# Conversations as infinite games 

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## Cooperative principles of conversation - Gricean maxims

- Sincerity: say what you believe to be true.
- Relation: make your contribution contribute to solving the goals of the conversation.
- Manner: make your conribution appropriate in complexity to requirements of conversational goals.
- Quantity: make your contribution as informative as required for the goals of the conversation.

But non-cooperative exchanges happen all the time

- Debates, courtroom examinations, political campaigns, bargaining situations...
- Natural to model them as games.


## Problems in modeling such exchanges

Implicatures: expressed content which is not part of the literal meaning of the sentence.

## Example

John ate some of the doughnuts.
This sentence implicates that John did not eat all the doughnuts.

Implicatures are also cancellable/corrigible!
Example
John ate some of the donuts. In fact, he ate them all.

## Problems in modeling such exchanges contd.

Ambiguity:
Example
(a) Phil: Did you see Valentino this week?
(b) Janet: Valentino has mononucleosis.

## Signaling games

- Sender-Receiver.
- Sender has a type $t$ and knows his own type.
- Sends a message $m$ from a finite set $M$ of messages.
- Receiver observes the message but not the type.
- Chooses an action a from a finite set $A$ of actions.
- Players receive payoffs based on $t, m$ and $a$.


## Signaling games to model non-cooperative conversations

In zero-sum signaling games, the optimal equilibrium is a babbling equilibrium (Crawford and Sobel)

- Suppose $A$ and $B$ are playing a signaling game where the interests are strictly opposed.
- Then if A conveys a message, it is in her rational interest to do so.
- If it is in A's interest, it is not in B's interest to react to the message, and so she should ignore it. Thus A's message is just noise.
- If communication has a cost, then A should not send the message to $B$, since she knows that $B$ rationally will ignore it.


## Rationality and optimal action

The situation is not limited to signaling. Consider the game of buying and selling with nonaligned preferences.


Figure: Buying watches off the street

- Higher payoff for the seller with low-quality object, but the converse for the buyer.
- Best option (equalibrium): $(L, \neg b)$. Noncooperative.


## Rationality and optimal action contd.

- No incentive for truthful signal.
- No incentive to believe.
- No incentive to buy; so no incentive to communicate either.
- If I try to sell you a watch on the street claiming it is Rolex, you are not likely to believe me.
- The puzzle: Generally, when preferences are not aligned, there is no cooperation at equilibrium. Then why bother to communicate?


## Puzzling observations

- In situations where the agents' interests are broadly opposed (eg. political debates, marital disputes), people still act somewhat cooperatively.
- They answer more often than not their interlocutor's questions.
- You'd better attend to what your opponent says and gauge exactly what might be meant if you hope to win a debate.


## Bronston and the Prosecutor (Solan and Tiersma, 2005)

(a) Prosecutor: Do you have an account in the bank, Mr. Bronston?
(b) Bronston: No, sir.
(c) Prosecutor: Have you ever?
(d) The company had an account there for about six months, in Zurich.

Remark: Bronston's overall interests are opposed to that of the Prosecutor. Yet he is somewhat cooperative, and exploits the implicatures of (d) to answer his questions.

## Asymmetric bargaining and exchange/trust games

- Asymmetric: speaker places his fate in the hands of the hearer when making a request or asking a question.
- Trust games depict a scenario where Player $X$ has an initial option to defer the decition to player Y for a potentially larger payoff for both.


Figure: The centipede game

- Backward induction says Player X will exit at the first move so no conversation is predicted.
- Repeated games with reputation effects still fall prey to the backward induction argument.


## Avoiding the above problems: infinite games

- Play as if conversations had no set end. That is, conversations are countably infinite sequences of 'utterances'.
- Finite sequences can be modeled as infinite sequences (with a repeating null element).
- Conversational goals can be modeled as sequences satisfying certain objectives. Eg. The prosecutor wants Bronston to commit either to having an account or to not having one. But he will be happy if he EVENTUALLY gets an answer (LTL).


## Banach-Mazur and Gale-Stewart games

- Banach-Mazur game over an alphabet $X$, denoted $\mathrm{BM}\left(X^{\omega}\right.$, win $)$, win $\subset X^{\omega}$

$$
x_{1} x_{2}
$$

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x_{1} x_{2} x_{3} x_{4} x_{5}
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x_{1} x_{2} x_{3} x_{4} x_{5} x_{6} x_{7} x_{8}
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x_{1} x_{2} x_{3} x_{4} x_{5} x_{6} x_{7} x_{8} \cdots
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No turn-structure.

- Gale-Stewart game over an alphabet $X$, denoted $\mathrm{GS}\left(X^{\omega}\right.$, win $)$, $\operatorname{win} \subset X^{\omega}$


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$$
x_{1} x_{2} x_{3} x_{4}
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x_{1} x_{2} x_{3} x_{4} x_{5} x_{6} x_{7} x_{8} \cdots
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Involves turn-structure.

- A game is determined if one or the other player always has a winning strategy.
- A Banach-Mazur game $\operatorname{BM}\left(X^{\omega}\right.$, win $)$ where win is Borel is determined - follows from the Banach-Mazur theorem.
- A Gale-Stewart game $\operatorname{GS}\left(X^{\omega}\right.$, win $)$ where win is Borel is determined (Martin 75).


## Message-Exchange games

- Games for conversations combine elements from both Banach-Mazur and Gale-Stewart games.
- Message-Exchange game over an alphabet $X$, denoted $\operatorname{ME}\left(X^{\omega}\right.$, win $_{0}$, win $\left._{1}\right)$, win $_{0}, \operatorname{win}_{1} \subset X^{\omega}$

$$
\left\langle x_{1} x_{2}, 0\right\rangle
$$

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$$
\left\langle x_{1} x_{2}, 0\right\rangle\left\langle x_{3} x_{4} x_{5}, 1\right\rangle
$$

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\left\langle x_{1} x_{2}, 0\right\rangle\left\langle x_{3} x_{4} x_{5}, 1\right\rangle\left\langle x_{6}, 0\right\rangle
$$

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$$

- A Message-Exchange game is zero-sum if $\operatorname{win}_{1}=\overline{\operatorname{win}_{0}}$.

Theorem
A zero-sum Message-Exchange game $\operatorname{ME}\left(X^{\omega}\right.$, win $)$ where win is Borel is determined.

## Message-Exchange games for conversations

- Our vocabulary consists of discourse units, discourse relations and formulas describing elementary discourse contents.
- $\mathrm{win}_{0}$ and $\mathrm{win}_{1}$ represent the conversational goals.
- In addition we require conversational goals to be (i) consistent (ii) responsive (iii) coherent and (iv) satisfying CNEC

Theorem
If win satisfies the above constraints then it is $\Pi_{3}^{0}$-hard.

## Current and future work

- Can characterize the complexity of the conversational goals and strategies in terms of the complexities of the winning set.
- The situation between Bronston and the Prosecutor can be explained using conversational Message-Exchange game $\operatorname{ME}\left(X^{\omega}\right.$, win $_{0}$, win $\left._{1}\right)$ such that $\operatorname{win}_{0} \cap \operatorname{win}_{1} \neq \emptyset$.
- This leads to a phenomenon called 'misdirection'.

Our goals

- How to deal with implicatures, ambiguities.
- Need to incorporate epistemic entities, dynamic update of epistemic models (ME games over epistemic models??)
- Design automated programs for winning a debate, a political campaign, a courtroom case...

