Gossipers and epistemic logic

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To Be Announced! Synthesis of Epistemic Protocols
17 – 21 August 2015, Leiden

The Gossip Problem

Problem:

- six friends each with a secret
- they can call each other to exchange every secret they know
- how many calls to spread all secrets among all friends?

Solution:

- the minimal number of calls is 8 ($2n - 4$ in the general case)
- example of sequence: 12, 34, 56, 13, 45, 16, 24, 35

Dynamic Epistemic Logic of Propositional Assignment and Observation

Language:

$$\alpha ::= p \mid S_i\, \alpha \mid JS\, \alpha$$
$$\varphi ::= \alpha \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_i\varphi \mid CK\varphi \mid [\pi]\varphi$$
$$\pi ::= +\alpha \mid -\alpha \mid \pi;\pi \mid \pi \sqcup \pi \mid \varphi?$$

with p a propositional variable and i an agent.

Indistinguishability relations:

$V \sim_i V'$ iff $\forall\alpha, S_i\,\alpha \in V$ implies $V(\alpha) = V'(\alpha)$

$V \sim_{Agt} V'$ iff $\forall\alpha, JS\,\alpha \in V$ implies $V(\alpha) = V'(\alpha)$

Intuitively:

$$K_i\alpha \leftrightarrow S_i\,\alpha \wedge \alpha$$
$$K_i\neg\alpha \leftrightarrow S_i\,\alpha \wedge \neg\alpha$$

Gossip Problem with DEL-PAO

Call between i and j:

$$C_{ij} = (S_i\, s_1?; +S_j\, s_1 \sqcup \neg S_i\, s_1?); \ldots ; (S_i\, s_6?; +S_j\, s_6 \sqcup \neg S_i\, s_6?);$$
$$(S_j\, s_1?; +S_i\, s_1 \sqcup \neg S_j\, s_1?); \ldots ; (S_j\, s_6?; +S_i\, s_6 \sqcup \neg S_j\, s_6?)$$

With $s_i \in V$ and $S_i\, s_i \in V$ for every $1 \leq i \leq 6$:

$$V \models [C_{12}; C_{34}; C_{56}; C_{13}; C_{45}; C_{16}; C_{24}; C_{35}] \bigwedge_{1 \leq i \leq 6} K_i \left(\bigwedge_{1 \leq j \leq 6} s_j \right)$$

$$V \models \left\langle \Big(\bigsqcup_{1 \leq i,j \leq 6, i \neq j} \neg S_i\, s_j?; C_{ij} \Big)^8 \right\rangle \bigwedge_{1 \leq i \leq 6} K_i \left(\bigwedge_{1 \leq j \leq 6} s_j \right)$$

$$V \models \left[\Big(\bigsqcup_{1 \leq i,j \leq 6, i \neq j} C_{ij} \Big)^7 \right] \neg \bigwedge_{1 \leq i \leq 6} K_i \left(\bigwedge_{1 \leq j \leq 6} s_j \right)$$

Ongoing work: higher-order knowledge

Objective:

$$\bigwedge_{1\leq i\leq 6} K_i \left(\bigwedge_{1\leq j\leq 6} s_j \right) \Longrightarrow \bigwedge_{1\leq i,j\leq 6} K_i K_j \left(\bigwedge_{1\leq k\leq 6} s_k \right)$$

Call:

$(S_i\, s_1?; +S_j\, s_1 \sqcup \neg S_i\, s_1?); \ldots \Longrightarrow$

$(S_i\, s_1?; +S_j\, s_1; +S_i\, S_j\, s_1; +S_j\, S_i\, s_1 \sqcup \neg S_i\, s_1?);$

and $\forall k, (S_i\, S_k\, s_1 \wedge S_k\, s_1?; +S_j\, S_k\, s_1 \sqcup \neg(S_i\, S_k\, s_1 \wedge S_k\, s_1)?); \ldots$

Conjecture (experimental): $3n - 6$ calls for n agents

... and so on.