A DYNAMIC TEST METHOD FOR THE ENERGY CONSUMPTION OF SMALL HOUSES
Model Set-up, Test Design, Measurements, Parameter Estimation and Simulation

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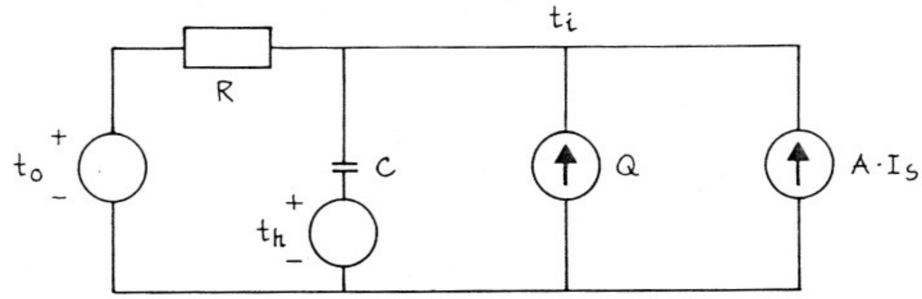
If you want to assess the heat loss from a house, you can determine its overall heat loss coefficient (W/°C): the heat loss (W) to the surroundings divided by the temperature difference (°C) between the inside and the outside. This is often done by turning off the normal heating system of the house and replacing it with electric resistance heaters. These heaters are often controlled by thermostats in order to achieve a constant indoor temperature. You then hope for a period with a fairly constant outdoor temperature and no (or little) solar radiation. Often, you must wait for days or even weeks to obtain this.

Rather than carrying out the traditional heat loss coefficient calculation, you can perform a regression analysis to obtain this coefficient and other equivalent thermal parameters. Wind speed and wind direction may be included when applying this method. A problem in this connection is that (choosing the indoor temperature as the dependent variable) the independent variables outdoor temperature, solar radiation and the power input from the heaters are strongly correlated. Also, you will achieve no information about the heat capacity of the house due to the constant indoor temperature unless the insolation causes overheating.

If on the other hand the electric resistance heaters are controlled by some randomised signal the correlations between the power input from the heaters and the other input variables will be very poor. Also, the randomised power input will cause more variation in the indoor temperature than in the case of thermostat control. This will produce information about the heat capacity of the house whether the insolation causes overheating or not.

Simple Models

Different simple models have been proposed over the years (1,2,3,4,5). In (6) this thermal-electric analogue circuit was used:



structions based on a masonite beam insulated with 300 mm of mineral wool. The ground floor contains the test space. An insulated partition wall with 95 mm of mineral wool divides this space into an east room and a west room of 60 m^2 each. The internal surfaces of ceiling and walls in both rooms are 12 mm gypsum boards. In the middle of both rooms structural and wind bracing cross frame constructions with 12 mm wood fibre boards are positioned. The floor in the east room is made of 75 mm of concrete. The floor in the west room is made of 22 mm of chip board on top of 53 mm of expanded polystyrene. The windows make up 15% of the floor area, 10% facing south and 5% facing north. All windows are triple glazed. The heat capacity of the east room has been increased by adding 96 m^2 of concrete flags 50 mm thick. The flags are positioned on four racks making up a 48 m^2 concrete partition wall 100 mm thick. Thermally speaking, the east room is 'heavy' and the west room is 'light'. Both rooms have ventilation systems with cross-flow plate type heat exchangers.

In this test house we have carried out a series of experiments in which the energy supply from the electric resistance heaters (3 x 500 W) in both rooms were controlled by PRBSs. The two sequences were displaced by half a period length to avoid correlation between the temperatures of the two rooms. Depending on n and T, the PRBS will excite different amounts of active heat capacity (different time constants) in the house. The choice of n and T is therefore important.

Figure 2 shows a set of data from an experiment carried out in the test house from 10 October 1983 till 14 October 1983. Here the PRBS parameters

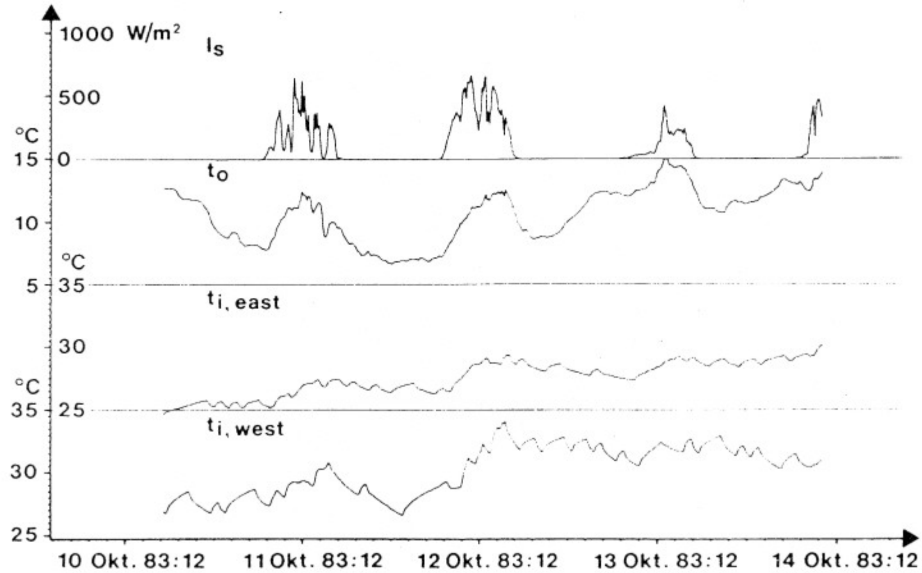


Figure 2. Data from both rooms of the low-energy test house and outdoor parameters. Scans every 10 minutes. PRBS not shown.

are: n=6 and T=1 hour. The door between the two rooms was closed and the ventilation systems were turned off and sealed. The influence of the wind is neglegted because of the extreme tightness of the house. The air change rate has been measured by means of the tracer gas decay method yielding a mere 0.005 a.c.h. $t_{i,east}$ and $t_{i,west}$ are the indoor temperatures measured in the north side of the rooms and t_o is the outdoor temperature measured in the shade on the north side of the house. I_s is the solar radiation on a south-facing vertical surface. Scans every 10 minutes.

to set up the following equation:

$$\frac{t_i - t_o}{R} + C \cdot (\dot{t}_i - \dot{t}_h) - (Q + A \cdot I_s) = 0 \qquad (1)$$

where

- t_i is the indoor temperature (^oC),
- t_o is the outdoor temperature (^oC),
- t_h is a temperature step generator (^oC),
- I_s is the solar radiation (kW/m^2) on a vertical surface facing south,
- R is the reciprocal of the overall heat loss coefficient ($^oC/kW$),
- C is the equivalent thermal mass ($kWh/^oC$),
- Q is the power input from the heaters (kW) and
- A is the area (m^2) of an equivalent 'solar window' corresponding to 100% transparency.

The wind parameters are omitted for simplicity. The dots (•) over t_i and t_h denote the time derivatives of these quantities. The voltage t_h represents a step generator which is introduced to take the fast indoor temperature variations right after each heat supply on/off into account. The step generator has a non-zero value when the heaters are on only. The above Equation 1 is Kirchhoff's current law for the node with the voltage t_i. This law states that the sum of the branch currents is 0 at every instant of time.

Experiments and Measurements

A pseudo random binary sequence (PRBS) is a deterministic series of digits, 'a's and '-a's. Transition between levels is allowed at times T, 2•T, 3•T, ... only but does not necessarily take place. Constructed in a sensible manner, this sequence forms a signal with a period of $(2^n-1) \cdot T$. n is the order of the signal and T is the shortest interval of time during which the signal is constant. The longest interval of time during which the signal is constant is n•T. This signal has an important statistical characteristic similar to that of 'white noise': it has no autocorrelation (except for lags equal to the period length), (7). An example of a PRBS with n=4 and T=5 hours is shown in Figure 1 along with the thermal response of a room (here the levels are 0 and 1.5 kW; outdoor parameters are not shown).

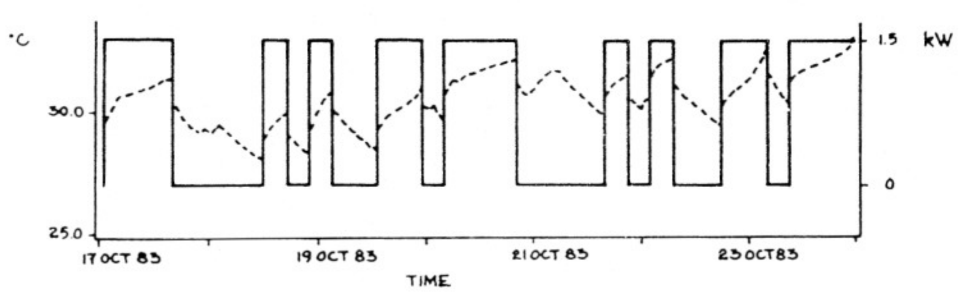


Figure 1. Example of a PRBS with n=4 and T=5 hours.

The low-energy test house at the Technical University of Denmark is a one-storey, wood-built house with a crawl space and a roof space. The floor, the walls and the ceiling of the ground floor are light sandwich con-

Parameter Estimation

The step-generator mentioned in Equation 1 has a value, H, when the heat supply is on. When the heat supply is off its value is 0. Re-writing Equation 1 for estimation and estimating H on the basis of the first three scans after each heat supply on/off rather than the first scan only (this simply gave the best fit), we arrive at:

$$t_{i,t+1} = t_{i,t} - \frac{t_{i,t} - t_{o,t}}{R \cdot C} \cdot dt + \frac{Q_t + A \cdot I_{s,t+1}}{C} \cdot dt + H \cdot \frac{1}{|Q|} \cdot (Q_t - Q_{t-3}) \qquad (2)$$

where R, C, A and H are the model parameters. To determine which lags of the independent variables to use in the model, one can use a technique described in (8) called prewhitening.

A period of $(2^n-1) \cdot T$ = 63 hours from 5 PM on 10 October 1983 till 8 AM on 13 October 1983 containing 378 observations is chosen for further analysis. By means of a non-linear ordinary least squares estimation method, the results quoted in Table 1 are obtained. The t-test statistic is the estimate divided by the standard error. r^2 expresses the relative variance reduction and sse (sum of squared errors) is the sum of squared differences between measured and estimated indoor temperatures.

Table 1. Estimated parameters for the model described by Equation 2.

East Room 'heavy'	R $^oC/kW$	C $kWh/^oC$	A m^2	H ^oC
Estimate	20.3	4.00	2.87	0.0955
Std.err.	1.3	0.30	0.40	0.0046
t-value	15.24	13.51	7.14	20.58

r^2 = 0.9989
sse = 0.5685 $^oC^2$
Residual variance = 0.001512 $^oC^2$

West Room 'light'	R $^oC/kW$	C $kWh/^oC$	A m^2	H ^oC
Estimate	21.80	1.487	2.69	0.1513
Std.err.	0.78	0.068	0.20	0.0075
t-value	27.82	22.01	13.20	20.25

r^2 = 0.9991
sse = 1.449 $^oC^2$
Residual variance = 0.003854 $^oC^2$

The t-test statistic expresses whether or not the estimate is significantly different from 0. Normally, the estimate is considered significant if t>2. The large values of r^2 are basically due to the fact that the indoor temperatures are strongly autocorrelated ($t_{i,t+1}$ depends strongly on $t_{i,t}$). The variance reduction when differencing once ($t_{i,t+1}-t_{i,t}$) is 0.9950 for $t_{i,east}$ and 0.9940 for $t_{i,west}$. For $t_{i,east}$ the variance reduction from one differencing to 'full model' is 0.7892 and for $t_{i,west}$ it is 0.8439. If scan intervals increase to 30 minutes the variance reduction when differencing once will decrease (for $t_{i,east}$ to 0.9606 and for $t_{i,west}$ to 0.9517) and the variance reduction from one differencing to 'full model' will increase (for $t_{i,east}$ to 0.9179 and for $t_{i,west}$ to 0.9265; the entire variance reductions are 0.9967 for $t_{i,east}$ and 0.9964 for $t_{i,west}$). The estimated values for R, C, A and H at 30 minute scans all lie within the standard errors shown in Table 1.

Calculations of R and C based on physical constants of the building materials employed (and for R also on actual temperature differences) give values from 24.7 to 27.7 ^{o}C/kW and 7.0 kWh/^{o}C for the east room, and from 23.0 to 26.9 ^{o}C/kW and 1.8 kWh/^{o}C for the west room. For both rooms, the south-facing, transparent window areas are 4.9 m^2. Taking the reflection at low incidence angles into account, one would expect around 60% of the incident solar radiation on the windows to be transmitted to the house. Thus, an A of 2.9 m^2 is expected for both rooms. Solar heat gain through the north facing windows is neglegted.

The estimated values of R are satisfactory. As the entire thermal mass will not participate in the temperature fluctuations, it is more difficult to comment on the estimated values of C. The estimates lie between 0 and the values based on physical constants for the building materials and the value for the east room is the larger. The estimated time constants (R·C) are 81.3 (±8.0) hours for the east room and 32.4 (±1.9) hours for the west room. The estimated values for A are excellent. Still the effect of the solar radiation is clearly seen in the residuals.

Simulation

In (6) Equation 2 was used to simulate the indoor temperatures of both rooms dynamically. Here, we shall model the power input. Inspecting Equation 2 it is clearly seen that Q_t will become a function of $t_{i,t+1}$ and $I_{s,t+1}$ among others. This may seem impossible at first sight. However, it merely expresses that the expected power input depends on the wanted future indoor temperature and the expected solar radiation (from some model). Thus, modelling the power input in this way implies construction of a control strategy for the indoor temperature and a model for the solar radiation. A procedure which is obviously easier to handle is simply to replace Q_t in the third term on the right hand side of Equation 2 by Q_{t+1} (this is <u>not</u> in accordance with the results from the prewhitening).

Re-writing Equation 2 in this manner to obtain a model for the power input, one arrives at an expression that causes a dynamic simulation with this model to diverge. If the last term in Equation 2 is simply omitted one achieves:

$$Q_{t+1} = C\cdot\frac{t_{i,t+1}-t_{i,t}}{dt} + \frac{t_{i,t}-t_{o,t}}{R} - A\cdot I_{s,t+1} \tag{3}$$

Equation 3 was used to simulate the cumulative energy consumptions ($\Sigma Q_{t+1} \cdot dt$) in both rooms from 5 PM on 10 October 1983 till 10 AM on 14 October 1983. This was done using three different sets of parameters (all omitting H in the simulation process): a) parameters estimated from Equation 2 omitting H in the estimation process also (results not quoted), b) the parameters quoted in Table 1, and c) parameters estimated from Equation 2 with Q_t in the third term on the right hand side replaced by Q_{t+1} (results not quoted).

Table 2 shows mean values and standard deviations of the residuals from the simulations in both east and west rooms using all three sets of parameters. In all three cases a good agreement between measured and simulated cumulative energy consumptions is achieved when using the simple model. The model described by Equation 3 and the parameters from Table 1 (omitting H) is seen to be the best in the sense that the differences between measured and simulated cumulative energy consumptions have small mean values as well as small standard deviations. This is highly satisfactory as the parameter estimates from this model (by and large) are the ones that are closest to the values expected from physical considerations. Figure 3 shows the measured and the simulated cumulative energy consumptions and the differences between the two using the model described by Equation 3 with the parameters from Table 1 (omitting H in the simulation process).

Table 2. Mean values and standard deviations of the residuals from the simulations.

Model	East Room		West Room	
	Mean kWh	Std.dev. kWh	Mean kWh	Std.dev. kWh
a) (results not quoted)	2.78	1.12	-0.59	0.97
b) Table 1, omitting H	1.22	1.13	-0.61	0.91
c) (results not quoted)	0.86	1.52	-0.89	1.15

Conclusions

The measurement-based method applied has been able to simulate the cumulative energy consumptions satisfactorily for a four day period. A similar procedure could probably be used also in smaller sections of larger buildings with little or no modification.

The amount of heat to be supplied by the electric resistance heaters replacing the normal heating system of the house, requires some advance information about the heat loss from the house. This information is often represented by the year of construction. If the heat supply is not suitable, the indoor temperatures will not be reasonably stationary; stationarity in this context denotes the characteristic, that the mean level of a variable is constant in time. The two time series that have been analysed

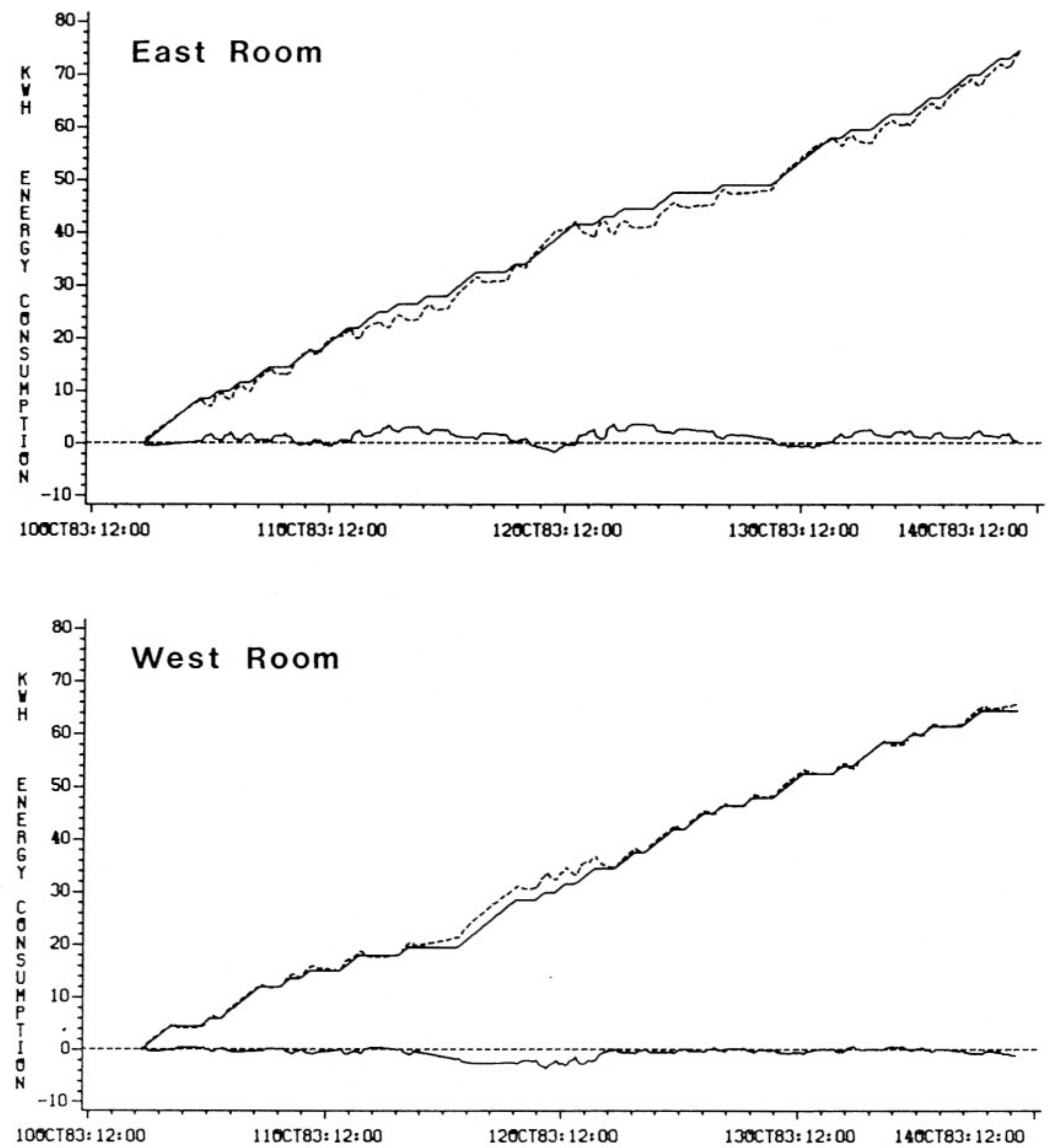


Figure 3. Measured and simulated cumulative energy consumptions and the differences between the two using the model described by Equation 3 with the parameters from Table 1 (omitting H in the simulation process).

here are not stationary (see Figure 2), and still the estimations went well. However, the change of the temperature level in the west room causes the residuals of the simulations to increase drastically. This is the case also for the dynamic temperature simulations reported in (6).

Due to the large variations in the indoor temperature, one might be forced to recommend no habitation during measurements. Besides, habitation will introduce variations which are hard to model. All measurements carried out so far have taken place in a low-energy test house at the Technical University of Denmark.

Hopefully, future work some of which is essential to the method as such will show

- if more components and therefore more time constants in a thermal-electric analogue circuit can be estimated,

- if other methods of estimation (for instance 'maximum likelihood') will increase the quality of the model,

- if another solar input taking account of windows facing other directions than south as well as different sun angles can be found, and

- if a different type of model based on distributed rather than lumped elements can be set up.

Acknowledgements

I should like to thank Bjarne Kjær Nielsen of the Institute of Mathematical Statistics and Operations Research at the Technical University of Denmark for his assistance.

The work reported here is funded by the Danish Ministry of Energy.

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SUMMARY

Allan Aasbjerg Nielsen: A Dynamic Test Method for the Energy Consumption of Small Houses. The paper describes a dynamic test method for setting up a model for the thermal performance/the energy consumption of a small house. The overall heat loss coefficient and other equivalent thermal parameters are determined. The model equation is based on the analogy between a simple passive electrical circuit and a house considered a simple one-dimensional, one-zone thermal system. A such method is believed to be capable of assessing the value of energy conserving measures. Estimates of the equivalent thermal parameters before and after carrying out the measures contain information about the improvements achieved. Using a pseudo-randomised power input one ensures that the correlations between the heat supply and the other input data are poor. A non-linear ordinary least squares method is used to estimate the components of the circuit modelling the house. The ability of the model to simulate the measured energy consumption is good considering its simplicity.

RESUME

Allan Aasbjerg Nielsen: Une Méthode de Test dynamique de la Consommation d'Énergie de petites Maisons. Ce rapport décrit une méthode de test dynamique dans le but d'établir un modèle pour la performance thermique/la consommation d'énergie d'une petite maison. Le coefficient de déperdition de chaleur global ainsi que d'autres paramètres thermiques équivalents sont déterminés. L'équation du modèle se base sur l'analogie entre un circuit électrique passif simple et une maison considerée comme un système thermique simple à une dimension et à une seule zone. On croit qu'une telle méthode est à même d'évaluer les mesures d'économie d'énergie. Les estimats des paramètres thermiques équivalents avant et après l'exécution des mesures contiennent des informations sur les améliorations obtenues. En employant une entrée d'effet électrique faite demi-arbitraire on s'assure que les corrélations entre l'admission de chaleur et les autres données d'entrée sont pauvres. Une méthode non-linéaire des moindres carrés ordinaire est employée pour estimer les éléments du circuit qui modèlent la maison. La capacité du modèle de simuler la consommation mesurée est bonne étant donné sa simplicité.

KURZFASSUNG

Allan Aasbjerg Nielsen: Eine dynamische Test Methode für den Energieverbrauch kleiner Häuser. Dieser Artikel beschreibt eine dynamische Test Methode für die Herstellung eines Modells für die thermische Leistung/den Energieverbrauch kleiner Haüser. Die Gesamtwärmedurchgangszahl und andere äquivalente thermische Parameter sind festgestellt worden. Die Modellgleichung gründet sich auf die Analogie zwischen einem einfachen, passiven, elektrischen Kreislauf und einem Haus als ein einfaches, eindimensionales, thermisches System mit einer Zone betrachtet. Eine solche Methode ist wahrscheinlich imstande den Wert energiewirtschaftlicher Veranstaltungen zu quantifizieren. Estimation der äquivalenten thermischen Parameter vor und nach der Ausführung der Veranstaltungen gibt Auskunft über die gewonnene Verbesserung. Bei der Anwendung eines pseudozufälliggemachten Effekt Eintritts sichert man sich, dass die Korrelationen zwischen der Wärmezuleitung und den anderen Eintritt Daten gering sind. Eine nichtlineare ordinäre Methode der kleinsten Quadrate ist angewendet, um die Komponente des Kreislaufs, der das Haus modelliert, zu estimieren. Die Fähigkeit des Modells, den gemessenen Energiverbrauch zu simulieren, ist gut, wenn man sich seine Einfachheit bedenkt.