belt in Southern Spain with the Geophysical Environmental Research Imaging Spectrometer (GERIS). The data obtained from such a scanning device are very useful in *e.g.* mineral exploration and environmental surveillance. Following the transformation from the original image space into the MNF space, a Fourier transformation of the MNFs (which are ordered by signal-to-noise ratio) will show more and more noise content. Also, the strong striping in primarily the visual bands of the scanner will be very conspicuous in the Fourier domain of only a few MNFs. We automatically detect the peaks in the Fourier spectra representing this striping, and if so desired we replace them by an iterated local mean value. Transforming back into the MNF space by the inverse Fourier transformation gives restored MNFs and transforming back into the original image space gives restored original bands. If we want to remove salt-and-pepper noise also, we can replace the noise-only MNFs by their mean value before transforming back into the original image space. This noise removal is very important along with atmospheric correction of the data before performing physically oriented analysis.

1.0 INTRODUCTION

Remote sensing imaging spectrometers measure the reflected/emitted energy in a number of discrete wavelength intervals. This “repetition” of the measurement at different wavelengths induces a high degree of redundancy in the data. This can be used for dimensionality reduction, noise reduction and data compression.

The exploitation of the spectral redundancy in the data is traditionally carried out by means of the celebrated principal components (PC) transformation. This is a pixel-wise operation that does not take the spatial structure of image data into account. Any permutation of the pixels in an image sequence will produce the same principal components. This is conceptually unsatisfactory. A spatial element should enter into our analysis. As opposed to the principal components that are new orthogonal variables with maximum variance, the maximum noise fractions are orthogonal linear transforms of the original variables that have the the highest signal-to-noise ratio. Also, PC analysis rarely produces components that show decreasing image quality with increasing component number. It is perfectly imaginable that certain types of noise have

[1]Presented at the First International Airborne Remote Sensing Conference and Exhibition, Strasbourg, France, 11–15 September 1994.

higher variance than certain types of signal components.

The GERIS data analyzed here are corrupted by a heavy two and four line banding. Such a regular noise pattern is well detected by Fourier methods. Also, the MNF transformation tends to isolate the striping in only few factors as a type of signal. The idea of this paper is now to combine the MNF transformation and well-known Fourier destriping techniques by first transforming from original spectral space into MNF space, from MNF space into Fourier space of MNFs, clean hopefully only a few MNFs isolating the striping, and then transforming via MNF space back into original spectral space again.

2.0 DATA DEPENDENT ORTHOGONAL TRANSFORMATIONS

First we will briefly consider the theory of principal components (see also Anderson, 1984). Second, two procedures for transformation of multivariate data given a spatial grid (images) with the purpose of isolating signal from noise and data compression are considered. These are the minimum/maximum autocorrelation factors transformation, which was first described by Switzer and Green (1984) and the maximum noise fractions transformation which was described by Green *et al*. (1988). An application of a version of the MNF transformation called Noise-Adjusted Principal Components to GERIS data is given in Lee *et al*. (1990). MAF and MNF analyses are also described in Conradsen *et al*. (1985, 1991) and Larsen (1991).

2.1 PRINCIPAL COMPONENTS

The principal components of a stochastic multivariate variable are linear transformations which produce uncorrelated variables of decreasing importance (*i.e*. decreasing variance). Let us consider a multivariate data set of p bands with grey levels $Z_i(\boldsymbol{x}), i = 1, \ldots, p$, where $\boldsymbol{x}$ denotes the coordinates of the sample, and the dispersion is $\mathrm{Cov}\{\boldsymbol{Z}(\boldsymbol{x})\} = \boldsymbol{\Sigma}$, where $\boldsymbol{Z}^T = [Z_1, \ldots, Z_p]$. The principal components are linear transforms

$$Y_i(\boldsymbol{x}) = \boldsymbol{a}_i^T \boldsymbol{Z}(\boldsymbol{x}), i = 1, \ldots, p \tag{1}$$

given by unit vectors $\boldsymbol{a}_i$, that have the property that the variance of $Y_i(\boldsymbol{x})$ is the highest among linear transforms orthogonal to $\boldsymbol{a}_j, j = 1, \ldots, i-1$. The variance of $Y_i(\boldsymbol{x})$ is the following Rayleigh coefficient

$$\mathrm{Var}\{\boldsymbol{a}_i^T \boldsymbol{Z}\} = \boldsymbol{a}_i^T \boldsymbol{\Sigma} \boldsymbol{a}_i = \frac{\boldsymbol{a}_i^T \boldsymbol{\Sigma} \boldsymbol{a}_i}{\boldsymbol{a}_i^T \boldsymbol{a}_i}. \tag{2}$$

This shows that the vectors $\boldsymbol{a}_i$ are eigenvectors of $\boldsymbol{\Sigma}$ and the variance of $Y_i(\boldsymbol{x})$ is equal to the i'th eigenvalue, λ_i, of $\boldsymbol{\Sigma}$. The first principal component is the linear transform of the original variables with the highest variance. The i'th principal component is the linear transform of the original variables that has the highest variance subject to the constraint that is uncorrelated with the first $i-1$ principal components. If we seek the i variables describing as much as possible of the total variance of the original variables the solution is the first i principal components. The fraction of the total variance described by these is given by

$$\frac{\lambda_1 + \cdots + \lambda_i}{\lambda_1 + \cdots + \lambda_i + \cdots + \lambda_p}. \tag{3}$$

A drawback of principal components analysis is that its result depends on the unit of measurement of the original variables. This problem can be eliminated by considering the standardized variables instead, *i.e*. performing the PC analysis on the correlation matrix instead of on the dispersion matrix.

2.2 MAXIMUM NOISE FRACTIONS

As we will show later by example, principal components do not always produce components of decreasing image quality. Maximizing variance across bands is not an optimal approach for image data. Neither is it an obvious one. Instead we will maximize a measure of image quality, namely a signal-to-noise ratio. This should ensure the desired ordering in terms of image quality.

The MNF transformation can be defined in several ways. It can be shown that the same set of eigenvectors is obtained by procedures that maximize the signal-to-noise ratio and the noise fraction. The procedure was first introduced by Green *et al*. (1988) where the authors in continuation of an earlier work by Switzer and Green (1984) choose the latter. Hence the name maximum noise fractions. First we will deduce the maximum noise fractions transformation and then – with the purpose of eliminating noise – we will sketch methods for estimating the covariance structure of the signal and the noise.

Let us as before consider a multivariate data set of p bands with grey levels $Z_i(\boldsymbol{x}), i = 1, \ldots, p$, where $\boldsymbol{x}$ denotes the coordinates of the sample. We will assume an additive noise structure

$$\boldsymbol{Z}(\boldsymbol{x}) = \boldsymbol{S}(\boldsymbol{x}) + \boldsymbol{N}(\boldsymbol{x}), \tag{4}$$

where $\boldsymbol{Z}^T = [Z_1, \ldots, Z_p]$. Also, we will assume that $\boldsymbol{S}(\boldsymbol{x})$ and $\boldsymbol{N}(\boldsymbol{x})$ are uncorrelated signal and noise components. Thus

$$\text{Cov}\{\boldsymbol{Z}(\boldsymbol{x})\} = \boldsymbol{\Sigma} = \boldsymbol{\Sigma}_S + \boldsymbol{\Sigma}_N, \tag{5}$$

where $\boldsymbol{\Sigma}_S$ and $\boldsymbol{\Sigma}_N$ are the covariance matrices for $\boldsymbol{S}(\boldsymbol{x})$ and $\boldsymbol{N}(\boldsymbol{x})$ respectively. Note that the techniques described in this section can in principle be applied to multiplicative noise also by first taking logarithms of the observations.

We define the signal-to-noise ratio (SNR) of the i'th band as

$$\frac{\text{Var}\{S_i(\boldsymbol{x})\}}{\text{Var}\{N_i(\boldsymbol{x})\}}, \tag{6}$$

the ratio of the signal variance and the noise variance. We define the maximum noise fractions transformation as the linear transformations

$$Y_i(\boldsymbol{x}) = \boldsymbol{a}_i^T \boldsymbol{Z}(\boldsymbol{x}), i = 1, \ldots, p \tag{7}$$

such that the signal-to-noise ratio for $Y_i(\boldsymbol{x})$ is maximum among all linear transforms orthogonal to $Y_j(\boldsymbol{x}), j = 1, \ldots, i-1$. Furthermore we shall assume that the vectors $\boldsymbol{a}_i$ are normed so that

$$\boldsymbol{a}_i^T \boldsymbol{\Sigma} \boldsymbol{a}_i = 1, i = 1, \ldots, p. \tag{8}$$

Maximization of the noise fraction leads to the opposite numbering, namely a numbering that gives increasing image quality with increasing component number. The SNR for $Y_i(\boldsymbol{x})$ is

$$\frac{\text{Var}\{\boldsymbol{a}_i^T \boldsymbol{S}(\boldsymbol{x})\}}{\text{Var}\{\boldsymbol{a}_i^T \boldsymbol{N}(\boldsymbol{x})\}} = \frac{\boldsymbol{a}_i^T \boldsymbol{\Sigma} \boldsymbol{a}_i}{\boldsymbol{a}_i^T \boldsymbol{\Sigma}_N \boldsymbol{a}_i} - 1. \tag{9}$$

If we work on the noise fraction instead, we get

$$\frac{\text{Var}\{\boldsymbol{a}_i^T \boldsymbol{N}(\boldsymbol{x})\}}{\text{Var}\{\boldsymbol{a}_i^T \boldsymbol{Z}(\boldsymbol{x})\}} = \frac{\boldsymbol{a}_i^T \boldsymbol{\Sigma}_N \boldsymbol{a}_i}{\boldsymbol{a}_i^T \boldsymbol{\Sigma} \boldsymbol{a}_i}. \tag{10}$$

In both cases we will, however, find the vectors $\boldsymbol{a}_i$ as eigenvectors to the real, symmetric, generalized eigenproblem

$$\det\{\boldsymbol{\Sigma}_N - \lambda \boldsymbol{\Sigma}\} = 0. \tag{11}$$

Thus the SNR for $Y_i(\boldsymbol{x})$ is given by $1/\lambda_i - 1$, where λ_i is the eigenvalue of $\boldsymbol{\Sigma}_N$ with respect to $\boldsymbol{\Sigma}$. Using the ordering obtained from maximization of the SNR the first maximum noise fraction is the linear transform of the original variables with the highest signal-to-noise ratio. The i'th maximum noise fraction is the linear transform of the original variables that has the highest signal-to-noise ratio subject to the constraint that is uncorrelated with the first $i - 1$ maximum noise fractions.

An important characteristic of the MNF transformation which is not shared by the PC transformation is the invariability to linear scaling (the signal-to-noise *ratio* is maximized).

When calculating minimum/maximum autocorrelation factors we find the eigenvectors of $\boldsymbol{\Sigma}_\Delta$ with respect to $\boldsymbol{\Sigma}$, where $\boldsymbol{\Sigma}_\Delta$ is the covariance matrix of the difference between an image and the same image shifted spatially by a displacement vector Δ.

2.2.1 Estimation of the Noise Covariance Matrix

The central problem in the calculation of the MNF transformation is the estimation of the noise with the purpose of generating a covariance matrix that approximates $\boldsymbol{\Sigma}_N$. It is in this process we will make use of the spatial characteristics of the image. We list four methods

- Simple differencing. The noise is estimated as the difference between the current and a neighboring pixel. In this case we refer to $\boldsymbol{\Sigma}_N$ as $\boldsymbol{\Sigma}_\Delta$ and we refer to the new factors as minimum/maximum autocorrelation factors.
- Causal SAR. The noise is estimated as the residual in a simultaneous autoregressive (SAR) model involving *e.g.* the neighboring pixels to the W, NW, N and NE of the current pixel.
- Differencing with the local mean. More pixels could be entered in to the estimation by differencing between the current pixel and the local mean.
- Differencing with local median. Mean filters blur edges and other details. This could be avoided by using the local median instead of the local mean.

2.2.2 Periodic Noise

As satellite images and images obtained from airborne scanners often are corrupted by striping we will consider methods for eliminating this type of noise. As periodic noise such as striping often has a high degree of spatial correlation, it will often be considered as signal by the MAF and MNF transformations. It should be noted that periodic noise can be very disturbing as the regular pattern catches the viewer's eyes.

A "naïve" bandwise Fourier filtering may corrupt significant parts of the relevant signal. Therefore we shall minimize the amount of filtering by eliminating the noise by filtering out the relevant peaks in the Fourier domain of the MNFs that isolate the striping and other types of noise. If we also filter or zero noise-only MNFs we eliminate salt-and-pepper noise. In order not to create an inverse pattern by setting the Fourier values to zero we keep the phase and fill the magnitude values by an iterative procedure that takes means of neighboring values.

3.0 CASE STUDY – GERIS DATA FROM SOUTHERN SPAIN

The GER imaging spectrometer actually consists of three spectrometers, that view the ground through the same aperture via an optoelectronic scanning device. The three spectrometers record a total of 63 bands through the visible (31 bands), nearinfrared (4 bands) and shortwave-infrared (28 bands) wavelength range between 0.47 and 2.45 μm. The spectral resolution in the visible region between 0.47 and 0.84 μm is 12.3 nm. In the nearinfrared (NIR) region from 1.40 nm to 1.90 nm it is much broader, around 120 nm. In the shortwave-infrared (SWIR) region between 2.00 and 2.45 μm the spectral resolution is 16.2 nm. This whiskbroom scanner uses a rotating mirror perpendicular to the flight direction to scan a line of 512 pixels with a scan angle of 45° to either side of the flight track. A flight altitude of 3000 m and an aperture setting of 2.5 mrad leads to a nominal pixel size of 7.5 m. The data are stored in 16 bit; the dynamic range is 12 bit. After recording the data are corrected for aircraft roll by use of the roll data recorded by a gyroscope hard mounted on the scanner optics. Because of the rotating mirror the GERIS data are corrupted by heavy two and four line banding. This banding comes from differences in optical properties (among other things dirt and oil) of the rotating mirror surfaces. The data used here are from a testsite in southern Spain. A 512×512 extract is analyzed.

3.1 TRANSFORMATIONS OF THE GERIS DATA

Transformations of the GERIS data were made using ordinary principal components and maximum noise fractions transformations. In the MNF case the noise was estimated as residuals of SAR models involving the W and N neighbors of the current pixel. Figure 1 shows principal components 1 through 6 (row-wise). It is evident that the principal components transformation does not yield the desired component ordering of image quality: clearly, PC5 contains more signal than PC4. Figure 2 shows maximum noise fractions 1 through 6 (row-wise). These give a much more satisfying ordering of image quality. One clearly sees an increasing spatial frequency with component number. An inspection of all 62 PCs and MNFs (the original band 28 is omitted) shows that this is true for higher order components also. Figure 3 shows correlations between the 62 original bands (band 28 omitted) and the first 10 MNFs. The different behavior of the three spectrometers is clearly seen. The number on the ordinate accompanying the MNF identification is the SNR.

3.2 FILTERING THE GERIS MNF IMAGES

We will now concern ourselves with the filtering of the components obtained by applying the MNF transformation. The advantage of performing the clean-up in the MNF Fourier space rather than in the original image band Fourier space is that individual bands that contribute only little to the noise are cleaned to a lesser extent and *vice versa*. Also, we only need to process signal components.

In Figure 4 the un-filtered and the filtered MNF4 are seen. As the human eye is very easily disturbed by striping, the visual impression of this new component is much better.

The MNF transformation is a good tool for elimination of salt-and-pepper noise as it makes a decomposition of spatial frequency. Assuming the components of high spatial frequency to be noise, we can set these to their mean values before transforming back into the original image space, thus eliminating this noise. In Figure 5 we show the original channel 1 and channel 1 obtained by calculating the MNFs, retaining the first 20 MNFs, performing the Fourier filtering by removing peaks in the magnitude of the Fourier transforms corresponding to the two and four line banding individually, and transforming back into image space. This procedure will remove not only salt-and-pepper noise but also other types of noise that might be isolated in higher order MNFs (MNF21 and up).

4.0 CONCLUSIONS

Following the transformation from the original image space into the MNF space, a Fourier transformation of the MNFs (which are ordered by signal-to-noise ratio) shows more and more noise content. We automatically detect the peaks in the MNF Fourier spectra representing the striping present in the data, and we replace them by an iterated local mean value. Transforming back into the MNF space by the inverse Fourier transformation gives restored MNFs and transforming back into the original image space gives restored original bands. In order to remove salt-and-pepper noise also, we replace the noise-only MNFs (MNF21 to MNF62) by their mean values before transforming back into the original image space. The results presented show that when the taks is noise removal preprocessing, the combination of the MNF transformation and Fourier techniques is a successful one. The two and four line banding, and the salt-and-pebber noise in the original GERIS channels have been removed by the techniques presented.

5.0 ACKNOWLEDGMENTS

We wish to thank Professor Knut Conradsen, head of the IMM and the IMSOR Image Analysis Group, for providing an inspiring environment, professionally and socially. Dr. Michael Grunkin, IMM, wrote the software applied to locate and replace peaks in the Fourier spectra.

The work reported here is a small part of a large research program funded by the European Commission under contract No. MA2M-CT90-0010.

6.0 REFERENCES

T.W. Anderson: *An Introduction to Multivariate Statistical Analysis*. 2nd Edition. John Wiley and Sons, p. 675, 1984.

K. Conradsen, B.K. Nielsen and T. Thyrsted: "A Comparison of Min/Max Autocorrelation Factor Analysis and Ordinary Factor Analysis". In *Proceedings from Symposium in Applied Statistics*, Technical University of Denmark, pp. 47–56, 29–30 January 1985.

K. Conradsen and B.K. Ersbøll: *Data Dependent Orthogonal Transformations of Multichannel Image Data*. Technical Report, IMSOR, Technical University of Denmark, p. 35, 1991.

K. Conradsen, B.K. Ersbøll and A.A. Nielsen: "Noise Removal in Multichannel Image Data by a Parametric Maximum Noise Fractions Estimator". In *Proceedings of 24th International Symposium on Remote Sensing of Environment*, Rio de Janeiro, Brazil, Vol. 1, pp. 403–416, 27–31 May 1991.

K. Conradsen, A.A. Nielsen, K. Windfeld, B.K. Ersbøll, R. Larsen, K. Hartelius and C.K. Olsson: *Application and Development of New Techniques Based on Remote Sensing, Data Integration and Multivariate Analysis for Mineral Exploration. Technical Annex*. Institute of Mathematical Statistics and Operations Research, EC Contract No. MA2M-CT90-0010, p. 96, 1993.

GAF, MAYASA, IMSOR and DLR: *Application and Development of New Techniques Based on Remote Sensing, Data Integration and Multivariate Analysis for Mineral Exploration. Final Report*. The Commission of the European Communities, Contract No. MA2M-CT90-0010, p. 117, 1993.

A.A. Green, M. Berman, P. Switzer and M.D. Craig: "A Transformation for Ordering Multispectral Data in Terms of Image Quality with Implications for Noise Removal". *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 26, No. 1, pp. 65–74, January 1988.

R. Larsen: *MAF og andre transformationer i remote sensing*, in Danish. Masters Thesis 36/91, IMSOR, Technical University of Denmark, 1991.

J.B. Lee, A.S. Woodyatt and M. Berman: "Enhancement of High Spectral Resolution Remote-Sensing Data by a Noise-Adjusted Principal Components Transform". *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 28, No. 3, pp. 295–304, May 1990.

P. Switzer and A.A. Green: *Min/Max Autocorrelation Factors for Multivariate Spatial Imagery*. Technical Report No. 6, Department of Statistics, Stanford University, p. 10, 1984.

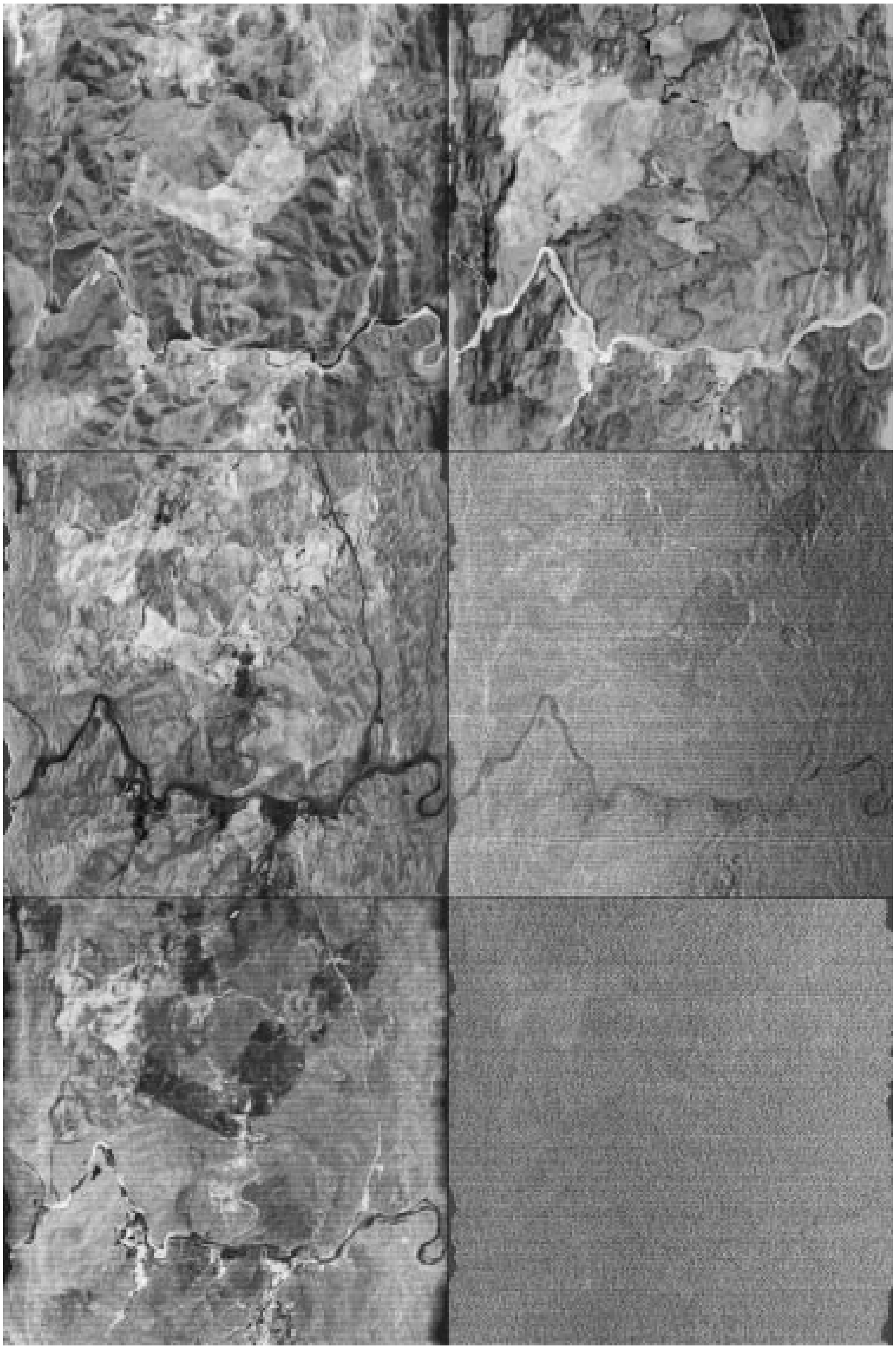

Figure 1: Principal Components 1 to 6

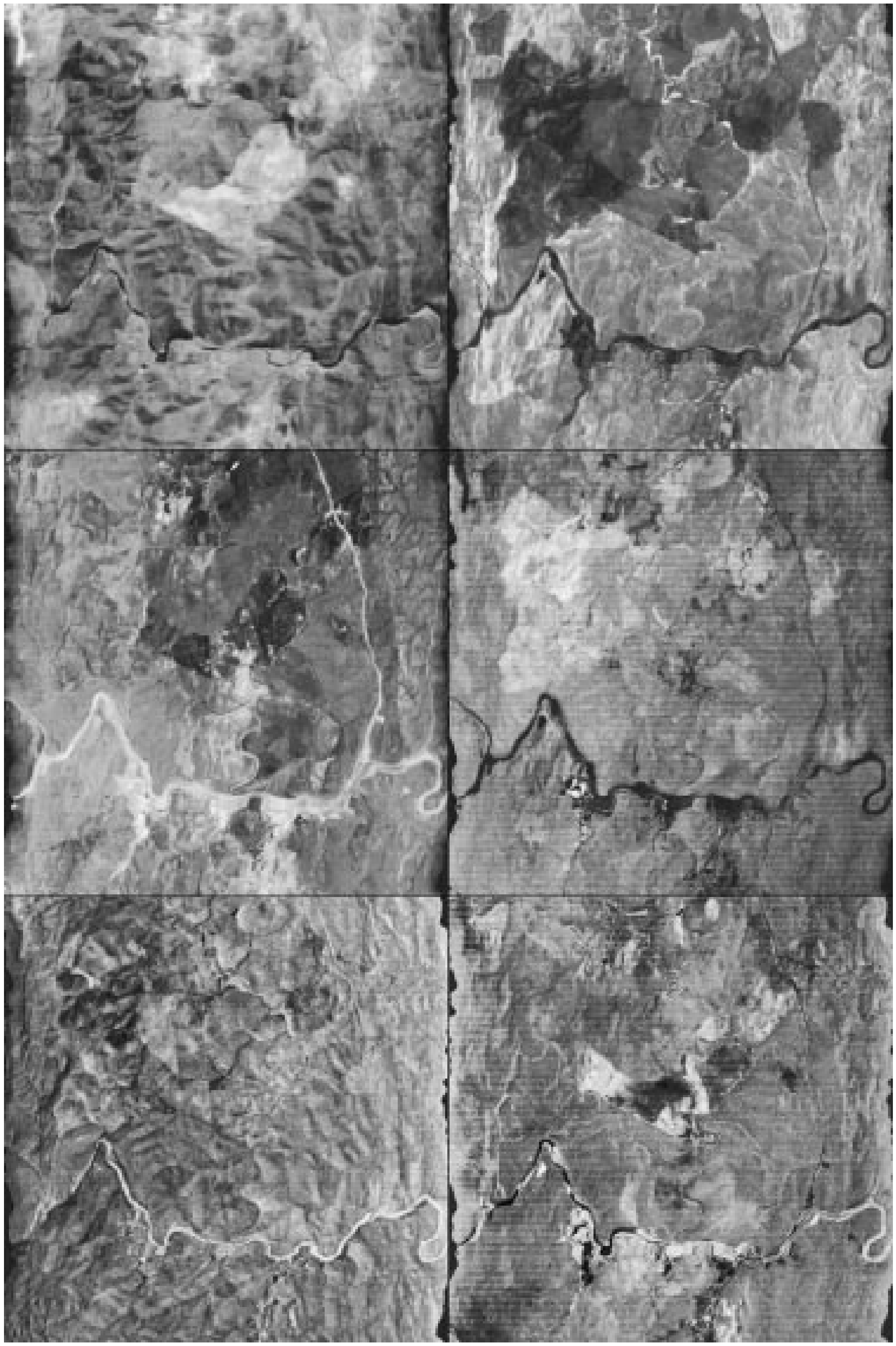

Figure 2: Maximum Noise Fractions 1 to 6

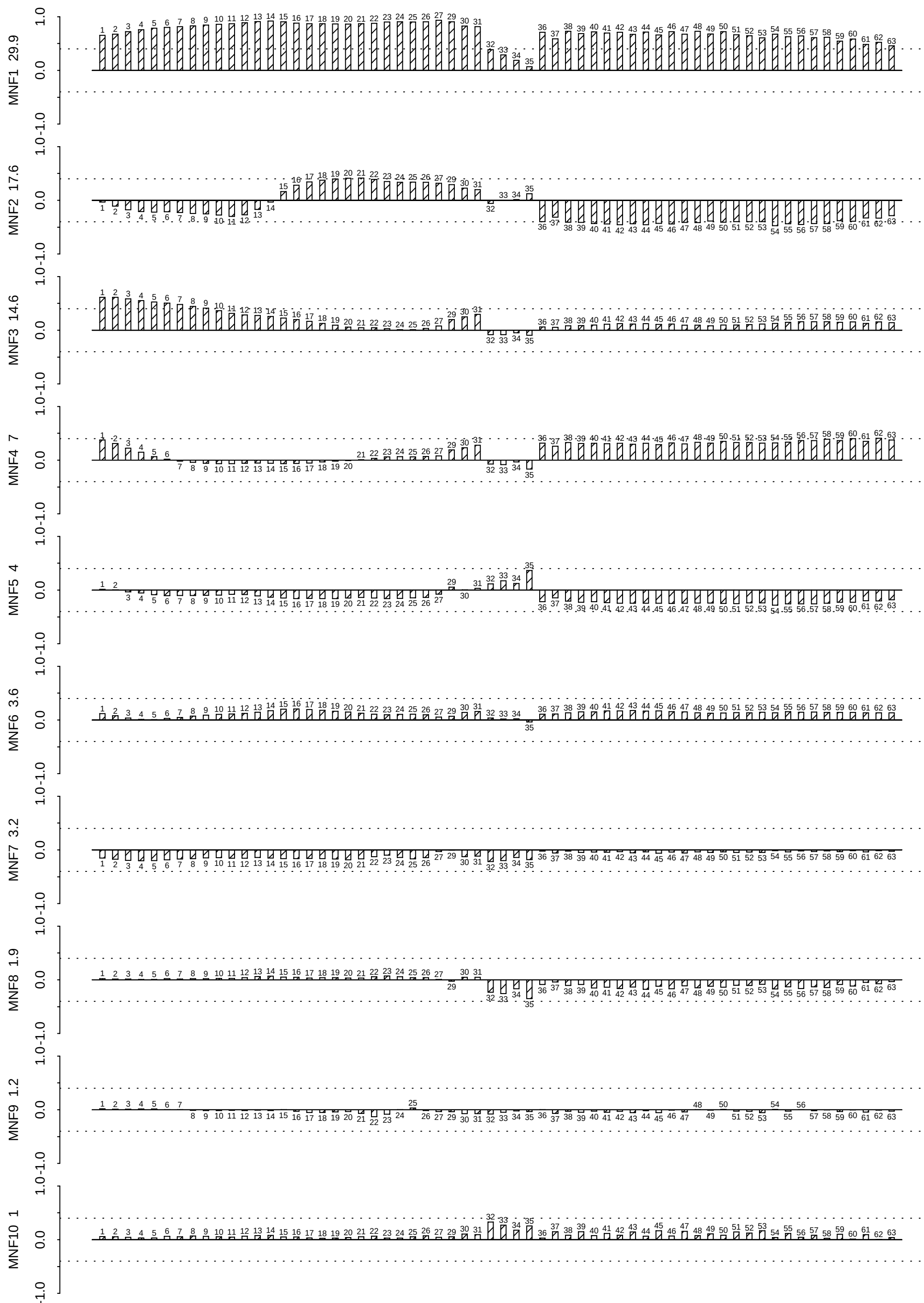

Figure 3: Correlations between Original Bands and MNFs

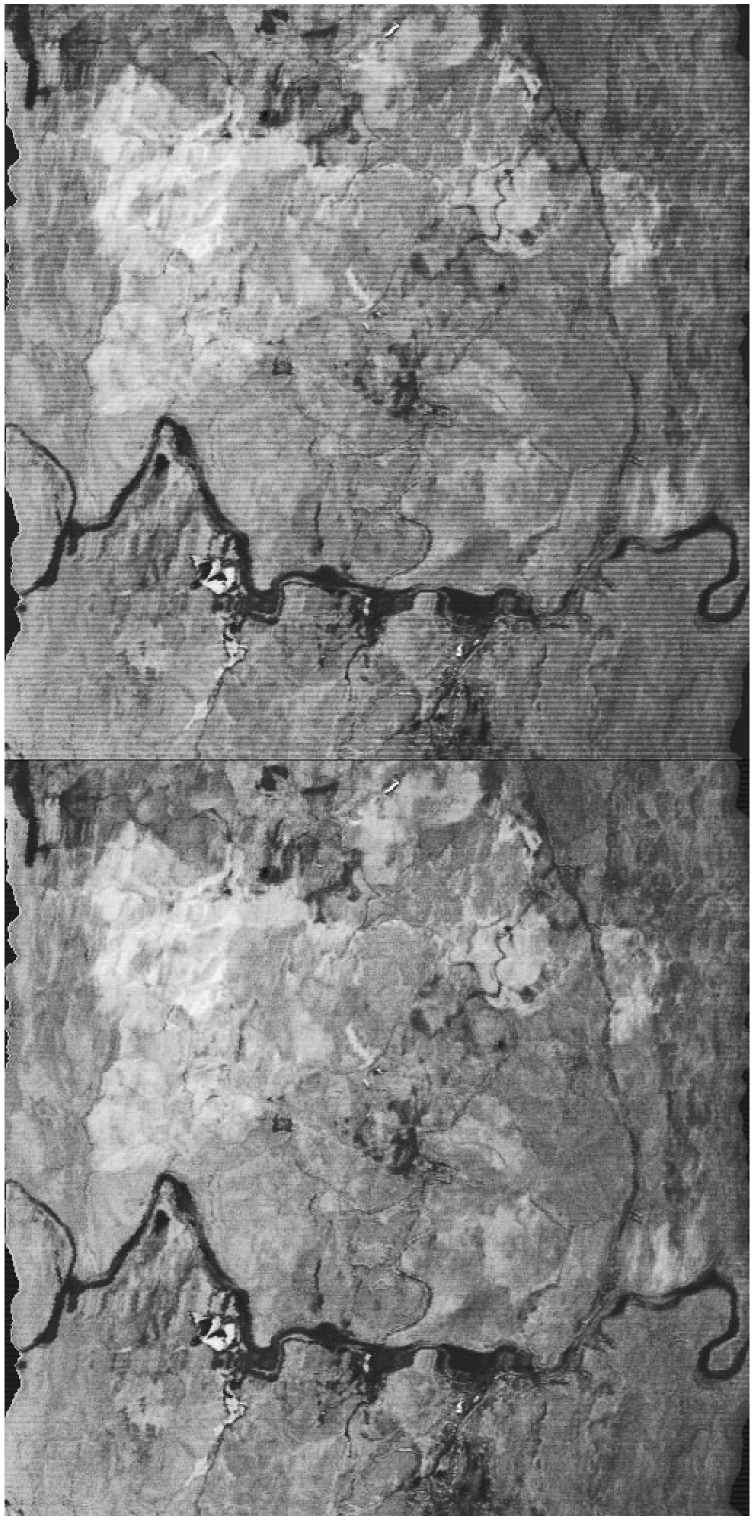

Figure 4: Original and Filtered MNF4