Complexity Results in Epistemic Planning

Thomas Bolander, DTU Compute, Tech Univ of Denmark
Joint work with: Martin Holm Jensen and Francois Schwarzentruber
Automated planning (or, simply, planning):

- Given is a planning task consisting of: 1) initial state; 2) finite set of actions; 3) goal formula.
- The aim is to compute a plan: a sequence of actions that leads from the initial state to a state satisfying the goal formula.

Example.
Goal: \(\text{On}(A,B) \land \text{On}(B,C)\).

Complexity Results in Epistemic Planning – p. 2/9
Epistemic planning

Epistemic planning: Planning where agents can reason about their own and other agents’ beliefs as part of the planning process.

Epistemic planning application examples:

Games with strong epistemic components (Cluedo, Hanabi, etc.). What will the other agents know if I choose to announce that I have this card?

Robots or intelligent software assistants taking and giving instructions. “Fetch a cup of coffee. The beans are in the cupboard.”

Cryptographic protocols. How can agent $a$ get to know $\varphi$ without agent $b$ knowing?
Our framework for Epistemic Planning

Epistemic planning: Our framework for planning with epistemic reasoning based on Dynamic Epistemic Logic (DEL).

From classical planning to epistemic planning: Replace the propositional logic underlying classical planning by DEL.

<table>
<thead>
<tr>
<th>States</th>
<th>Classical planning</th>
<th>Epistemic planning</th>
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<tbody>
<tr>
<td></td>
<td>models of prop. logic</td>
<td>models of MA epist. logic</td>
</tr>
<tr>
<td>Goal formula</td>
<td>formula of prop. logic</td>
<td>formula of MA epist. logic</td>
</tr>
<tr>
<td>Actions</td>
<td>induced by action schemas</td>
<td>action models of DEL</td>
</tr>
</tbody>
</table>

Epistemic planning can deal with: non-determinism, partial observability, sensing actions, multiple agents, arbitrary nestings of beliefs about beliefs.
Epistemic planning tasks and plan existence problem

Epistemic planning task: Planning task in epistemic planning.

Plan existence problem for class of epistemic planning tasks $X$: “Given an planning task in $X$, does there exist a plan for it?”.

Our paper: Complexity results for the plan existence problem for various classes of epistemic planning tasks.
Example

Consider the epistemic planning task with

1) **Initial state:**

\[ w_1 : p \quad \xrightarrow{\alpha_1} \quad w_2 \]

2) **Actions:**

\[ \alpha_1 = e_1 : p \quad e_2 : \top \]

\[ \alpha_2 = e_1 : p \quad e_2 : \top \]

\[ \alpha_3 = e_1 : p \]

(\(\alpha_1\): privately announcing p to a; \(\alpha_2\): privately announcing p to b; \(\alpha_3\): publicly announcing p to both agents)

3) **Goal formula:**

\[ K_a p \land K_b p \land \neg K_a K_b p \land \neg K_b K_a p \]

A **plan** for this task is \(\alpha_1, \alpha_2\). Another plan is \(\alpha_2, \alpha_1\). Also \(\alpha_1, \alpha_2, \alpha_1\) and \(\alpha_1, \alpha_1, \alpha_2\) are plans, etc.
Complexity of plan existence in epistemic planning

- **The bad news**: The plan existence problem in epistemic planning is undecidable. [Bolander and Andersen, 2011]

- **The even worse news**: The plan existence problem of non-factual epistemic planning (changing only beliefs, no ontic effects) is undecidable. [Aucher and Bolander, 2013]

- **Some slightly good news**: The plan existence problem in epistemic planning with **propositional preconditions** is decidable (in NON-ELEMENTARY). [Yu et al., 2013]

**This paper**: Getting lower complexities for further restricted (but still practically relevant) classes of planning tasks.
Actions are graphs

- Different graph structures allow different actions to be formed (e.g. public announcements like in Cluedo = singletons).
- We study how the underlying graph structure impact complexity of plan existence.

public announcement  private announcement  semi-private announcement
# Summary of complexity results for plan existence

<table>
<thead>
<tr>
<th>Underlying graphs of actions</th>
<th>Types of epistemic actions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Non-factual, propositional preconditions</td>
</tr>
<tr>
<td>Chains</td>
<td>NP-complete</td>
</tr>
<tr>
<td>Trees</td>
<td>PSPACE-complete</td>
</tr>
<tr>
<td>Graphs</td>
<td>in EXPSPACE</td>
</tr>
</tbody>
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↑ in this paper ↑
APPENDIX

Why study very expressively restricted fragments?

Motivation for studying complexity of very restrictive fragments of epistemic planning:

• Still relevant for many interesting applications (e.g. Cluedo only involves public announcements = singleton action models).
• Where does the complexity come from?
• Constructing search heuristics for planning engines (relaxed problems).
Undecidability in Epistemic Planning.
In Proceedings of the Twenty-Third International Joint Conference on Artificial Intelligence (IJCAI) pp. 27–33.

Automata Techniques for Epistemic Protocol Synthesis.

On the Complexity of Dynamic Epistemic Logic.
In TARK.

Journal of Applied Non-Classical Logics 21, 9–34.

Epistemic and Doxastic Planning.
DTU Compute PHD-2014.

DEL planning and some tractable cases.
Exploring the Iterated Update Universe. 

Multi-Agent Epistemic Explanatory Diagnosis via Reasoning about Actions. 
In IJCAI, (Rossi, F., ed.), IJCAI/AAAI.