Program Models and Semi-Public Environments

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Semi-Public Environments

- A computation (possibly non-deterministic) is executed publicly. Intuitively "One of the deterministic programs $\pi_1, \pi_2, \ldots, \pi_n$ will be randomly executed" is announced.
- Data is distributed privately, in the sense that not all agents are able to see the value of all variables.

Vision is the agent's way to 'break' some (or all) of the non-determinism involved.

Program Models & Semi-Public Environments

In the intersection of at least three research areas.

- **Interpreted systems**:
  Interpreted Systems / Epistemic Model = Vision / Program Model
- **Dynamic Logic**:
  DL has tests & assignments as basic programs, and non-deterministic choice, sequential composition, iteration, as a way to combine them. It is also a way to study *ontic* change.
- **Dynamic Epistemic Logic**:
  Program models are a modification of action models. Used for *epistemic* change.

The logic of Semi-Public Environments (I)

- Language:

$Ag = \{1, \ldots, m\}$, $Var = \{x_1, \ldots, x_n\}$

$$\mathsf{M}_\pi, \mathsf{w} \;::=\; \mathsf{M}_{!\varphi_0}, \mathsf{e} \mid \mathsf{M}_{\updownarrow x}, \mathsf{a} \mid (\mathsf{M}_\pi \cup \mathsf{M}_\pi), \mathsf{w} \mid (\mathsf{M}_\pi ; \mathsf{M}_\pi), \mathsf{w}$$

$$\varphi \;::=\; \top \mid x_j \mid V_i x_j \mid \neg\varphi \mid \varphi \wedge \varphi \mid [\mathsf{M}, \mathsf{w}]\varphi \mid K_i\varphi$$

$V_i x_j$ is to express that agent i ‘sees’ variable x_j.

The logic of Semi-Public Environments (II)

- **Epistemic Model**:

$$M = \langle W, R, V, f \rangle, \text{ where}$$

1. W is a finite (possibly empty) set of states;
2. $f : W \to \Theta$ assigns a valuation θ to each state in W;
3. $V : Ag \to 2^{Var}$ keeps track of the variables that agent i 'can see';
4. $R : Ag \to 2^{(W \times W)}$ assigns an accessibility relation to each agent $i \in Ag$. Each $R(i)$ is an equivalence relation.

The logic of Semi-Public Environments (III)

We define $\mathcal{A}^- = \mathcal{A} \setminus \{\bot\}$, where $\mathcal{A}$ is the carrier set of the Lindenbaum-Tarski algebra of $\mathcal{L}_0$ (the propositional language).

- **Program Point**
  An element $w = (\mathsf{pre}(w), \mathsf{tgl}(w)) \in \mathcal{A}^- \times \mathcal{P}(Var)$ is called a *program point*.
- **Program Model**
  Any finite set of program points M is called a *program model*.

For all $\mathsf{u}, \mathsf{w} \in \mathsf{M}$: $\mathsf{w} \approx_i^V \mathsf{u}$ iff $(\mathsf{tgl}(\mathsf{w}) \triangle \mathsf{tgl}(\mathsf{u})) \cap V(i) = \emptyset$

The logic of Semi-Public Environments (IV)

- **Model product**
  Let $M = \langle W, R, V, f \rangle$ be an epistemic model, and M be a program model. $(M \times \mathsf{M})$ is the epistemic model $M' = \langle W', R', V', f' \rangle$ defined as follows:
  1. $W' = \{(w, \mathsf{w}) \mid w \in W, \mathsf{w} \in \mathsf{M} \ \& \ (M, w) \models pre(\mathsf{w})\}$;
  2. $(w, \mathsf{w}) R'_i (u, \mathsf{u})$ iff $w R_i u$ and $\mathsf{w} \approx_i^V \mathsf{u}$;
  3. $V' = V$;
  4. $f'((w, \mathsf{w})) = \updownarrow(\mathsf{tgl})(\mathsf{w})(f(w))$.

The logic of Semi-Public Environments (V)

- **Truth definition for** $[\mathsf{M}, \mathsf{w}]\varphi$
  $(M, w) \models [\mathsf{M}, \mathsf{w}]\varphi$ iff
  $(M, w)[\![\mathsf{M}, \mathsf{w}]\!](M', w')$ implies $(M', w') \models \varphi$

- $(M, w)[\![\mathsf{M}, \mathsf{w}]\!](M', w')$ iff

$$(M, w) \models pre(\mathsf{w}) \text{ and } (M', w') = (M \times \mathsf{M}, (w, \mathsf{w}))$$

Inductively Defined Program Models

- $\mathsf{M}_{!\varphi_0} = (\varphi_0, \emptyset)$
- $\mathsf{M}_{\updownarrow x} = (\top, \{x\})$
- $\mathsf{M}_{\pi_1 \cup \pi_2} = \mathsf{M}_{\pi_1} \cup \mathsf{M}_{\pi_2}$

In contrast to PDL and DEL $\not\models ([\pi_1]\varphi_1 \wedge [\pi_2]\varphi_2) \rightarrow [\pi_1 \cup \pi_2](\varphi_1 \vee \varphi_2)$

- $\mathsf{M}_{\pi_1;\pi_2} = \{\mathsf{w} \in \mathcal{A}^- \times \mathcal{P}(Var) \mid \exists \mathsf{w}_1 \in \mathsf{M}_{\pi_1}, \mathsf{w}_2 \in \mathsf{M}_{\pi_2}$ such that $\mathsf{pre}(\mathsf{w}) = \mathsf{pre}(\mathsf{w}_1) \wedge \updownarrow (\mathsf{tgl}(\mathsf{w}_1))(\mathsf{pre}(\mathsf{w}_2))$ & $\mathsf{tgl}(\mathsf{w}) = \mathsf{tgl}(\mathsf{w}_1) \triangle \mathsf{tgl}(\mathsf{w}_2)\}$

Again note that $[\pi_1; \pi_2]\varphi \not\equiv [\pi_1][\pi_2]\varphi$.

The logic of Semi-Public Environments - Axiomatisation

Propositional Component	
φ	if φ is a prop. tautology
Epistemic Component	
$V_i x \to (K_i x \vee K_i \neg x)$	seeing implies knowing
$V_i x \to K_j V_i x$	vision is common knowledge
$K_i(\varphi \to \psi) \to (K_i \varphi \to K_i \psi)$	K-axiom
$K_i \varphi \to \varphi$	veridicality (truth axiom)
$K_i \varphi \to K_i K_i \varphi$	positive introspection
$\neg K_i \varphi \to K_i \neg K_i \varphi$	negative introspection
Rules of Inference	
if $\vdash \phi$ and $\vdash (\phi \to \psi)$ then $\vdash \psi$	modus ponens
if $\vdash \phi$ then $\vdash K_i \phi$	knowledge-necessitation
if $\vdash \phi$ then $\vdash [\mathsf{M}, \mathsf{w}]\phi$	program-necessitation
if $\vdash \psi_1 \leftrightarrow \psi_2$ then $\vdash \varphi[\psi_1/\psi] \leftrightarrow [\psi_2/\psi]$	substitution of equivalents

The logic of Semi-Public Environments - Axiomatisation

Dynamic Component ($\varphi_0 \in \mathcal{L}_0,\ \varphi, \psi \in \mathcal{L}$)

$[\mathsf{M},\mathsf{w}]\varphi_0 \leftrightarrow (pre(\mathsf{w}) \rightarrow \updownarrow (tgl(\mathsf{w}))(\varphi_0)$	ontic change
$[\mathsf{M},\mathsf{w}]V_i x \leftrightarrow (pre(\mathsf{w}) \rightarrow V_i x)$	vision permanence
$[\mathsf{M},\mathsf{w}]\neg\varphi \leftrightarrow (pre(\mathsf{w}) \rightarrow \neg[\mathsf{M},\mathsf{w}]\varphi)$	program and negation
$[\mathsf{M},\mathsf{w}](\varphi \wedge \psi) \leftrightarrow ([\mathsf{M},\mathsf{w}]\varphi \wedge [\mathsf{M},\mathsf{w}]\psi)$	program and conjunction

The logic of Semi-Public Environments - Axiomatisation

$$[\mathsf{M},\mathsf{w}]K_i\varphi \leftrightarrow \left(pre(\mathsf{w}) \rightarrow \bigwedge_{V\in\mathcal{V}_i} (\chi_V \rightarrow \bigwedge_{\mathsf{w}\approx_i^V \mathsf{u}} K_i[\mathsf{M},\mathsf{u}]\varphi)\right)$$

"Program and Knowledge"

$$\chi_V = \bigwedge_{\substack{i\in Ag,\\ x\in V(i)}} V_i x \ \wedge \bigwedge_{\substack{i\in Ag,\\ x\notin V(i)}} \neg V_i x.$$

Expanding the setting of Semi-Public Environments

- Infinite domain for the variables, e.g., natural numbers.
- Programs that change the agents vision.
- Higher order vision; i sees that j sees variable x.
- Preconditions that are not necessarily propositional.
- Vision not just of variables but more complex formulas.
- Changing the fact that vision is common knowledge.
- Changing the fact that the program executed is common knowledge.
- Introducing the notion that programs are executed by someone within the system, i.e., an agent.

Epistemic Planning

- Given a well-specified epistemic state as input, and a desirable epistemic-state as output, what is the process of transforming the input into the output?
- Sequential Composition works catalytically.

Thank You!