Stochastic Adaptive Control (02421)

Lecture 7

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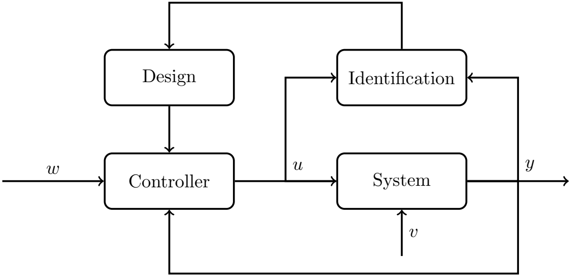
Section for Dynamical Systems, DTU Compute

Lecture Plan

1. Systems theory
2. Stochastics
3. State estimation - Kalman filter 1
4. State estimation - Kalman filter 2
5. Optimal control 1 - internal models
6. External models
7. **Prediction + optimal control 1 - external models**
8. Optimal control 2 - external models
9. System identification 1
10. System identification 2
11. System identification 3 + model validation
12. Adaptive control 1
13. Adaptive control 2



Today's Agenda

- Polynomials and transfer function truncation
- Diophantine equation
- External prediction
- External control: Minimum variance
- External control: Pole-zero

Follow-up from last time: Q3

We consider the system:

$$x_{k+1} = \begin{bmatrix} 1.0000 & -0.5000 \\ 0.4000 & -0.7000 \end{bmatrix} x_k + \begin{bmatrix} 1.0000 \\ 0.3000 \end{bmatrix} e_k, \quad e_k \sim N(0,3) \tag{1}$$

$$y_k = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k + 0.5512 e_k \tag{2}$$

The DC-gain is then

$$K_{dc} = C(I - A)^{-1} G + F = 8.3012 \tag{3}$$

While the AC-gain is found solving the dlyap for a given input variance:

$$\sigma_{out}^2 = C \text{dlyap}(A, G\sigma_{in}^2 G') C' + D\sigma_{in}^2 D' \tag{4}$$

$$K_{ac} = \frac{\sigma_{out}^2}{\sigma_{in}^2} = \frac{13.1771}{3} = 4.3924 \tag{5}$$

Follow-up from last time

Questions?

Polynomials and Transfer functions

Let us consider the polynomials on the form

$$B(q^{-1}) = b_0 + b_1 q^{-1} + \dots + b_n q^{-n} \tag{6}$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + \dots + b_n z^{-n} \tag{7}$$

$$\tag{8}$$

for the time and frequency domains.

The polynomial has the order n, if $b_n \neq 0$ and $b_i = 0, \quad i > n$, and if $b_0 = 1$ the polynomial is said to be monic.
A transfer function $H(q)$, can be written with polynomials in an infinitely number of ways:

$$H(q) = \frac{B(q^{-1})}{A(q^{-1})} = \frac{C(q^{-1})B(q^{-1})}{C(q^{-1})A(q^{-1})} \tag{9}$$

Polynomials and Transfer functions

Our transfer function written as polynomials can be rewritten as

$$\frac{B(q^{-1})}{A(q^{-1})} = \frac{b_0 + b_1 q^{-1} + ... + b_n q^{-n}}{1 + a_1 q^{-1} + ... + a_n q^{-n}} \tag{10}$$

$$= b_0 + q^{-1} \frac{(b_1 - b_0 a_1) + (b_2 - b_0 a_2) q^{-1} + ... + (b_n - b_0 a_n) q^{-(n-1)}}{1 + a_1 q^{-1} + ... + a_n q^{-n}} \tag{11}$$

Using this we can define the transfer function:

$$H(q^{-1}) = \frac{B(q^{-1})}{A(q^{-1})} = g_0 + q^{-1} \frac{S_1(q^{-1})}{A(q^{-1})} \tag{12}$$

$$S_1(q^{-1}) = s_0 + s_1 q^{-1} + \ldots + s_{n_1} q^{-n_1} \tag{13}$$

$$g_0 = b_0 \quad s_i = b_{i-1} - b_0 a_{i-1} \tag{14}$$

where $n_1 = n - 1$ is the order of S_1

Polynomials and Transfer functions

If we repeat the rewriting for $\frac{S_1}{A}$, $\frac{S_2}{A}$, and so on:

$$H(q^{-1}) = g_0 + g_1 q^{-1} + \ldots + g_{m-1} q^{-(m-1)} + q^{-m} \frac{S_m(q^{-1})}{A(q^{-1})} \tag{15}$$

$$= G_m(q^{-1}) + q^{-m} \frac{S_m(q^{-1})}{A(q^{-1})} \tag{16}$$

This is known as the mth step truncation of the transfer function, where it can be shown that g_i is the ith coefficient of the impulse response $h(t) = \mathcal{F}^{-1}(H(z))$, and the coefficients of G_m being the truncated impulse response

From the definition of $H(q^{-1})$, we get the following relation:

$$B(q^{-1}) = A(q^{-1}) G_m(q^{-1}) + q^{-m} S_m(q^{-1}) \tag{17}$$

which is known as the simple Diophantine Equation.
The order of S_m is given by $\max(n_a - 1, n_b - m)$, and $m - 1$ for G_m.

How to solve the Diophantine equation

The simple Diophantine equation can be solved by iterations of:

$G = [\quad]; S = [B, 0];$ % Pad B with zeros to make S as long as A

for $i = 1 : m$

$G = [G, S(1)];$

$S = [S(2 : end) - S(1) * A(2 : end), 0];$

end

$S = S(1 : end - 1);$

here given in Matlab notation

Example: Solving the Diophantine equation

The simple Diophantine equation, can be solved by iterations of:

$$A = [1, 2, 3] \qquad B = [2, 3] \tag{18}$$
$$G = [] \qquad S = [B, 0] \tag{19}$$

$k = 1$

$$G = 2 \quad S_{loop} = [[3, 0] - 2 * ([2, 3]), 0] = [-1, -6, 0] \tag{20}$$
$$\tag{21}$$

$k = 2$

$$G = [2, -1] \quad S_{loop} = [[-6, 0] - (-1) * ([2, 3]), 0] = [-4, 3, 0] \tag{22}$$
$$S = S_{loop}(1 : end - 1) = [-4, 3] \tag{23}$$

What would S be if $B = [3, 4]$?

Think about it for yourself for one minute and
then discuss with the person next to you for two minutes.

Diophantine Equation

The Diophantine Equations are named after Diophantus of Alexandria (200 AD - 298 AD, became 84 year)

The general Diophantine is given by

$$C(q^{-1}) = A(q^{-1})R(q^{-1}) + B(q^{-1})S(q^{-1}) \tag{24}$$

and defined by the polynomials:

$$C(q^{-1}) = c_0 + c_1 q^{-1} + \cdots + c_{n_c} q^{-n_c} \tag{25}$$

$$B(q^{-1}) = \qquad b_1 q^{-1} + \cdots + b_{n_b} q^{-n_b}, \quad b_0 = 0 \tag{26}$$

$$A(q^{-1}) = 1 + \ a_1 q^{-1} + \cdots + a_{n_a} q^{-n_a} \tag{27}$$

The solutions R, S exist if and only if any common factors of A and B is shared with C.

The solutions of Diophantine is in general not unique:

$$R(q^{-1}) = R_0(q^{-1}) + B(q^{-1})F(q^{-1}) \tag{28}$$

$$S(q^{-1}) = S_0(q^{-1}) - A(q^{-1})F(q^{-1}) \tag{29}$$

A unique solution, exist if $n_r = n_b - 1$ and $n_s = max(n_a - 1, n_c - n_b)$.

Diophantine - Sylvester method

The solution to the general Diophantine can be computed by:

$$\left[\begin{array}{cccc|cccc} 1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ a_1 & 1 & \ddots & \vdots & b_1 & 0 & \ddots & \vdots \\ a_2 & a_1 & & 0 & b_2 & b_1 & & 0 \\ \vdots & \vdots & & 1 & \vdots & \vdots & & 0 \\ a_{n_a} & a_{n_a-1} & \dots & a_1 & b_{n_b} & b_{n_b-1} & \dots & b_1 \\ 0 & a_{n_a} & & \vdots & 0 & b_{n_b} & & \vdots \\ \vdots & & \ddots & a_{n_a-1} & \vdots & & \ddots & b_{n_b-1} \\ 0 & 0 & & a_{n_a} & 0 & 0 & & b_{n_b} \end{array}\right] \begin{bmatrix} r_0 \\ r_1 \\ \vdots \\ r_{n_r} \\ s_0 \\ s_1 \\ \vdots \\ s_{n_s} \end{bmatrix} = \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_{n_c} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (30)$$

Prediction in the ARMA Structure

Consider that we have a weakly stationary process y_t:

$$A(q^{-1})y_t = C(q^{-1})e_t \tag{31}$$

where e_t is a white noise signal $\mathbb{F}(0, \sigma^2)$, and that A, C are monic.

Then if we want to predict the mth step ahead, we can use truncation and the Diophatine equation, where we get that

$$y_{t+m} = \frac{C(q^{-1})}{A(q^{-1})}e_{t+m} = G_m(q^{-1})e_{t+m} + \frac{S_m(q^{-1})}{A(q^{-1})}e_t \tag{32}$$

From here the we can define the prediction and prediction error:

$$\hat{y}_{t+m|t} = \frac{S_m(q^{-1})}{A(q^{-1})}e_t = \frac{S_m(q^{-1})}{A(q^{-1})}\left(\frac{A(q^{-1})}{C(q^{-1})}y_t\right) = \frac{S_m(q^{-1})}{C(q^{-1})}y_t \tag{33}$$

$$\tilde{y}_{t+m|t} = G_m(q^{-1})e_{t+m} \tag{34}$$

where $\hat{y}_t$ and $\tilde{y}_t$ are independent. This method requires an inversely stable $C(q^{-1})$.

Prediction in the ARMAX structure

Let us consider the system:

$$A(q^{-1})y_t = q^{-k}B(q^{-1})u_t + C(q^{-1})e_t \tag{35}$$

where k is the delay of the control.

Then our output prediction for time m is given by

$$\hat{y}_{t+m|t} = \frac{1}{C(q^{-1})}(B(q^{-1})G_m(q^{-1})u_{t+m-k} + S_m(q^{-1})y_t) \tag{36}$$

$$\tilde{y}_{t+m|t} = G_m(q^{-1})e_{t+m} \tag{37}$$

This can be derived using the Diophantine:

$$C(q^{-1}) = A(q^{-1})G_m(q^{-1}) + q^{-m}S_m(q^{-1}) \tag{38}$$

where the order of G and S is $m-1$ and $\max(n_a-1, n_c-m)$, and $G(0)=1$

Proving the ARMAX prediction

The future output can be rewritten in terms of the Diophantine as

$$y_{t+m} = \frac{C(q^{-1})}{C(q^{-1})} y_{t+m} \tag{39}$$

$$= \frac{A(q^{-1})G_m(q^{-1}) + q^{-m}S_m(q^{-1})}{C(q^{-1})} y_{t+m} \tag{40}$$

$$= \frac{G_m(q^{-1})}{C(q^{-1})} A(q^{-1}) y_{t+m} + \frac{S_m(q^{-1})}{C(q^{-1})} y_t \tag{41}$$

By substituting the system description, we get

$$y_{t+m} = \frac{G_m(q^{-1})}{C(q^{-1})} (B(q^{-1})u_{t+m-k} + C(q^{-1})e_{t+m}) + \frac{S_m(q^{-1})}{C(q^{-1})} y_t \tag{42}$$

$$= \frac{G_m(q^{-1})B(q^{-1})}{C(q^{-1})} u_{t+m-k} + \frac{S_m(q^{-1})}{C(q^{-1})} y_t + G_m(q^{-1})e_{t+m} \tag{43}$$

$$= \hat{y}_{t+m|t} + \tilde{y}_{t+m|t} \tag{44}$$

Our prediction now depends on the control and noise, as well as the noise at the kth time step.

Prediction in the L structure

Let us consider the system:

$$A(q^{-1})y_t = q^{-k}\frac{B(q^{-1})}{F(q^{-1})}u_t + \frac{C(q^{-1})}{D(q^{-1})}e_t \quad (45)$$

where k is the delay of the control.

Then our output prediction for time m is given by

$$\hat{y}_{t+m|t} = \frac{1}{C(q^{-1})}\left(\frac{D(q^{-1})}{F(q^{-1})}B(q^{-1})G_m(q^{-1})u_{t+m-k} + S_m(q^{-1})y_t\right) \quad (46)$$

$$\tilde{y}_{t+m|t} = G_m(q^{-1})e_{t+m} \quad (47)$$

where the order of G and S is $m-1$ and $\max(n_a+n_d-1, n_c-m)$, respectively, and $G(0)=1$.

External Control

When designing controllers for a system on external form, such as

$$A(q^{-1})y_t = q^{-k}B(q^{-1})u_t + C(q^{-1})e_t \tag{48}$$

we are looking for a control law on the form:

$$R(q^{-1})u_t = Q(q^{-1})w_t - S(q^{-1})y_t + \bar{d} \tag{49}$$

The PID controller in the classical theory, would be equivalent to

$$u_t = -\frac{S(q^{-1})}{R(q^{-1})}y_t \tag{50}$$

While in optimal controllers, we minimize a cost J_t

$$\min_{u_t} J_t \tag{51}$$

Minimum Variance Control

Let us consider the system:

$$A(q^{-1})y_t = q^{-k}B(q^{-1})u_t + C(q^{-1})e_t \tag{52}$$

where B and C are assumed stable.
In order to achieve higher certainty of the output, we want to minimize the variance:

$$J_t = E\{y_{t+k}^2\} \tag{53}$$

For the simplicity of notation G and S will be solution to the Diophantine:

$$C(q^{-1}) = A(q^{-1})G(q^{-1}) + q^{-k}S(q^{-1}) \tag{54}$$

Minimum Variance Control

Remember that the mth prediction and error is given by:

$$\hat{y}_{t+m|t} = \frac{1}{C(q^{-1})}(B(q^{-1})G_m(q^{-1})u_{t+m-k} + S_m(q^{-1})y_t) \quad (55)$$

$$\tilde{y}_{t+m|t} = G_m(q^{-1})e_{t+m} \quad (56)$$

Our cost can then be given by

$$J_t = E\{y_{t+k}^2\} = E\left\{\left(\frac{1}{C(q^{-1})}(B(q^{-1})G_k(q^{-1})u_t + S_k(q^{-1})y_t)\right)^2\right\} \quad (57)$$

$$+ E\left\{\left(G_k(q^{-1})e_{t+k}\right)^2\right\} \quad (58)$$

The minimization is with respect to the control. Therefore, minimum variance is achieved if the first term is zero:

$$B(q^{-1})G_k(q^{-1})u_t = -S_k(q^{-1})y_t \quad (59)$$

Minimum Variance Control

Given a controller based on (59), the closed-loop and stationary version of the system is given by the prediction error:

$$y_t = G_k(q^{-1})e_t, \quad u_t = -\frac{S_k(q^{-1})}{B(q^{-1})G_k(q^{-1})}y_t = -\frac{S_k(q^{-1})}{B(q^{-1})}e_t \tag{60}$$

where the closed loop poles are given by BC.
In the stationary case, the variance of output and control is

$$Var(y_t) = \sigma^2 \sum_{i=0}^{k-1} g_i^2 \tag{61}$$

$$Var(u_t) = \int_{-\pi}^{\pi} \frac{S_k(e^{-jw})}{B(e^{-jw})} \frac{S_k(e^{jw})}{B(e^{jw})} dw\sigma^2 \tag{62}$$

Minimum Variance Control

Consider the L-structure, the control would then be given by

$$u_t = -\frac{F(q^{-1})}{D(q^{-1})} \frac{S_k(q^{-1})}{B(q^{-1})G_k(q^{-1})} y_t \tag{63}$$

The minimum variance controller has issues with the following:

1. set-points
2. constant disturbances
3. large control effort
4. non damped zeros (zeros outside to the unit circle)

MV_0 control

Let us consider, the case were we have a desired set-point

$$J_t = E\{(y_{t+k} - w_t)^2\} \tag{64}$$

and let us consider the ARMAX system with a constant disturbance:

$$A(q^{-1})y_t = q^{-k}B(q^{-1})u_t + C(q^{-1})e_t + d \tag{65}$$

The minimizing control is then given by

$$u_t = \frac{C(q^{-1})}{B(q^{-1})G_k(q^{-1})}w_t - \frac{S_k(q^{-1})}{B(q^{-1})G_k(q^{-1})}y_t - \frac{1}{B(q^{-1})}d \tag{66}$$

MV_0 control

The stationary closed-loop system is then given by

$$y_t = q^{-k} w_t + G_k(q^{-1}) e_t + \frac{G_k(q^{-1})}{C(q^{-1})}(1 - q^{-k}) d = q^{-k} w_t + G_k(q^{-1}) e_t \tag{67}$$

$$u_t = \frac{A(q^{-1})}{B(q^{-1})} w_t - \frac{S_k(q^{-1})}{B(q^{-1})} e_t - \frac{1}{B(q^{-1})} d \tag{68}$$

with the poles being given by BC.

MV_0 control

Considering the L-structure:

$$A(q^{-1})y_t = q^{-k}\frac{B(q^{-1})}{F(q^{-1})}u_t + \frac{C(q^{-1})}{D(q^{-1})}e_t + d \tag{69}$$

then the MV0 control becomes

$$u_t = \frac{F(q^{-1})}{B(q^{-1})}\frac{C(q^{-1})}{D(q^{-1})G_k(q^{-1})}w_t - \frac{S_k(q^{-1})}{B(q^{-1})}\frac{F(q^{-1})}{D(q^{-1})G_k(q^{-1})}y_t - \frac{F(q^{-1})}{B(q^{-1})}d \tag{70}$$

The closed-loop becomes

$$y_t = q^{-k}w_t + G_k(q^{-1})e_t \tag{71}$$

$$u_t = \frac{F(q^{-1})A(q^{-1})}{B(q^{-1})}w_t - \frac{S_k(q^{-1})}{B(q^{-1})}\frac{F(q^{-1})}{D(q^{-1})}e_t - \frac{F(q^{-1})}{B(q^{-1})}d \tag{72}$$

MV_0 control

If the disturbance d and our setpoint w_t is zero, then MV_0 control becomes the minimum variance control

The MV_0 controller still has some issues with the following:

1. large control effort
2. non damped zeros

Example of MV0 controller

Consider the ARMAX model

$$y_t - 1.7y_{t-1} + 0.7y_{t-2} = u_{t-1} + 0.5u_{t-2} + \varepsilon_t + 1.5\varepsilon_{t-1} + 0.9\varepsilon_{t-2}. \quad (73)$$

We want to design a feedback strategy such that

$$\mathtt{E}\Big[\,(y_{t+1} - 1)^2\,\Big], \quad (74)$$

is minimal. We have the following polynomials

$$\begin{aligned} A(q^{-1}) &= 1 - 1.7q^{-1} + 0.7q^{-2}, \quad B(q^{-1}) = 1 + 0.5q^{-1} \\ C(q^{-1}) &= 1 + 1.5q^{-1} + 0.9q^{-2}, \quad d = 0 \end{aligned} \quad (75)$$

We also have that the input lag is $k = 1$.

Example of MV0 controller

From the Diophantine equation we find that

$$1 + 1.5q^{-1} + 0.9q^{-2} = \left(1 - 1.7q^{-1} + 0.7q^{-2}\right) G(q^{-1}) + q^{-1}S(q^{-1}). \quad (76)$$

We also have the conditions that $G(0) = 1$, ord$[G] = k - 1$ and ord$[S] = \max(n_a - 1, n_c - k)$. Using the polynomials defined previously we find

$$\text{ord}[G] = 0, \qquad \text{ord}[S] = 1. \quad (77)$$

Consequently,

$$1 + 1.5q^{-1} + 0.9q^{-2} = 1 - 1.7q^{-1} + 0.7q^{-2} + s_1q^{-1} + s_2q^{-2} \quad (78)$$

$$1.5 = -1.7 + s_1, \qquad 0.9 = 0.7 + s_2 \quad (79)$$

Thus, by matching coefficients, we obtain the solution

$$G(q^{-1}) = 1, \quad (80)$$

$$S(q^{-1}) = s_1 + s_2q^{-1} = 3.2 + 0.2q^{-1} \quad (81)$$

Example of MV0 controller

From the Diophantine equation we find that

$$1 + 1.5q^{-1} + 0.9q^{-2} = \left(1 - 1.7q^{-1} + 0.7q^{-2}\right) G(q^{-1}) + q^{-1} S(q^{-1}). \quad (82)$$

Consequently,

$$1 + 1.5q^{-1} + 0.9q^{-2} = 1 - 1.7q^{-1} + 0.7q^{-2} + s_1 q^{-1} + s_2 q^{-2} \quad (83)$$

$$1.5 = -1.7 + s_1, \qquad 0.9 = 0.7 + s_2 \quad (84)$$

Thus, by matching coefficients, we obtain the solution

$$G(q^{-1}) = 1, \quad (85)$$

$$S(q^{-1}) = s_1 + s_2 q^{-1} = 3.2 + 0.2q^{-1} \quad (86)$$

What would S be if $A(q^{-1}) = 1 - 1.5q^{-1} + 0.8q^{-2}$ instead of $A(q^{-1}) = 1 - 1.7q^{-1} + 0.7q^{-2}$?

Think about it for yourself for one minute and then discuss with the person next to you for one minute.

Example of MV0 controller

MV0 control law:

$$u_t = \frac{A(q^{-1})}{B(q^{-1})} w_t - \frac{S_k(q^{-1})}{B(q^{-1})} e_t - \frac{1}{B(q^{-1})} d \tag{87}$$

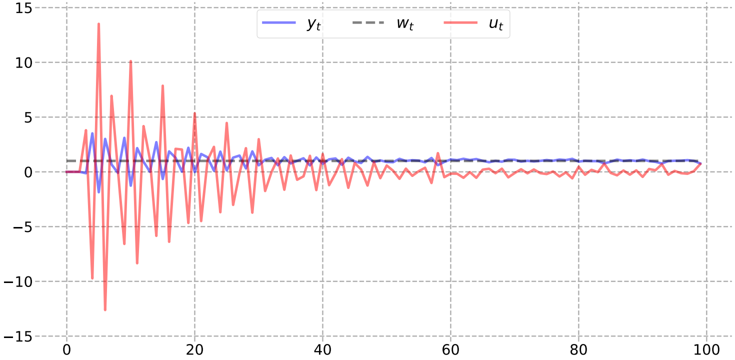
The optimal controller is therefore given by

$$(1 + 0.5q^{-1})u_t = (1 - 1.7q^{-1} + 0.7q^{-2})w_t - (3.2 + 0.2q^{-1})y_t, \tag{88}$$

For $w_t = 1$, we can rearrange in order to find the control law

$$u_t = -0.5u_{t-1} - 3.2y_t - 0.2y_{t-1}. \tag{89}$$

Example of MV0 controller



Notice the control relative to the output

MV_1 control

Let us consider, the case were we don't want too much control action:

$$J_t = E\{(y_{t+k} - w_t)^2 + \rho u_t^2\} \tag{90}$$

and let us consider the ARMAX system:

$$A(q^{-1})y_t = q^{-k}B(q^{-1})u_t + C(q^{-1})e_t + d \tag{91}$$

The minimizing control is then given by the equation:

$$\left(B(q^{-1})G_k(q^{-1}) + \frac{\rho}{b_0}C(q^{-1})\right)u_t = C(q^{-1})w_t - S_k(q^{-1})y_t - G_k(q^{-1})d \tag{92}$$

MV_1 control

Again let us consider, the stationary closed-loop case:

$$y_t = q^{-k}\frac{B(q^{-1})}{B(q^{-1}) + \frac{\rho}{b_0}A(q^{-1})}w_t + \frac{B(q^{-1})G_k(q^{-1}) + \frac{\rho}{b_0}C(q^{-1})}{B(q^{-1}) + \frac{\rho}{b_0}A(q^{-1})}e_t + \frac{\frac{\rho}{b_0}}{B(q^{-1}) + \frac{\rho}{b_0}A(q^{-1})}d \tag{93}$$

$$u_t = \frac{A(q^{-1})}{B(q^{-1}) + \frac{\rho}{b_0}A(q^{-1})}w_t - \frac{S_k(q^{-1})}{B(q^{-1}) + \frac{\rho}{b_0}A(q^{-1})}e_t - \frac{1}{B(q^{-1}) + \frac{\rho}{b_0}A(q^{-1})}d \tag{94}$$

If our system has $A(1) \neq 0$ (no pure integrator), then for a non-zero setpoint, MV_1 contains a stationary error.

MV_{1a} control

A work around for the stationary error is to consider the control change:

$$J_t = E\{(y_{t+k} - w_t)^2 + \rho(u_t - u_{t-1})^2\} \tag{95}$$

The resulting control is then given by

$$\left(B(q^{-1})G_k(q^{-1}) + \frac{\rho}{b_0}C(q^{-1})\Delta\right)u_t = C(q^{-1})w_t - S_k(q^{-1})y_t - G_k(q^{-1})d \tag{96}$$

$$\Delta = 1 - q^{-1} \tag{97}$$

with the stationary case being given by

$$y_t = q^{-k}\frac{B(q^{-1})}{B(q^{-1}) + \frac{\rho}{b_0}\Delta A(q^{-1})}w_t + \frac{B(q^{-1})G_k(q^{-1}) + \frac{\rho}{b_0}\Delta C(q^{-1})}{B(q^{-1}) + \frac{\rho}{b_0}\Delta A(q^{-1})}e_t \tag{98}$$

$$u_t = \frac{A(q^{-1})}{B(q^{-1}) + \frac{\rho}{b_0}\Delta A(q^{-1})}w_t - \frac{S_k(q^{-1})}{B(q^{-1}) + \frac{\rho}{b_0}\Delta A(q^{-1})}e_t \tag{99}$$

$$- \frac{1}{B(q^{-1}) + \frac{\rho}{b_0}\Delta A(q^{-1})}d \tag{100}$$

PZ-control

Another approach to limiting the control effort is to reduce the requirements of following the set-point. We do this by introducing a filter:

$$\tilde{w}_t = q^{-k}\frac{B_m(q^{-1})}{A_m(q^{-1})}w_t \tag{101}$$

$$y_{t+k} - \tilde{w}_{t+k} \qquad \text{or} \qquad A_m(q^{-1})y_{t+k} - B_m(q^{-1})w_t \tag{102}$$

Our cost then becomes

$$J_t = E\{(A_m(q^{-1})y_{t+k} - B_m(q^{-1})w_t)^2\} \tag{103}$$

If we again consider the system

$$A(q^{-1})y_t = q^{-k}B(q^{-1})u_t + C(q^{-1})e_t + d \tag{104}$$

the control is then defined as

$$u_t = \frac{C(q^{-1})B_m(q^{-1})}{B(q^{-1})G_k(q^{-1})}w_t - \frac{S_k(q^{-1})}{B(q^{-1})G_k(q^{-1})}y_t - \frac{1}{B(q^{-1})}d \tag{105}$$

with the Diophantine given as

$$A_m(q^{-1})C(q^{-1}) = A(q^{-1})G_k(q^{-1}) + q^{-k}S_k(q^{-1}) \tag{106}$$

PZ-control

The stationary closed-loop then becomes:

$$y_t = q^{-k}\frac{B_m(q^{-1})}{A_m(q^{-1})}w_t + \frac{G_k(q^{-1})}{A_m(q^{-1})}e_t \tag{107}$$

$$u_t = \frac{A(q^{-1})B_m(q^{-1})}{B(q^{-1})A_m(q^{-1})}w_t - \frac{S_k(q^{-1})}{B(q^{-1})A_m(q^{-1})}e_t - \frac{1}{B(q^{-1})}d \tag{108}$$

Then, as with the MV-controllers, the PZ-control has an issue with

1. non-damped zeros

Questions

Questions?