A DYNAMIC TEST METHOD FOR THE THERMAL PERFORMANCE OF SMALL HOUSES
Model Set-up, Test Design, Measurements, Parameter Estimation and Simulation

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ABSTRACT

Key Words: thermal response of small houses (single-family residences)
thermal-electric analogy (RC-circuits)
pseudo random binary sequence (PRBS)
active heat capacity
time constants
regression/time series analysis
low-energy test house (at the Technical University of Denmark)

The paper describes a dynamic test method for setting up a model for the thermal performance of a small house. The heat loss coefficient and other equivalent thermal parameters are determined. The model equations are based on the analogy between a house considered a simple one-dimensional, single-cell thermal system and a simple RC-circuit.

A method like this is believed to be useful when wanting to assess the value of an energy conserving measure. Estimates of the equivalent thermal parameters before and after carrying out the measure hold information about the improvements achieved.

Rather than having thermostats control the amount of energy needed to maintain a fixed temperature level, we supply the house with an amount of energy governed by a pseudo random binary sequence, PRBS. The energy is supplied by a number of electric resistance heaters replacing the normal heating system of the house. Depending on its parameters the PRBS will excite different layers of active heat capacity (time constants) in the house. The choice of parameters is therefore important. This type of signal will produce information about the dynamic characteristics of the house rather than the static ones achieved with the 'normal' constant temperature approach. Using a PRBS, we ensure that the correlations between the heat supply and the other input parameters are as poor as possible.

Having measured the indoor temperature, the outdoor parameters (temperature and solar radiation) and the amount of heat supplied by the replacing electric heating system, we used a computer program to estimate the components of the RC-circuit modelling the house. This was done using time series analysis. The residuals were checked by means of standard statistical methods. The estimates were used in a dynamic simulation of the thermal performance of the house. The ability of the model to simulate the measured indoor temperature dynamically is good considering its simplicity.

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INTRODUCTION

If you want to assess the heat loss from a house, you can determine the heat loss coefficient (W/°C): the heat loss (W) to the surroundings divided by the temperature difference (°C) between the inside and the outside. This is often done by turning off the normal heating system of the house and replacing it with electric resistance heaters. These heaters are often controlled by thermostats in order to achieve a constant indoor temperature. You then hope for a period with a fairly constant outdoor temperature and no (or little) solar radiation. Often, you must wait for days or even weeks to obtain this.

Rather than carrying out the traditional heat loss coefficient calculation, you can perform a regression analysis to obtain this and other equivalent thermal parameters. Usually, wind speed and maybe also wind direction is included in this analysis.

A problem arises in connection with the regression analysis: choosing the indoor temperature as the dependent variable, you will find that the input variables outdoor temperature, solar radiation and the energy supplied from the heaters are strongly correlated. Also, you will achieve no information about the heat capacity of the house due to the constant indoor temperature, unless the insolation causes overheating.

If you, on the other hand, have your electric resistance heaters controlled by a deterministic, pseudo random signal, the correlations between the energy input from the heaters and the other input variables will be as poor as possible. Also, the pseudo random energy input will cause more variation in the indoor temperature than in the case of thermostat control. This will produce information about the heat capacity of the house independent of the insolation.

SIMPLE MODELS

Based on an early work by Korsgaard, 1960 /1/, Saxhof and Aasbjerg Nielsen, 1982 /2/ used this simple regression model:

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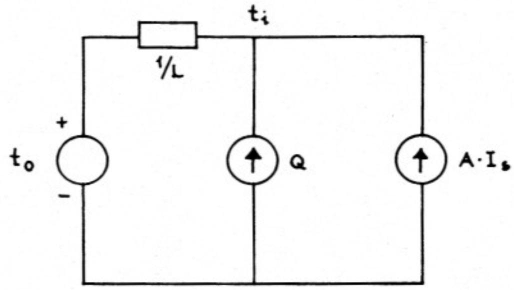
$$Q = L \cdot (t_i - t_o) - A \cdot I_s \tag{1}$$

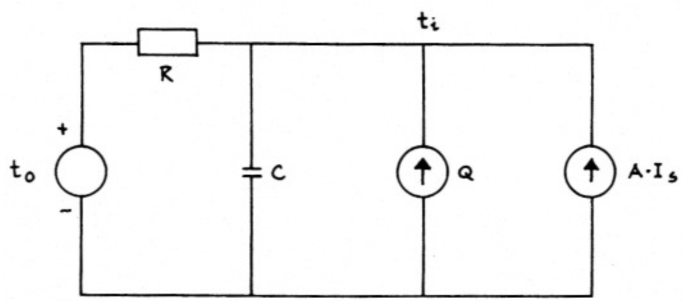
where

- Q is the power input from the heaters (kW),
- L is the heat loss coefficient (kW/°C),
- t_i is the indoor temperature (°C),
- t_o is the outdoor temperature (°C),
- A is the area (m^2) of an equivalent 'solar window' corresponding to 100% transparency and
- I_s is the solar radiation (kW/m^2) on a vertical surface facing south.

The wind parameters are omitted for simplicity. In a thermal-electric analogy, Equation 1 corresponds to this simple circuit -- Kirchhoff's current law* for the node with the voltage t_i gives Equation 1:



This model does not take the heat capacity of the house into account. For dynamic analysis, the following model is better, see also Sonderegger, 1978 /3/ or Oldengarm and Euser, 1981 /4/:



Kirchhoff's current law gives:

$$\frac{t_i - t_o}{R} + C \cdot \dot{t}_i - (Q + A \cdot I_s) = 0 \tag{2}$$

where

- R is the reciprocal of the heat loss coefficient (°C/kW) and
- C is the equivalent thermal mass (kWh/°C).

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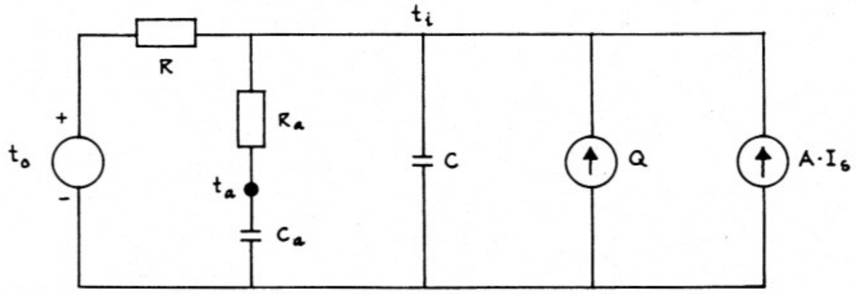
* Kirchhoff's current law states that the sum of the branch currents is 0 at every instant of time.

Compared to the large models sometimes seen for analog computer analysis, this is very simple. A somewhat more complicated model is:



Kirchhoff's current law for the two nodes with voltages t_i and t_a gives:

$$\frac{t_i - t_o}{R} + \frac{t_i - t_a}{R_a} + C \cdot \dot{t}_i - (Q + A \cdot I_s) = 0 \tag{3}$$

$$-\frac{t_i - t_a}{R_a} + C_a \cdot \dot{t}_a = 0 \tag{4}$$

<u>where</u>

t_a is the temperature in the accumulating layer and
R_a is the heat resistance from the indoor air to this layer.

Of course, you can add more resistors and capacitors as you like, thus increasing the complexity of the model.

EXPERIMENTS AND MEASUREMENTS

A pseudo random binary sequence (PRBS) is a deterministic series of digits, 'a's and '-a's. Transition between levels is allowed at times T, 2·T, 3·T, ... only but does not necessarily take place. Constructed in a sensible manner, this sequence forms a signal with a period of $(2^n-1) \cdot T$. n is the order of the signal and T is the shortest interval of time during which the signal is constant. The longest interval of time during which the signal is constant is n·T*. See also Letherman, Palin and Park, 1982 /5/. An example of a PRBS with n=4 and T=5 hours is shown in Figure 1 along with the thermal response of a room (here the levels are 0 and 1.5 kW; outdoor parameters are not shown).

The low-energy test house at the Technical University of Denmark is a one-storey, wood-built house with a crawl space and a roof space. The floor,

* This signal has an important statistical characteristic similar to that of 'white noise': it has no autocorrelation except for lags equal to the period length.

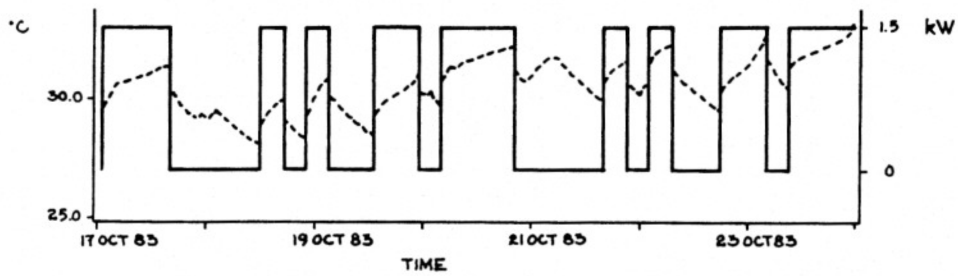


Figure 1. Example of a PRBS with n=4 and T=5 hours.

the walls and the ceiling of the ground floor are light sandwich constructions based on a masonite beam insulated with 300 mm of mineral wool. The ground floor contains the test space. An insulated partition wall with 95 mm of mineral wool divides this space into an east room and a west room of 60 m^2 each. The internal surfaces of ceiling and walls in both rooms are 12 mm gypsum boards. In the middle of both rooms structural and wind bracing cross frame constructions with 12 mm wood fibre boards are positioned. The floor in the east room is made of 22 mm of chip board on top of 53 mm of expanded polystyrene. The floor in the west room is made of 75 mm of concrete. The windows make up 15% of the floor area, 10% facing south and 5% facing north. All windows are triple glazed. The heat capacity of the east room has been increased by adding 96 m^2 of concrete flags with a thickness of 50 mm. The flags are positioned on four racks making up a 48 m^2 concrete partition wall with a thickness of 100 mm. Thermally speaking, the east room is 'heavy' and the west room is 'light'. Both rooms have ventilation systems with cross-flow plate type heat exchangers.

In this test house which is described by Rasmussen and Saxhof, 1982 /6/, we have carried out a series of experiments in which the energy supply from the electric heaters (3 x 500 W) in both rooms were controlled by PRBSs. The two sequences were displaced by half a period length to avoid correlation between the temperatures of the two rooms. Depending on n and T, the PRBS will excite different layers of active heat capacity in the house. The choice of n and T is therefore important.

Figure 2 shows a set of data from an experiment carried out in the test house from October 10, 1983 till October 14, 1983. Here the PRBS parameters are: n=6 and T=1 hour. The door between the two rooms was closed and the ventilation systems were turned off and sealed. The influence of the wind is neglegted because of the extreme tightness of the house. The air change rate has been measured to be .005 a.c.h. This was done by means of the tracer gas decay method. $t_{i,east}$ and $t_{i,west}$ are the indoor temperatures measured in the north side of the rooms and t_o is the outdoor temperature measured in the shade on the north side of the house. I_s is the solar radiation on a south-facing vertical surface. Scans every 10 minutes.

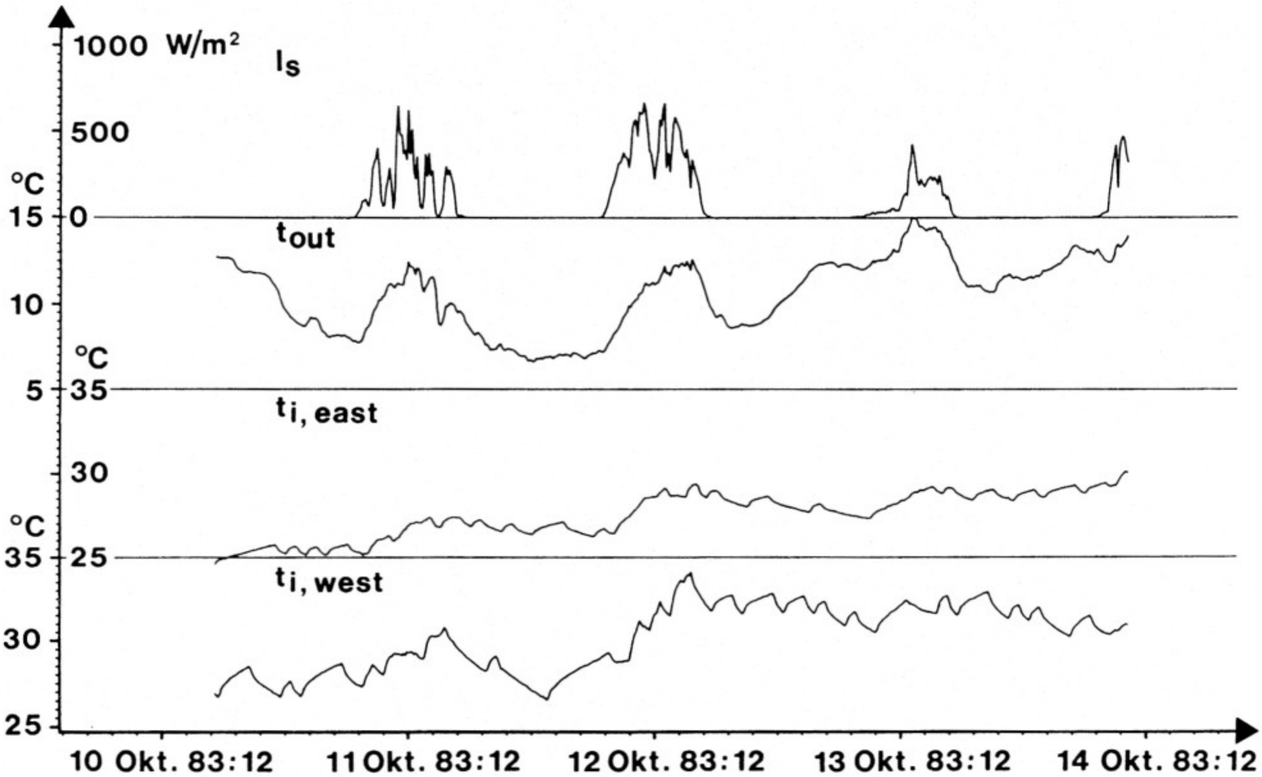


Figure 2. Data from both rooms of the low-energy test house and outdoor parameters. Scans every 10 minutes. PRBS not shown.

PARAMETER ESTIMATION

It was difficult to estimate the parameters of Equations 3 and 4. This problem might be overcome by measuring a representative of the temperature of the accumulating layer, t_a, as done by Pryor, Burns and Winn, 1980 /7/. Another reason for the difficulty could be the fact that for this time series, the number of observations containing information primarily about the heat capacity of the air, C, is low compared to number of observations containing information about the thermal capacity of the accumulating layers, C_a. This problem might be overcome by designing a special 'optimal sequence' for the system in question (Goodwin and Payne, 1977 /8/). Here, we shall confine ourselves to estimating the parameters R, C and A in the simpler Equation 2 re-written as: to estimating the parameters R, C and A in the simpler Equation 2 re-written as:

$$t_{i,t+1} = t_{i,t} - \frac{t_{i,t} - t_{o,t}}{R \cdot C} \cdot dt + \frac{Q_t + A \cdot I_{s,t+1}}{C} \cdot dt \qquad (5)$$

To determine which lags of the independent variables to use in the model, one can use a technique described by Box and Jenkins, 1976 /9/ called prewhitening*.

A period of $(2^n-1)\cdot T$ = 63 hours from October 10, 5 PM, 1983 till October 13, 8 AM, 1983 containing 378 observations is chosen for further analysis. By means of a procedure using a non-linear ordinary least squares estimation method, the following results are obtained:

Table I. Estimated parameters for model described by Equation 5.

East room 'heavy'	R °C/kW	C kWh/°C	A m^2
Estimate	21.6	2.30	1.43
Std.err.	1.2	.12	.27
t-value	17.38	18.46	5.20
r^2 = .9977 AIC = -1,088.7 sse = 1.219 °C^2 Residual variance = .003234 °C^2			

West room 'light'	R °C/kW	C kWh/°C	A m^2
Estimate	23.00	1.006	1.75
Std.err.	.84	.038	.17
t-value	27.29	26.35	10.08
r^2 = .9980 AIC = -742.5 sse = 3.047 °C^2 Residual variance = .008082 °C^2			

The t-test statistic is the estimate divided by the standard error. r^2 expresses the relative variance reduction, AIC is Akaike's information criterion (Akaike, 1974 /10/) and sse (sum of squared errors) is the sum of squared differences between measured and estimated indoor temperatures**.

* First, make a model of your independent variables (for instance a 5th order autoregressive model). Then use this model to transform your dependent variable. Cross correlate the residuals from the model of the input with the correspondingly transformed output and obtain an approximation to the impulse response function for your system. This function yields the appropriate lags.

**The t-test statistic expresses whether or not the estimate is significantly different from 0. Normally, the estimate is considered significant if t is greater than 2. -- The large values of r^2 are basically due to the fact

Calculations of R based on actual temperature differences and physical constants of the building materials employed, give values from 24.7 to 27.7 °C/kW for the east room and from 23.0 to 26.9 °C/kW for the west room. Calculations of C based on physical constants for the building materials employed, give 7.0 kWh/°C for the east room and 1.8 kWh/°C for the west room. For both rooms, the south-facing, transparent window areas are 4.9 m^2. Taking the reflection at low incidence angles into account, one will expect around 60% of the incident solar radiation on the windows to be transmitted to the house. Thus, an A of 2.9 m^2 is expected for both rooms. Solar heat gain through the north facing windows is neglegted.

The estimated values of R are satisfactory. As the entire thermal mass will not participate in the temperature fluctuations, it is more difficult to comment on the estimated values of C. The estimates lie between 0 and the values based on physical constants for the building materials, and the value for the east room is the larger. The estimated time constants (R·C) are 49.6 hours for the east room and 23.1 hours for the west room. The estimated values for A are unsatisfactory as they make up only around half of the expected values.

Closer investigation of the residuals shows that they have large values as the internal heat supplies are turned on/off, that they do not follow Gaussian distributions, that they are autocorrelated and that they hold more information about low-frequency components of the indoor temperature variation.

Inspecting the indoor temperatures at internal-heat-supply-on/off, one finds that drastic changes occur during the first 20 to 30 minutes after on/off. We therefore add a step-generator, t_h, between the capacitor and ground to the model described by Equation 2. This step-generator has a value, H, when the heat supply is on. If the heat supply is off, the step-generator is 0. This causes Equation 2 to change into:

$$\frac{t_i - t_o}{R} + C \cdot (\dot{t}_i - \dot{t}_h) - (Q + A \cdot I_s) = 0 \qquad (6)$$

Re-writing Equation 6 for estimation and estimating H on the basis of the first three scans (approximately 30 minutes) after each on/off rather than the first scan only, we arrive at:

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that the indoor temperatures are strongly autocorrelated ($t_{i,t+1}$ depends strongly on $t_{i,t}$). The variance reduction when differencing once ($t_{i,t+1} - t_{i,t}$) is .9950 for $t_{i,east}$ and .9940 for $t_{i,west}$. For $t_{i,east}$ the variance reduction from one differencing to 'full model' is .5490 and for $t_{i,west}$ it is .6726. If scan intervals increase (to for instance 30 minutes) the variance reduction when differencing once will decrease (for $t_{i,east}$ to .9606 and for $t_{i,west}$ to .9517). -- AIC is expected to be at minimum when the number of parameters is the right one.

$$t_{i,t+1} = t_{i,t} - \frac{t_{i,t}-t_{o,t}}{R \cdot C} \cdot dt + \frac{Q_t + A \cdot I_{s,t+1}}{C} \cdot dt + H \cdot \frac{1}{|Q|} \cdot (Q_t - Q_{t-3}) \qquad (7)$$

Using the model described in Equation 7, one achieves:

Table II. Estimated parameters for model described by Equation 7.

East room 'heavy'	R °C/kW	C kWh/°C	A m^2	H °C
Estimate	20.3	4.00	2.87	.0955
Std.err.	1.3	.30	.40	.0046
t-value	15.24	13.51	7.14	20.58
r^2 = .9989 AIC = -1,374.1 sse = .5685 °C^2 Residual variance = .001512 °C^2				
West room 'light'	**R °C/kW**	**C kWh/°C**	**A m^2**	**H °C**
Estimate	21.80	1.487	2.69	.1513
Std.err.	.78	.068	.20	.0075
t-value	27.82	22.01	13.20	20.25
r^2 = .9991 AIC = -1,020.5 sse = 1.449 °C^2 Residual variance = .003854 °C^2				

The estimated values for R and C are still satisfactory though the values for R are lower than before. The values for A are now much better. The estimated time constants are 81.3 hours for the east room and 32.4 hours for the west room*. Investigation of the residuals also shows that this is a better model. The effect of the solar radiation is clearly seen in the residuals, though. Tests show that the reductions in the sum of squared differences

* For this model the variance reductions from one differencing to 'full model' are .7892 for $t_{i,east}$ and .8439 for $t_{i,west}$. If scan intervals increase (to for instance 30 minutes) the variance reductions from one differencing to 'full model' also increase (for $t_{i,east}$ to .9179 and for $t_{i,west}$ to .9265; the entire variance reductions are .9967 for $t_{i,east}$ and .9964 for $t_{i,west}$; the estimated values for R, C, A and H at 30 minute scans all lie within the standard errors shown in the tables).

between the measured and the estimated indoor temperatures, sse, when adding the step-generator, are strongly significant.

Figure 3 shows the measured and the estimated indoor temperatures, the measured ones differenced once and the residuals from the 'full model'.

SIMULATION

The model described in Equation 7 is used to simulate the indoor temperatures in both rooms from October 10, 5 PM, 1983 till October 14, 10 AM, 1983. Figure 4 shows the measured and the simulated indoor temperatures, and the differences between the two. As opposed to the estimated indoor temperatures mentioned in the section on estimation above, the simulated indoor temperatures are calculated by inserting $t_{i,t}$ from the last simulation step in the right hand side of Equation 7 rather than the value from the measured series. A good agreement between measured and simulated indoor temperatures is achieved when using this simple model.

CONCLUSIONS

Based on a simple thermal-electric analogy, a simple empirical model for the indoor temperatures in two rooms of a low-energy test house is found. This is done by means of time series analysis methods and pseudo random heat input (PRBS). Under the imposed heat supplies, the two rooms are well described temperature- and energywise by means of only 4 parameters each. In principle, these 4 parameters can be estimated without any knowledge of the constructions of the house. Measurements from a period of 63 hours are used to estimate the parameters of the model. Scans every 10 minutes. Model parameters are the heat loss coefficient, the thermal mass and other equivalent thermal parameters of the house. Knowing the input variables solar radiation, outdoor temperature and heat supply from the heaters, we have simulated the indoor temperature satisfactorily for 4 days. A similar procedure could probably be used also in smaller sections of larger buildings with little or no modification.

The amount of heat supplied by the electric resistance heaters replacing the normal heating system of the house, requires some advance information about the heat loss from the house. If the heat supply is not suitable, the indoor temperatures will not be reasonably stationary*. The two time series that have been analyzed here are not stationary (see the figures), and still the estimations went well. The required advance information mentioned is often represented by the year of construction.

* Stationarity in this context denotes the characteristic, that the mean level of a variable is constant in time.

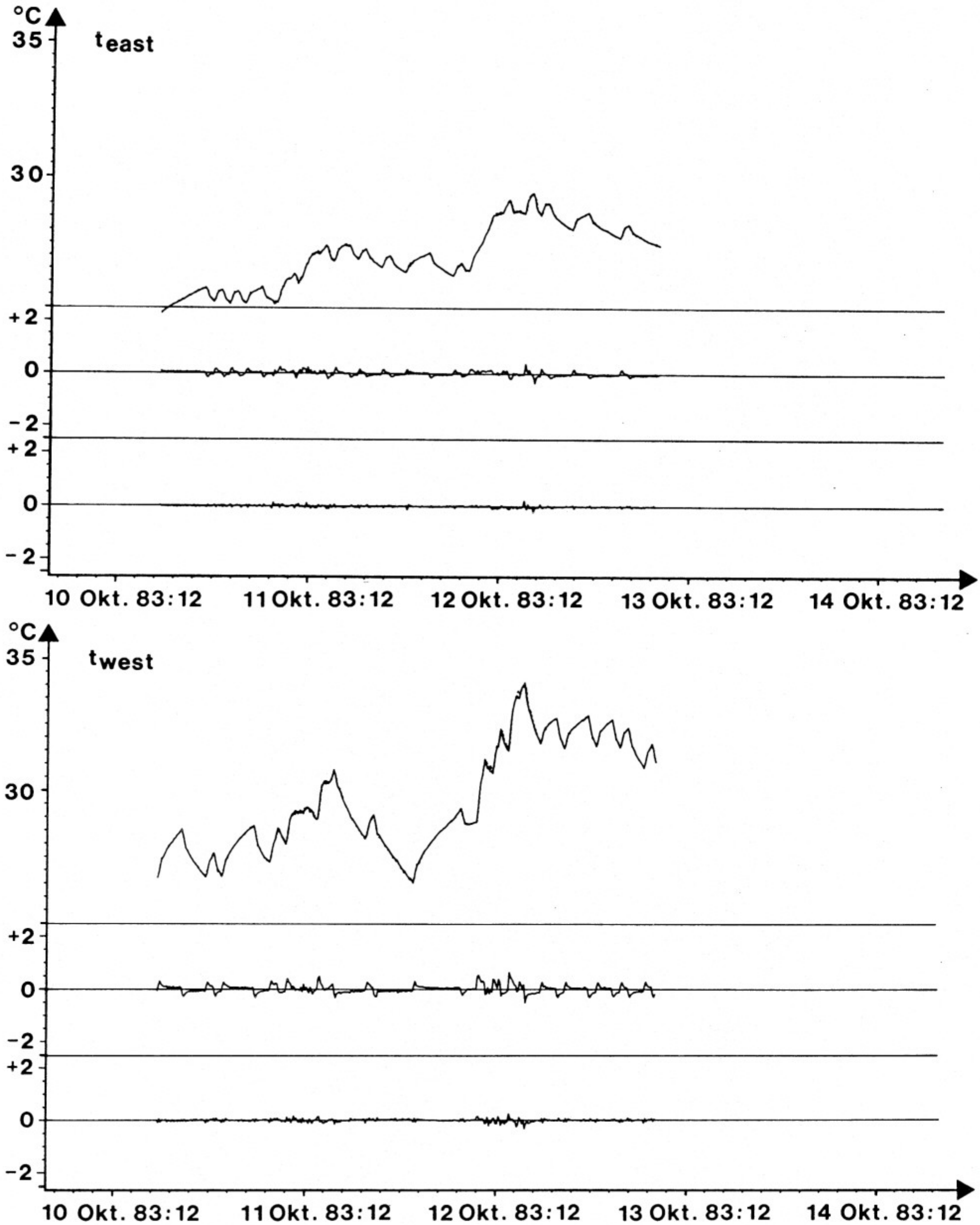

Figure 3. Measured and estimated indoor temperatures, the measured ones differenced once and the residuals from the 'full model'.

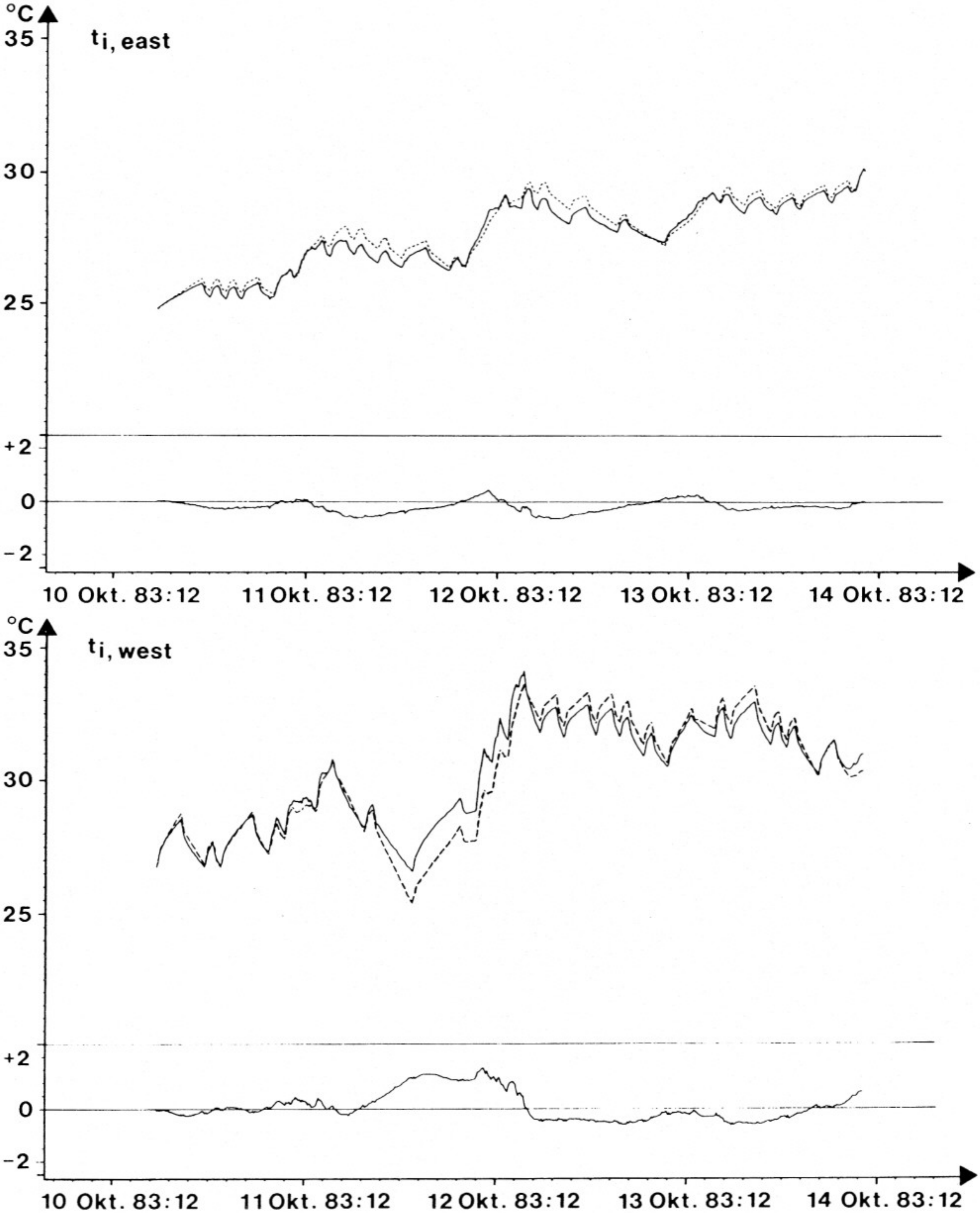

Figure 4. Measured and simulated indoor temperatures, and the differences between the two.

Due to the large variations in the indoor temperature, one might be forced to recommend no habitation during measurements. Besides, habitation will introduce variations which are hard to model.

All measurements carried out so far have taken place in a low-energy test house at the Technical University of Denmark.

Hopefully, future work, some of which is essential to the method as such, will show

- if the time required to perform the test can be reduced,
- if more components in a thermal-electric analog circuit can be estimated,
- if another scan interval is better for setting up a model like this,
- if it is possible to find a pseudo random signal for the power input to simulate the indoor temperature from an experiment with thermostat controlled indoor temperature satisfactorily,
- if it is possible to design an optimal signal for the power input to minimize the standard errors of the estimates,
- if other methods of estimation (for instance 'maximum likelihood') will increase the quality of the model,
- if the influence of the wind can be modelled satisfactorily for non-low-energy houses,
- if another solar input taking account of windows facing other directions than south as well as different sun angles can be found.

ACKNOWLEDGEMENTS

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Also, we should like to thank our colleages Mogens R. Byberg (project manager) and Rolf G. Djurtoft for their help at various stages of the job.

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