Conformant Planning with Probability

A dynamic epistemic framework

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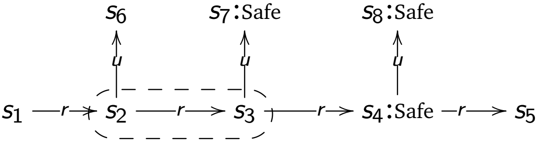
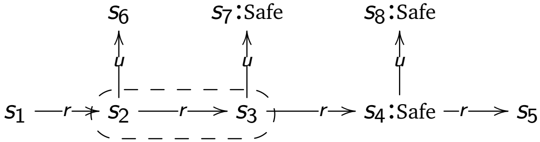
An Example of Conformant planning

Scenario in *Mission Impossible*

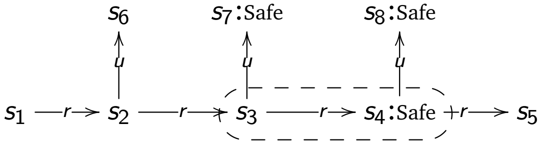
A rookie spy sneaks in a building. Suddenly someone spots him and pulls the alarm. Now he must run to a safe place. However, in panic he gets lost...



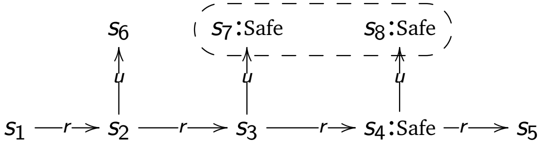
The initial model is $\mathcal{M}$:



After he moves right, the model is updated to $\mathcal{M}|^{r}$:



After he moves right and up, the model is updated to $\mathcal{M}|^{ru}$:



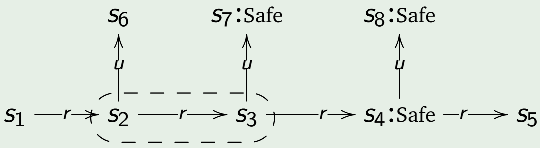
Definition (Uncertainty Map)

An *uncertainty map* (UM) $\mathcal{M} = \langle \mathcal{N}, U \rangle$ consists of a Kripke model $\mathcal{N}$ and an uncertainty set U.

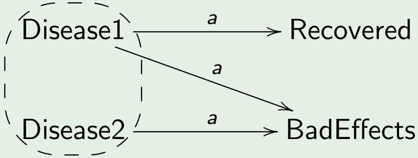
Definition (Conformant Planning)

Given an UM $\mathcal{M}$ and a goal set G, a conformant plan consists of a sequence of actions that is guaranteed to achieve the goal regardless of the uncertainty in the initial state and in the nondeterministic effects of actions.

Example (ru is a conformant plan for $\mathcal{M}$ with $G = \{s_4, s_8, s_9\}$)

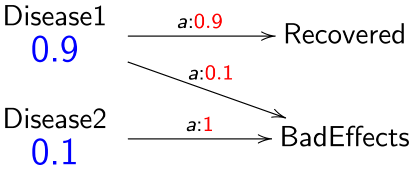


Example ($\mathfrak{M}$ is an UM with $G = \{\text{Recovered}\}$)



In the paper [Yu,Li,Wang TARK2015], we build a dynamic epistemic logic EPDL and reduce the existence of a conformant plan to a model checking problem of EPDL.

Conformant planning with probability



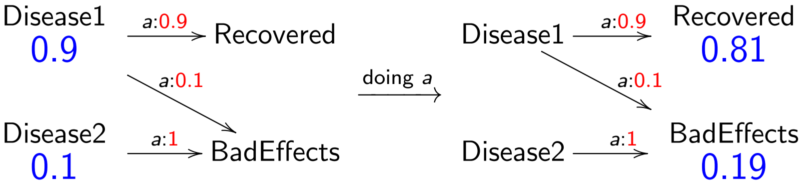
Definition (Probabilistic Uncertainty Map)

A Probabilistic Uncertainty Map PUM $\mathfrak{M}$ is a tuple $\langle W^{\mathfrak{M}}, E^{\mathfrak{M}}, \{\mathrm{P}^{\mathfrak{M}}_{(a,s)} \mid a \in E^{\mathfrak{M}}(s)\}, \mathrm{P}^{\mathfrak{M}}, V^{\mathfrak{M}} \rangle$ such that

- $W^{\mathfrak{M}} \neq \emptyset$, and $E^{\mathfrak{M}} : W^{\mathfrak{M}} \to \mathcal{P}(\mathrm{A})$,
- $\mathrm{P}^{\mathfrak{M}}_{(a,s)} : W^{\mathfrak{M}} \to [0,1]$ such that $\sum_{t \in W^{\mathfrak{M}}} \mathrm{P}^{\mathfrak{M}}_{(a,s)}(t) = 1$,
- $\mathrm{P}^{\mathfrak{M}} : W^{\mathfrak{M}} \to [0,1]$ such that $\sum_{s' \in W^{\mathfrak{M}}} \mathrm{P}^{\mathfrak{M}}_{s}(s') = 1$,
- $V^{\mathfrak{M}} : \mathrm{P} \to \mathcal{P}(W^{\mathfrak{M}})$.

For any $s \in W^{\mathfrak{M}}$, $(\mathcal{M}, s)$ is a pointed PUM.

After doing a, the agent's belief degree will be updated.



Definition (Update)

Given $\mathfrak{M}, s$ and $a \in E^{\mathfrak{M}}(s)$, we define $\mathrm{P}^{\mathfrak{M}}|^a : W^{\mathfrak{M}} \to [0,1]$ as for each $t \in W^{\mathfrak{M}}$,

$$\mathrm{P}^{\mathfrak{M}}|^a(t) = \frac{\sum_{\{s' \in W^{\mathfrak{M}} | a \in E^{\mathfrak{M}}(s')\}} \mathrm{P}^{\mathfrak{M}}(s') \times \mathrm{P}^{\mathfrak{M}}_{(a,s')}(t)}{\sum_{\{s' \in W^{\mathfrak{M}} | a \in E^{\mathfrak{M}}(s')\}} \mathrm{P}^{\mathfrak{M}}(s')}.$$

$\mathfrak{M}|^a$ is almost the same as $\mathfrak{M}$ except that $\mathrm{P}^{\mathfrak{M}|^a} = \mathrm{P}^{\mathfrak{M}}|^a$.

Definition (Language)

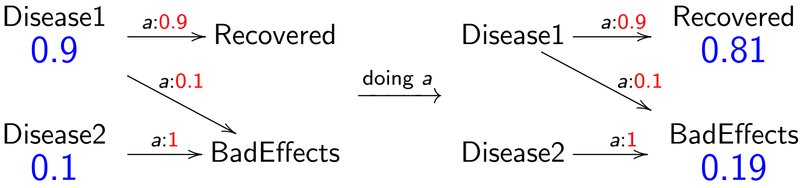
$$\phi ::= p \mid \neg\phi \mid (\phi \wedge \phi) \mid \langle a \rangle_{\geq q}\phi \mid B_{\geq q}\phi$$

Definition (Dynamic semantics)

Given positively pointed PUM $\mathcal{M}, s$, the truth relation is defined as follows:

$$\begin{array}{rcl}
\mathfrak{M}, s \vDash p & \iff & s \in V^{\mathfrak{M}}(p) \\
\mathfrak{M}, s \vDash \neg\phi & \iff & \mathfrak{M}, s \nvDash \phi \\
\mathfrak{M}, s \vDash \phi \wedge \psi & \iff & \mathfrak{M}, s \vDash \phi \text{ and } \mathfrak{M}, s \vDash \psi \\
\mathfrak{M}, s \vDash \langle a \rangle_{\geq q}\phi & \iff & a \in E^{\mathfrak{M}}(s) \text{ and } \mathrm{P}^{\mathfrak{M}}_{(a,s)}(\llbracket\phi\rrbracket^{\mathfrak{M}|^a}) \geq q \\
\mathfrak{M}, s \vDash B_{\geq q}\phi & \iff & \mathrm{P}^{\mathfrak{M}}(\llbracket\phi\rrbracket^{\mathfrak{M}}) \geq q
\end{array}$$

A weak plan



Let the goal is to find an action sequence σ such that after doing σ the belief degree of being recovered is more than 80%, then a is a solution. We can also check that

$$\mathfrak{M}, Disease1 \models \langle a \rangle B_{\geq 0.8} Recovered$$