An epistemic logic of "knowing what"

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TBA workshop, Lorentz center, Aug. 18 2015

Background: beyond “knowing that”

A logic of “knowing what”

Conclusions

Why knowledge matters (in plans and protocols)

We use knowledge, belief and probability to organize certainty and uncertainty (due to initial assumptions, non-deterministic actions, parallel compositions, malicious behaviours, partial observations...)

$$\texttt{Uncertain or false} \quad \overset{\textit{plan/protocol}}{\longrightarrow} \quad \texttt{Certain and ture}$$

- Goals
- Branching conditions
- Initial assumptions
- ...

Standard Epistemic Logic

Propositional modal logics about reasoning about propositional knowledge (and belief) [von Wright 1951, Hintikka 1962]

- Syntax: $\mathcal{K}_i\varphi$ expresses "agent *i knows that* φ"
- Semantics: knowledge as elimination of uncertainty
- Proof system: (normal) modal logics ([$\mathbb{S}4$, $\mathbb{S}5$])
- $\forall$ (semantic) vs. $\exists$ (syntactic)
- Powerful when combined with other modalities: Epistemic Temporal Logics, Dynamic Epistemic Logic, ATL+E, STIT+E, Epistemic Situation Calculus etc.

Beyond "knowing that": motivation

Knowledge is not only expressed in terms of "knowing that" (even restricted to the context of protocols and plans):

- Jérôme *knows whether* the component *i* is OK.
- Yoram *knows what* the maximal number is.
- Valentin *knows how* to 'announce' the card distribution safely.
- Ron's robot *knows who* ordered the water.
- Sheila *knows why* the radio is not working.
- Yanjing *knows where* to look for examples.
- ...

Linguistically: "know" takes embedded questions but "believe" does not: factive verbs; ambiguity...

Philosophically: reducible to "knowledge-that"?

Logically: how to reason about "knowing X"?

Computationally: efficient knowledge representation, and automated reasoning about "knowing X"

Beyond knowing that: research agenda

In fact, "knowing who" was discussed by Hinttikka (1962) in terms of first-order modal logic: $\exists x K_i(Hans = x)$. "Knowing the answer of the embedded question."

Our agenda:

- Take a know-X construction as a single modality, e.g., pack $\exists x K_i(Hans = x)$ into $Kwho_i Hans$.
- Give an intuitive semantics according to some interpretation.
- Axiomatize the logics with (combinations of) those operators.
- Dynamify those logic with knowledge updates.
- Automate the inferences.
- Come back to philosophy and linguistics with new insights.

The (potential) advantages of modal logics of knowing X

- Natural and succinct to express the desired properties;
- Limited expressive power and moderate complexity;
- Formal notion of consistency of knowledge bases;
- Proof theoretic and model checking tools;
- Capture the essence of the relevant reasoning by axioms;
- Philosophically and linguistically promising semantics;
- Some new insights to bring back to Phil. and Ling...

Beyond knowing that: (technical) difficulties

- not normal:
  - $\nvdash Kw(p \to q) \land Kw\ p \to Kw\ q$
  - $\nvdash Khow\varphi \land Khow\psi \to Khow(\varphi \land \psi)$
  - $\vdash \varphi \nRightarrow \vdash Kwhy\varphi$
- not strictly weaker: $\vdash Kw\varphi \leftrightarrow Kw\neg\varphi$
- combinations of quantifiers and modalities: $\exists x \Box \varphi(x)$;
- the axioms depend on the special schema of φ essentially;
- weak language vs. rich model: hard to axiomatize;
- fragments of FO/SO-modal language: decidability?
- new uses of Kripke models.

Beyond knowing that: some results

Some of our results:

- Knowing whether (non-contingency): model theory and complete axiomatizations of its logics over various frame classes [Fan, Wang & van Ditmarsch: AiML14, RSL 15]; neighbourhood semantics [Fan & vD: ICLA15]
- Knowing what: axiomatization and decidability for conditionally knowing what logic over FO epistemic models [Wang & Fan: IJCAI13, AiML14][Xiong 14][Ding 15]
- Knowing how: philosophical discussion [Lau 15]; alternative non-possible-world semantics [Wang ICLA15]; a logic of 'knowing how' [Wang LORI15]

"Knowing what" operator $\mathcal{K}v_i$ proposed by [Plaza 89]

ELKv is defined as (where $c \in C$):

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathcal{K}_i\varphi \mid \mathcal{K}v_i c$$

ELKv is interpreted on FO-epistemic models with constant domain $\mathcal{M} = \langle S, D, \{\sim_i \mid i \in \mathbf{I}\}, V, V_C\rangle$ where D is a *constant* domain, V_C assigns to each (non-rigid) $c \in C$ a $d \in D$ on each $s \in S$:

$$\mathcal{M}, s \vDash \mathcal{K}v_i c \iff \text{for any } t_1, t_2: \text{ if } s \sim_i t_1, s \sim_i t_2, \text{ then } V_C(c, t_1) = V_C(c, t_2).$$

ELKv can express "i knows that j knows the password but i doesn't know what exactly it is" by $\mathcal{K}_i\mathcal{K}v_j c \wedge \neg\mathcal{K}v_i c$.

The interaction between the two operators is crucial: it cannot be treated as $\mathcal{K}_i\mathcal{K}_j p \wedge \neg\mathcal{K}_i p$.

Knowing what operator $\mathcal{K}v_i$ proposed by [Plaza 89]

To handle the *Sum and Product* puzzle, Plaza extended ELKv with announcement operator (call it PALKv):

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathcal{K}_i\varphi \mid \mathcal{K}v_i c \mid \langle\varphi\rangle\varphi$$

Plaza proposed some axioms for PALKv on top of $\mathbb{S}5$ ($\mathbb{PALKV}_p$).

Theorem (Wang & Fan IJCAI13)

$\langle p\rangle\mathcal{K}v_i c \wedge \langle q\rangle\mathcal{K}v_i c \to \langle p \vee q\rangle\mathcal{K}v_i c$ is not derivable in $\mathbb{PALKV}_p$, thus $\mathbb{PALKV}_p$ is not complete w.r.t. $\vDash$ on FO-epistemic models.

By defining a suitable bisimulation notion we can show that PALKv is not reducible to ELKv.

Conditionally knowing what

Axiomatizing PALKv is indeed hard. We propose a conditional generalization of $\mathcal{K}v_i$ operator (call the language ELKv^r):

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathcal{K}_i\varphi \mid \mathcal{K}v_i(\varphi, c)$$

where $\mathcal{K}v_i(\varphi, c)$ says "agent i knows what c is *given* φ". Everyday knowledge is usually conditional.

$$\boxed{\begin{array}{lll} \mathcal{M}, s \vDash \mathcal{K}v_i(\varphi, c) & \Leftrightarrow & \text{for any } t_1, t_2 \in S \text{ such that } s \sim_i t_1 \text{ and } s \sim_i t_2 : \\ & & \mathcal{M}, t_1 \vDash \varphi \& \mathcal{M}, t_2 \vDash \varphi \text{ implies } V_C(c, t_1) = V_C(c, t_2) \end{array}}$$

Let PALKv^r be:

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \mathcal{K}_i\varphi \mid \mathcal{K}v_i(\varphi, c) \mid \langle\varphi\rangle\varphi$$

PALKv^r looks more expressive than PALKv but in fact they are equally expressive.

Theorem (Wang & Fan 13)

The comparison of the expressive power of those logics are summarized in the following (transitive) diagram:

$$\begin{array}{ccc} \text{ELKv}^r & \longleftrightarrow & \text{PALKv}^r \\ \uparrow & & \updownarrow \\ \text{ELKv} & \longrightarrow & \text{PALKv} \end{array}$$

where ELKv and ELKv^r are the announcement-free fragments of PALKv and PALKv^r.

We can simply forget about Plaza's PALKv and use ELKv^r!

System $\mathbb{ELKV}^r$

Axiom Schemas	
TAUT	all the instances of tautologies
DISTK	$\mathcal{K}_i(p \to q) \to (\mathcal{K}_i p \to \mathcal{K}_i q)$
T	$\mathcal{K}_i p \to p$
4	$\mathcal{K}_i p \to \mathcal{K}_i \mathcal{K}_i p$
5	$\neg \mathcal{K}_i p \to \mathcal{K}_i \neg \mathcal{K}_i p$
$\mathtt{DISTKv}^r$	$\mathcal{K}_i(p \to q) \to (\mathcal{K}v_i(q, c) \to \mathcal{K}v_i(p, c))$
$\mathtt{Kv}^r 4$	$\mathcal{K}v_i(p, c) \to \mathcal{K}_i \mathcal{K}v_i(p, c)$
$\mathtt{Kv}^r \bot$	$\mathcal{K}v_i(\bot, c)$
$\mathtt{Kv}^r \vee$	$\hat{K}_i(p \wedge q) \wedge \mathcal{K}v_i(p, c) \wedge \mathcal{K}v_i(q, c) \to \mathcal{K}v_i(p \vee q, c)$

Rules	
MP	$\dfrac{\varphi, \varphi \to \psi}{\psi}$
NECK	$\dfrac{\varphi}{\mathcal{K}_i \varphi}$
SUB	$\dfrac{\varphi}{\varphi[p/\psi]}$
RE	$\dfrac{\psi \leftrightarrow \chi}{\varphi \leftrightarrow \varphi[\psi/\chi]}$

$\mathcal{K}v_i(\varphi, c)$ can be viewed as $\exists x \mathcal{K}_i(\varphi \to c = x)$ where x is a variable and c is a *non-rigid* constant.

A $\mathcal{K}v_i$ operator packages a quantifier, a modality, an implication and an equality together: a blessing and a curse.

To build a suitable canonical FO-epistemic model with a constant domain, we need to saturate each maximal consistent set with:

- counterparts of atomic formulas such as $c = x$
- counterparts of $\mathcal{K}_i(\varphi \to c = x)$

By using axioms in the modal language, we need to make sure these extra bits are consistent with the maximal consistent sets and canonical relations.

Lemma

Each maximal consistent set can be properly saturated with those counterparts.

Lemma

Each saturated MCS including $\hat{K}\varphi$ has a saturated φ-successor.

Lemma

Each saturated MCS including $\neg Kv_i(\varphi, c)$ has two saturated φ-successors which disagree about the value of c.

Axiom $\mathtt{Kv}^r\vee$: $\hat{K}_i(p \wedge q) \wedge \mathcal{K}v_i(p, c) \wedge \mathcal{K}v_i(q, c) \to \mathcal{K}v_i(p \vee q, c)$ plays an extremely important role.

Theorem (Wang & Fan AiML14)

$\mathbb{ELKV}^r$ *is sound and strongly complete for* ELKv^r.

We can axiomatize multi-agent PALKv^r by adding the following reduction axiom schemas (call the resulting system $\mathbb{SPALKV}^r$):

`!ATOM`	$\langle\psi\rangle p \leftrightarrow (\psi \wedge p)$
`!NEG`	$\langle\psi\rangle\neg\varphi \leftrightarrow (\psi \wedge \neg\langle\psi\rangle\varphi)$
`!CON`	$\langle\psi\rangle(\varphi \wedge \chi) \leftrightarrow (\langle\psi\rangle\varphi \wedge \langle\psi\rangle\chi)$
`!K`	$\langle\psi\rangle\mathcal{K}_i\varphi \leftrightarrow (\psi \wedge \mathcal{K}_i(\psi \to \langle\psi\rangle\varphi))$
$\texttt{!Kv}^r$	$\langle\varphi\rangle\mathcal{K}v_i(\psi, c) \leftrightarrow (\varphi \wedge \mathcal{K}v_i(\langle\varphi\rangle\psi, c))$

Theorem (Xiong 14)

(Multi-agent) ELKv^r *on epistemic models is decidable.*

Theorem (Ding 14)

W.r.t. the class of all models: $\mathbb{ELKV}^r$ *without T,4,5 is complete and SAT problem of* ELKv^r *is PSPACE-complete.*

Conclusions

Systematic study of "knowing-X" in modal logic may lead us to:

- interesting non-normal 'modal' operators packaging quantifiers and modalities together;
- new use of Kripke model to accommodate non-normality;
- interesting new axioms;
- discovery of new decidable ("guarded") fragments of FO/SO-modal logic;
- knowledge representations closer to natural language.
- maybe useful for protocol and plans.

There are many things to be explored!

See the ESSLLI course page for more slides and pointers:
`http://www.phil.pku.edu.cn/personal/wangyj/esslli15/`

Thank you for your attention!