Synthesis of Epistemic Plans and Protocols with DEL

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Outline

The core of Dynamic Epistemic Logic (DEL)

- Dynamic Epistemic Logic (DEL) extends ordinary epistemic logic by the inclusion of **event models** to describe actions/events, and a **product update** operator that defines how epistemic models are updated as the consequence of executing actions described through event models.

- The methodology of DEL splits the task of representing the agents' beliefs and knowledge into **three parts**:
  1. **Epistemic Models** $(\mathcal{M}, w)$: representation of their beliefs about an initial situation;
  2. **Event Models** $(\mathcal{E}, e)$: representation of their beliefs about an event taking place in this situation;
  3. **Product Update** $\otimes$: representation of the way the agents update their beliefs about the situation after (or during) the occurrence of the event: $(\mathcal{M}, w) \otimes (\mathcal{E}, e)$.

1. One-shot epistemic plan existence

Input:

- $(\mathcal{M}, w)$ pointed epistemic model
- φ epistemic formula

Output:

- "yes" iff there is a pointed event model $(\mathcal{E}, e)$ such that $(\mathcal{M}, w) \otimes (\mathcal{E}, e) \models \varphi$

2. Epistemic plan existence

Input:

- $(\mathcal{M}, w)$ pointed epistemic model
- φ epistemic formula
- $\mathcal{S} = \{(\mathcal{E}_1, e_1), \ldots, (\mathcal{E}_n, e_n)\}$ finite set of pointed event models.

Output:

- "yes" iff there is an **epistemic plan** $(\mathcal{E}_{i_1}, e_{i_1}) \ldots (\mathcal{E}_k, e_{i_k}) \in \mathcal{S}^*$ such that $(((\mathcal{M}, w) \otimes (\mathcal{E}_{i_1}, e_{i_1})) \otimes \ldots) \otimes (\mathcal{E}_{i_k}, e_{i_k}) \models \varphi$.

3. Epistemic protocol existence

Input:

- $(\mathcal{M}, w)$ pointed epistemic model
- φ formula of CTL*K$_n$
- $\mathcal{S} = \{(\mathcal{E}_1, e_1), \ldots, (\mathcal{E}_n, e_n)\}$ finite set of pointed event models.

Output:

- "yes" iff there is an **epistemic protocol** Prot $\subseteq (\mathcal{M}, w)\{(\mathcal{E}_1, e_1), \ldots, (\mathcal{E}_n, e_n)\}^*$ such that Prot $\models \varphi$.

(An epistemic protocol is a set of epistemic plans.)

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Epistemic language

- $\mathbb{P}$ is a countable set of propositional letters called **atomic facts** which describe static situations,
- $\mathbb{G} := \{1, \ldots, m\}$ is a finite set of indices called **agents**.

We define the **epistemic language** $\mathcal{L}^{\mathbb{P}}$ inductively by the following grammar in BNF, where $p \in \mathbb{P}$ and $j \in \mathbb{G}$:

$$\mathcal{L}^{\mathbb{P}}: \quad \varphi \quad ::= \quad p \quad | \quad \neg\varphi \quad | \quad (\varphi \wedge \varphi) \quad | \quad \Box_j \varphi$$

We will use the following abbreviation: $\Diamond_j \varphi := \neg\Box_j\neg\varphi$, $\varphi \vee \psi := \neg(\neg\varphi \wedge \neg\psi)$ and $\varphi \rightarrow \psi := \neg\varphi \vee \psi$.

Epistemic model

A (pointed) epistemic model $(\mathcal{M}, w)$ represents how the actual world represented by w is perceived by the agents.

An **epistemic model** is a tuple $\mathcal{M} = (W, R_1, \ldots, R_m, V)$ where:

- W is a non-empty set of **possible worlds**,
- $R_j \subseteq W \times W$ is an **accessibility relation** on W, for each $j \in \mathbb{G}$,
- $V : \mathbb{P} \to 2^W$ is a **valuation** assigning to each propositional letter a subset of W.

We write $w \in \mathcal{M}$ for $w \in W$, and $(\mathcal{M}, w)$ is called a **pointed epistemic model** (w often represents the actual world). We denote by $\mathcal{C}^{\mathbb{P}}$ the set of pointed epistemic models. If $w \in W$,

$$R_j(w) := \{v \in W \mid (w, v) \in R_j\}.$$

Intuitively, wR_jv means that in world w agent j considers that world v might correspond to the actual world.

Epistemic logic

We define the **satisfaction relation** $\models \subseteq \mathcal{C}^{\mathbb{P}} \times \mathcal{L}^{\mathbb{P}}$ as follows. Let $\mathcal{M}$ be an epistemic model, $w \in \mathcal{M}$ and $\varphi, \psi \in \mathcal{L}^{\mathbb{P}}$. The truth conditions for the atomic facts and the connectives $\neg$, $\wedge$ and $\Box_j$ are defined as follows:

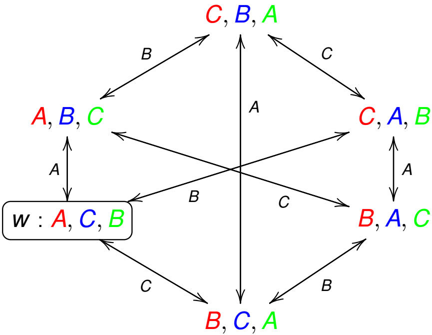
$$
\begin{array}{lll}
\mathcal{M}, w \models p & \text{iff} & w \in V(p) \\
\mathcal{M}, w \models \neg\psi & \text{iff} & \text{it is not the case that } \mathcal{M}, w \models \psi \\
\mathcal{M}, w \models \varphi \wedge \psi & \text{iff} & \mathcal{M}, w \models \varphi \text{ and } \mathcal{M}, w \models \psi \\
\mathcal{M}, w \models \Box_j\varphi & \text{iff} & \text{for all } v \in R_j(w), \text{ we have that } \mathcal{M}, v \models \varphi
\end{array}
$$

The triple $(\mathcal{L}^{\mathbb{P}}, \mathcal{C}^{\mathbb{P}}, \models)$ forms a logic called **epistemic logic**.

Card example

Ann (A), Bob (B) and Claire (C) play a card game with three cards: a green one, a red one and a blue one. Each of them has a single card but they do not know the cards of the other players.

Pointed epistemic model $(\mathcal{M}, w)$:



A: "Ann has the red card"

C: "Claire has the blue card"

B: "Bob has the green card"

$\mathcal{M}, w \models (A \wedge \Box_A A) \wedge (C \wedge \Box_C C) \wedge (B \wedge \Box_B B)$

$\mathcal{M}, w \models \Box_A(B \vee B) \wedge \Box_A(C \vee C)$

Outline

Event language

The propositional letters p_ψ describing events are called **atomic events** and range over:

$$\mathbb{A} = \{p_\psi : \psi \in \mathcal{L}^{\mathbb{P}}\}.$$

The reading of p_ψ is "an event of precondition ψ is occurring".

We define the **event language** $\mathcal{L}^{\mathbb{A}}$ inductively as follows:

$$\mathcal{L}^{\mathbb{A}} : \alpha \quad ::= \quad p_\psi \quad | \quad \neg\alpha \quad | \quad (\alpha \wedge \alpha) \quad | \quad \Box_j \alpha$$

where $\psi \in \mathcal{L}^{\mathbb{P}}$ and $j \in \mathbb{G}$. We use the same abbreviations as for $\mathcal{L}^{\mathbb{P}}$.

Event models

An **event model** is a tuple $\mathcal{E} = (W^\alpha, R_1^\alpha, \ldots, R_m^\alpha, \text{Pre})$ where:

- W^α is a non-empty set of possible events,
- $R_j^\alpha \subseteq W^\alpha \times W^\alpha$ is an accessibility relation on W^α, for each $j \in \mathbb{G}$,
- $\text{Pre} : W^\alpha \to \mathcal{L}^\mathbb{P}$ is a function assigning to each possible event a formula of $\mathcal{L}^\mathbb{P}$. The function Pre is called the **precondition function**.

We write $e \in \mathcal{E}$ for $e \in W^\alpha$, and $(\mathcal{E}, e)$ is called a **pointed event model** (e often represents the actual event). We denote by $\mathcal{C}^\alpha$ the set of pointed event models. If $e \in W^\alpha$,

$$R_j^\alpha(e) := \{f \in W^\alpha \mid (e, f) \in R_j^\alpha\}.$$

Let $P \subseteq \mathcal{L}^\mathbb{P}$ be finite. A **P-complete event model** $\mathcal{E}$ is an event model such that for all $e \in \mathcal{E}, \text{Pre}(e) \in P$.

Sometimes, event models contain a **postcondition function** $\text{Post} : W^\alpha \to \mathcal{L}^\mathbb{A}$ to deal with events that change propositional facts. These events are called **ontic events**.

Satisfaction relation

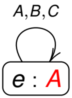
The truth conditions of the event language $\mathcal{L}^{\mathbb{A}}$ are identical to the truth conditions of the epistemic language $\mathcal{L}^{\mathbb{P}}$:

We define the **satisfaction relation** $\models \subseteq \mathcal{C}^{\alpha} \times \mathcal{L}^{\mathbb{A}}$ as follows. Let $\mathcal{M}$ be an event model, $e \in \mathcal{E}$ and $\alpha, \beta \in \mathcal{L}^{\mathbb{A}}$. The truth conditions for the atomic events and the connectives $\neg$, $\wedge$ and $\Box_j$ are defined as follows:

$$
\begin{array}{lll}
\mathcal{E}, e \models p_{\psi} & \text{iff} & \mathrm{Pre}(e) = \psi \\
\mathcal{E}, e \models \neg\alpha & \text{iff} & \text{it is not the case that } \mathcal{E}, e \models \alpha \\
\mathcal{E}, e \models \alpha \wedge \beta & \text{iff} & \mathcal{E}, e \models \alpha \text{ and } \mathcal{E}, e \models \beta \\
\mathcal{E}, e \models \Box_j\alpha & \text{iff} & \text{for all } f \in R_j(e), \mathcal{E}, f \models \alpha
\end{array}
$$

Card example 1

The first example corresponds to the event whereby Player A shows her card publicly to everybody.



The following statement holds:

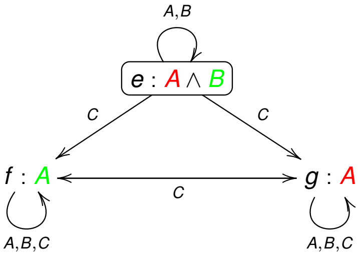
$$\mathcal{E}, e \models p_A \wedge \Box_A p_A \wedge \Box_B p_A \wedge \Box_C p_A \wedge \Box_A \Box_A p_A \wedge \Box_A \Box_B p_A \wedge \Box_A \Box_C p_A \wedge \Box_B \Box_A p_A \wedge \Box_B \Box_B p_A \wedge \Box_B \Box_C p_A \wedge \Box_C \Box_A p_A \wedge \Box_C \Box_B p_A \wedge \Box_C \Box_C p_A \wedge \ldots$$

It states that player A shows her red card and that players A, B and C ‘knoeit, that players A, B and C ‘knoethat each of them ‘knoeit, etc. . . in other words, there is **common knowledge** among players A, B and C that player A shows her red card:

$$\mathcal{E}, e \models p_A \wedge C p_A.$$

Card example 2

Assume that players A and B show their card to each other. As it turns out, C noticed that A showed her card to B but did not notice that B did so to A. Players A and B know this.



$$\mathcal{E}, e \models p_{A\wedge B} \wedge \Box_A p_{A\wedge B} \wedge \Box_B p_{A\wedge B} \wedge (\Diamond_C p_A \wedge \Diamond_C p_A \wedge \Box_C (p_A \vee p_A))$$

Outline

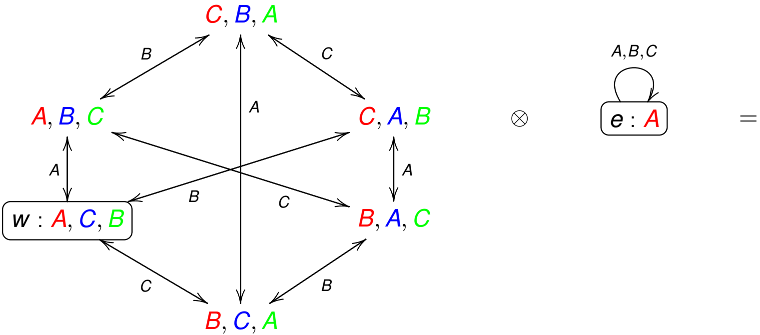
Product update

- $\mathcal{M} = (W, R_1, \ldots, R_m, I)$ is an epistemic model,
- $\mathcal{E} = (W^\alpha, R_1^\alpha, \ldots, R_m^\alpha, \mathrm{Pre})$ is an event model.

The **product update of $\mathcal{M}$ and $\mathcal{E}$** is the epistemic model $\mathcal{M} \otimes \mathcal{E} = (W^\otimes, R_1^\otimes, \ldots, R_m^\otimes, I^\otimes)$ defined as follows: for all $v \in W$ and all $f \in W^\alpha$,

- $W^\otimes = \{(v, f) \in W \times W^\alpha \mid \mathcal{M}, v \models \mathrm{Pre}(f)\}$,
- $R_j^\otimes(v, f) = \{(u, g) \in W^\otimes \mid u \in R_j(v) \text{ and } g \in R_j^\alpha(f)\}$,
- $I^\otimes(v, f) = I(v)$.

Card example 1



Card example 1

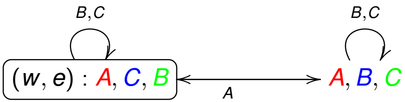


Figure : Situation after Ann has shown her card publicly

Card example 2

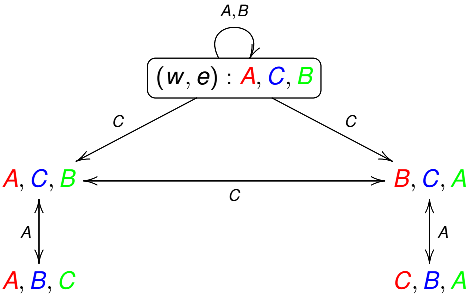


Figure : Situation after Ann and Bob have shown their cards to each other

In this resulting pointed epistemic model, the following statement holds:

$$(\mathcal{M}, w) \otimes (\mathcal{E}, e) \models (B \wedge \Box_A B) \wedge \Box_C \neg \Box_A B.$$

It states that player A ‘knows’ that player B has the green card but player C believes that it is not the case.

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One-shot epistemic plan synthesis: from $(\mathcal{M}, w)$ to ψ

Input:

- $(\mathcal{M}, w)$
- $\varphi \in \mathcal{L}^{\mathbb{P}}$
- $P \subseteq \mathcal{L}^{\mathbb{P}}$ finite.

Output: $\alpha \in \mathcal{L}^{\mathbb{A}}$ such that for all P-complete pointed event model $(\mathcal{E}, e)$,

$$\mathcal{E}, e \models \alpha$$

iff

$$(\mathcal{M}, w) \otimes (\mathcal{E}, e) \text{ is defined and } (\mathcal{M}, w) \otimes (\mathcal{E}, e) \models \varphi.$$

One-shot epistemic plan synthesis: from $(\mathcal{M}, w)$ to ψ

Let $P \subset \mathcal{L}^{\mathbb{P}}$ be finite. The formula $(\mathcal{M}, w) \oslash_P \varphi \in \mathcal{L}^{\mathbb{A}}$ is defined inductively as follows:

$$\begin{aligned}
(\mathcal{M}, w) \oslash_P p &= \begin{cases} Q_w & \text{if } \mathcal{M}, w \models p \\ \bot & \text{otherwise} \end{cases} \\
(\mathcal{M}, w) \oslash_P (\varphi \wedge \psi) &= ((\mathcal{M}, w) \oslash_P \varphi) \wedge ((\mathcal{M}, w) \oslash_P \psi) \\
(\mathcal{M}, w) \oslash_P \neg\varphi &= Q_w \wedge \neg((\mathcal{M}, w) \oslash_P \varphi) \\
(\mathcal{M}, w) \oslash_P \Box_j\varphi &= Q_w \wedge \bigwedge_{v \in R_j(w)} \Box_j (Q_v \rightarrow (\mathcal{M}, v) \oslash_P \varphi) .
\end{aligned}$$

where $Q_w = \bigvee \left\{ p_\varphi : \mathcal{M}, w \models \varphi \text{ and } \varphi \in P \right\}$.

Theorem (Aucher 2011)

Let $(\mathcal{M}, w)$ and let $\psi \in \mathcal{L}^{\mathbb{P}}$. Then, for all P-complete $(\mathcal{E}, e)$,

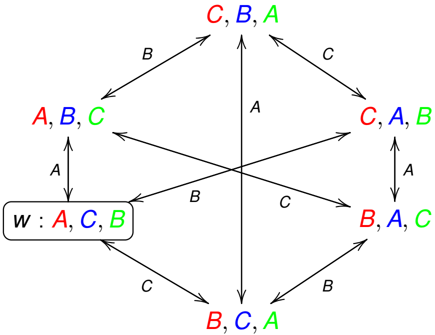
$$\mathcal{E}, e \models (\mathcal{M}, w) \oslash_P \psi$$

iff

$$(\mathcal{M}, w) \otimes (\mathcal{E}, e) \text{ is defined and } (\mathcal{M}, w) \otimes (\mathcal{E}, e) \models \psi.$$

Card example

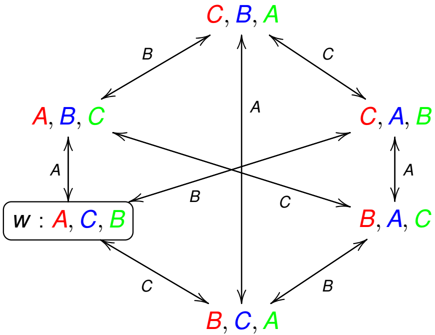
$P = \{A, C, B, \top\}$



$(\mathcal{M}, w) \oslash_P \Box_B(A \wedge C \wedge B) \leftrightarrow \Box_B(p_A \vee p_C)$ is valid

Card example

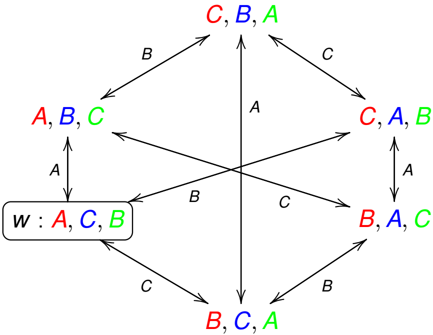
$P = \{A, C, B, \top\}$



$(\mathcal{M}, w) \oslash_P \neg\Box_A(A \wedge C \wedge B) \leftrightarrow \neg\Box_A(p_C \vee p_B)$ is valid

Card example

$P = \{A, C, B, \top\}$



$(\mathcal{M}, w) \oslash_P \Box_B \neg \Box_A (A \wedge C \wedge B) \leftrightarrow \Box_B \neg \Box_A (p_C \vee p_B)$ is valid

Card example

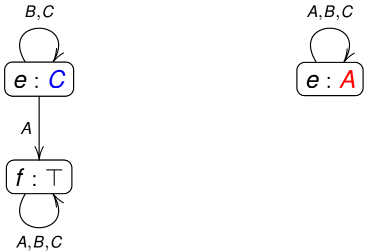


Figure : Claire shows her blue card privately to Bob (*left*) and Ann shows her red card publicly to Bob and Claire (*right*)

Both event models satisfy $(\mathcal{M}, w) \oslash_P \Box_B(A \wedge C \wedge B)$, $(\mathcal{M}, w) \oslash_P \neg\Box_A(A \wedge C \wedge B)$ and $(\mathcal{M}, w) \oslash_P \Box_B \neg\Box_A(A \wedge C \wedge B)$.

Outline

One-shot epistemic plan synthesis: from φ to ψ

Input:

- $\varphi \in \mathcal{L}^{\mathbb{P}}$
- $\psi \in \mathcal{L}^{\mathbb{P}}$
- $P \subseteq \mathcal{L}^{\mathbb{P}}$ finite.

Output: $\alpha \in \mathcal{L}^{\mathbb{A}}$ such that for all P-complete pointed event model $(\mathcal{E}, e)$,

$$\mathcal{E}, e \models \alpha$$

iff

for all $(\mathcal{M}, w)$ such that $\mathcal{M}, w \models \varphi$, if $(\mathcal{M}, w) \otimes (\mathcal{E}, e)$ is defined then $(\mathcal{M}, w) \otimes (\mathcal{E}, e) \models \psi$.

Characteristic formulas

Let P be a finite subset of Φ. We define inductively the sets S_n^P:

$$S_0^P = \left\{ \bigwedge_{p \in S_0} p \wedge \bigwedge_{p \notin S_0} \neg p : S_0 \subseteq P \right\}$$

$$S_{n+1}^P = \left\{ \delta_0 \wedge \bigwedge_{j \in \mathbb{G}} \left(\bigwedge_{\delta_n \in S_n^j} \Diamond_j \delta_n \wedge \Box_j \left(\bigvee_{\delta_n \in S_n^j} \delta_n \right) \right) : \delta_0 \in S_0^P, S_n^j \subseteq S_n^P \right\}.$$

A Characteristic formula δ provides a complete syntactic representation of an epistemic model $(\mathcal{M}, w)$ up to modal depth n and modulo bisimulation.

Lemma (Moss 2007)

Let $\varphi \in \mathcal{L}^{\mathbb{P}}$ be such that $deg(\varphi) \leq n$. Then, φ can be decomposed into characteristic formulas: there is $S \subseteq S_n^P$ such that

$$\varphi \leftrightarrow \bigvee_{\delta \in S} \delta \text{ is valid}$$

One-shot epistemic plan synthesis: from φ to ψ

Let $\varphi, \psi \in \mathcal{L}^{\mathbb{P}}$ and let P be a finite subset of $\mathcal{L}^{\mathbb{P}}$. Then, there are δs such that $\varphi \leftrightarrow \bigvee_{\delta \in E} \delta$ is valid. We define:

$$\varphi \oslash_P \psi \triangleq \bigvee \{\delta \oslash_P \psi : \delta \in S\}$$

where the formula $\delta \oslash_P \psi$ is defined inductively as follows:

$$\begin{array}{rcl}
\delta \oslash_P p & = & \begin{cases} Q_\delta & \text{if } \delta \to p \in \mathrm{K} \\ \bot & \text{otherwise.} \end{cases} \\
\delta \oslash_P (\varphi \wedge \psi) & = & (\delta \oslash_P \varphi) \wedge (\delta \oslash_P \psi) \\
\delta \oslash_P \neg\psi & = & Q_\delta \wedge \neg(\delta \oslash_P \psi) \\
\delta \oslash_P \Box_j \psi & = & Q_\delta \wedge \bigwedge_{\delta_j \in R_j(\delta)} \Box_j \left(Q_{\delta_j} \to \delta_j \oslash_P \psi \right).
\end{array}$$

where $Q_\delta = \bigvee \{p_\varphi : \delta \to \varphi \in \mathrm{K} \text{ and } \varphi \in P\}$. We also define:

$$\varphi[\oslash]_P \psi \triangleq \neg \left(\varphi \oslash_P \neg\psi\right).$$

One-shot epistemic plan synthesis

Theorem (Aucher 2011)

Let $\varphi, \psi \in \mathcal{L}^{\mathbb{P}}$ and let P be a finite subset of $\mathcal{L}^{\mathbb{P}}$. Then, for all P-complete pointed event model $(\mathcal{E}, e)$, it holds that

$$\mathcal{E}, e \models \varphi[\oslash]_P\psi \quad \text{iff} \quad \text{for all } (\mathcal{M}, w) \text{ such that } \mathcal{M}, w \models \varphi, \text{ if } \mathcal{M}, w \models Pre(e) \text{ then } (\mathcal{M}, w) \otimes (\mathcal{E}, e) \models \psi$$

Proposition

The time complexity of the algorithm synthesizing $(\mathcal{M}, w) \oslash_P \varphi$ is in $O(|P| \cdot |\varphi| \cdot d^n + k \cdot N)$ where $n = deg(\varphi)$, $k = |(\mathcal{M}, w)_{\upharpoonright n}|$, $N = \sum \{|\psi| : \psi \in P\}$ and $d = max\,\{|R_j(v)| : j \in Agt(\varphi), v \in (\mathcal{M}, w)_{\upharpoonright n}\}$.

Outline

Epistemic plan existence problem

Input:

- $(\mathcal{M}, w)$ pointed epistemic model,
- $\varphi \in \mathcal{L}^{\mathbb{P}}$,
- $\mathcal{S} := \{(\mathcal{E}_1, e_1), \ldots, (\mathcal{E}_n, e_n)\}$ finite set of event models.

Output:

- "yes" iff there is an epistemic plan $(\mathcal{E}_{i_1}, e_{i_1}) \ldots (\mathcal{E}_k, e_{i_k}) \in \mathcal{S}^*$ such that $((\mathcal{M}, w) \otimes (\mathcal{E}_{i_1}, e_{i_1}) \otimes \ldots) \otimes (\mathcal{E}_{i_k}, e_{i_k}) \models \varphi$

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Epistemic plan problem: undecidability

Theorem (Bolander & Andersen 2011)

The epistemic plan problem with postcondition is undecidable.

Theorem (Aucher & Bolander 2013)

The epistemic plan problem ***without*** *postcondition is undecidable, except with a single agent and with the logics K45 and S5.*

	Single-agent planning	Multi-agent planning
K	UD	UD
KT	UD	UD
K4	UD	UD
K45	D	UD
S4	UD	UD
S5	D	UD

Figure : D=Decidable, UD=UnDecidable

Outline

Epistemic plan problem: decidability

Theorem (Löwe & Al. 2011)

If sequential composition is idempotent and commutative over S, then the epistemic problem is decidable.

Theorem (Yu & Al. 2013)

The epistemic plan problem with propositional preconditions is decidable.

Complexity results

	Types of events		
Underlying graphs of event models	**Epistemic, propositional preconditions**	**Ontic, propositional preconditions**	**Ontic, epistemic preconditions**
Singletons	NP-complete [Bolander & Al. 2015]	PSPACE-hard [Jensen 2014]	PSPACE-hard [Jensen 2014]
Chains	NP-complete [Bolander & Al. 2015]	?	?
Trees	PSPACE-complete [Bolander & Al. 2015]	?	?
Graphs	in EXPSPACE [Bolander & Al. 2015]	in NON-ELEMENTARY [Yu & Al. 2013]	Undecidable [Bolander & Al. 2011]

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Branches, Trees, Forests

Υ is a set called the **tree alphabet**.

- A **tree** is a set of words $\tau \subseteq \Upsilon^+$ that is closed for nonempty prefixes, and for which there is an element $r = \tau \cap \Upsilon$, called the **root**, such that for all $x \in \tau$, $x = r \cdot x'$ for some $x' \in \Upsilon^*$.
- The words of a tree τ that cannot be extended are called **branches** and are denoted λ. We denote by $\lambda[i]$ the i^{th} element of λ and by $\lambda[i, j]$ the sequence of elements of λ between i and j.
- A **forest** is an union of trees.

Σ is a set called the **labeling alphabet**.

- A **labeled tree** is a pair $t = (\tau, l)$, where τ is a tree and $l : \tau \to \Sigma$ is a **labeling**.
- A **labeled forest** $\mathcal{U} = (u, l)$ is a set of labeled trees.

If $\lambda = e_1 e_2 \dots e_n \in \tau$, then $w(\lambda) := l(e_1)l(e_1 e_2) \dots l(e_1 \dots e_n) \in \Sigma^*$.

CTL*K$_n$

The set of well-formed CTL*K$_n$ formulas is given by the following grammar:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathbf{X}\varphi \mid \varphi\mathbf{U}\varphi \mid \forall\varphi \mid K_i\varphi \quad (p \in \mathbb{P}, i \in \mathbb{G})$$

The formula $\exists\varphi$ is an abbreviation for $\neg\forall\neg\varphi$.

The formulas φ are interpreted over nodes of a branch λ of one of the trees of a given forest $\mathcal{U}$: $\mathcal{U}, \lambda, m \models \varphi$.

Moreover, a binary relation $\leadsto_i$ between finite words over Σ is also given.

If $x, y \in \mathcal{U}$ and $i \in \mathbb{G}$, we write $x \leadsto_i y$ when $w(x) \leadsto_i w(y)$.

Truth conditions

Let $\mathcal{U}$ be a given forest, let λ be a branch of one of the trees of this forest and let $m \in \mathbb{N}$.

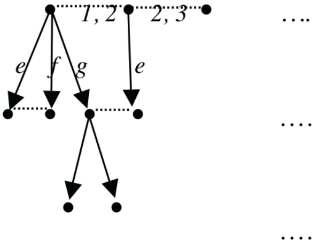
$$
\begin{array}{lll}
\mathcal{U}, \lambda, m \models p & \text{iff} & p \in I(\lambda[m]) \\
\mathcal{U}, \lambda, m \models \neg\varphi & \text{iff} & \text{it is not the case that } \mathcal{U}, \lambda, m \models \varphi \\
\mathcal{U}, \lambda, m \models \varphi \wedge \psi & \text{iff} & \mathcal{U}, \lambda, m \models \varphi \text{ and } \mathcal{U}, \lambda, m \models \psi \\
\mathcal{U}, \lambda, m \models \mathbf{X}\varphi & \text{iff} & \mathcal{U}, \lambda, m+1 \models \varphi \\
\mathcal{U}, \lambda, m \models \varphi \mathbf{U} \psi & \text{iff} & \text{there is } m' \geq m \text{ such that } \mathcal{U}, \lambda, m' \models \psi \text{ and} \\
 & & \text{for all } m \leq m'' < m',\ \text{we have } \mathcal{U}, \lambda, m'' \models \varphi \\
\mathcal{U}, \lambda, m \models \forall\varphi & \text{iff} & \text{for all } \lambda' \in \mathcal{U},\ \text{all } m' \in \mathbb{N} \text{ such that} \\
 & & \lambda[0,m] = \lambda'[0,m'],\ \text{we have } \mathcal{U}, \lambda', m' \models \varphi \\
\mathcal{U}, \lambda, m \models K_i\varphi & \text{iff} & \text{for all } \lambda' \in \mathcal{U},\ \text{all } m' \in \mathbb{N} \text{ such that} \\
 & & \lambda[0,m] \rightsquigarrow_i \lambda'[0,m'],\ \text{we have } \mathcal{U}, \lambda', m' \models \varphi
\end{array}
$$

Outline

DEL-forest

For an epistemic model $\mathcal{M} = (W, R_1, \ldots, R_m, V)$ and an event model $\mathcal{E} = (\mathsf{E}, R_1^\alpha, \ldots, R_m^\alpha, \mathsf{Pre}, \mathsf{Post})$, we define the family of epistemic models $\{\mathcal{M}\mathcal{E}^n\}_{n\geq 0}$ by letting $\mathcal{M}\mathcal{E}^0 = \mathcal{M}$, $\mathcal{M}\mathcal{E}^{n+1} = \mathcal{M}\mathcal{E}^n \otimes \mathcal{E}$.
If for each n, we denote $\mathcal{M}\mathcal{E}^n = (W^n, R_1^n, \ldots, R_m^n, V^n)$, then we define the **DEL-forest** $\mathcal{M}\mathcal{E}^* = (D, \rightsquigarrow_1, \ldots, \rightsquigarrow_m, V)$ by:

- $D = \bigcup_{n\geq 0} W^n$,
- $h \rightsquigarrow_i h'$ if there is some n such that $h, h' \in \mathcal{M}^n$ and $h\, R_i^n\, h'$,
- $V(p) = \bigcup_{n\geq 0} V^n(p)$.
  $V(p_e) = \{he \in D \mid h \in D\}$, for all $e \in \mathsf{E}$.
  $V(p_w) = \{we \in D \mid e \in \mathsf{E}\}$, for all $w \in W$.



Regular structure

A relational structure $\mathcal{S} = (D, \rightsquigarrow_1, \ldots, \rightsquigarrow_m, V)$ is a **regular structure** over a finite alphabet Σ if

- $D \subseteq \Sigma^*$ is a regular language over Σ
- $\rightsquigarrow_i \subseteq \Sigma^* \times \Sigma^*$ is a regular relation, for each i,
- $V(p) \subseteq D$ is a regular language.

Proposition

If $\mathcal{M}$ is an epistemic model and $\mathcal{E}$ is a propositional event model, then $\mathcal{M}\mathcal{E}^*$ is a regular structure.

Note: a finite set of pointed event models $\{(\mathcal{E}_1, e_1), \ldots, (\mathcal{E}_n, e_n)\}$ can be represented equivalently by a multi-pointed event model $(\mathcal{E}, \mathsf{E}')$, where $\mathsf{E}' := \{e_1, \ldots, e_n\}$.

Generating all the epistemic plans

Theorem (Aucher & Al. 2014)

The propositional epistemic planning problem is in k-EXPTIME for formulas of nesting depth k. Moreover, it is possible to build in the same time a finite word automaton $\mathcal{P}$ such that $\mathcal{L}(\mathcal{P})$ is the set of all solution plans.

- By the Proposition we obtain an automatic representation of the forest $\mathcal{M}\mathcal{E}^*$. The epistemic relations are given by finite state transducers. So, we can use the powerset construction of [Maubert 2014].
- If k is the maximal nesting depth of knowledge operators in φ, this construction yields an automaton $\widehat{\mathcal{A}}$ of size k-exponential, in which φ can be evaluated positionally.
- Keeping only transitions labelled by events in E', and choosing for accepting states those that verify φ, we obtain the automaton $\mathcal{P}$ that recognizes the set of solution plans.
- Solving the epistemic planning problem amounts to solving the nonemptiness problem for $\mathcal{L}(\mathcal{P})$; this can be done in time linear in the size of $\mathcal{P}$, which is k-exponential in the size of the input $(\mathcal{M}, \mathcal{E}, \mathrm{E}', \varphi)$.

Outline

3 generalizations of the epistemic planning problem

1. We no longer consider finite sequences of actions but **infinite sequences**. As a consequence, we need not stick to reachability objectives as in planning
2. We allow for any **epistemic temporal formula as objective**.
   The results for epistemic planning problem can be recovered by considering the following formula: (φ is the epistemic goal)
   $$\exists(p_w \vee \bigvee_{e \in E'} p_e)\mathbf{U}(\varphi \wedge (p_w \vee \bigvee_{e \in E'} p_e)).$$
3. We no longer look for a single series of events, but we try to **synthesize a protocol**, *i.e.* a set of plans.

Epistemic protocol problem

Input:

- $(\mathcal{M}, w)$ pointed epistemic model
- $\varphi \in \mathrm{CTL}^*\mathrm{K}_n$
- $\mathcal{S} := \{(\mathcal{E}_1, e_1), \ldots, (\mathcal{E}_n, e_n)\}$ finite set of propositional event models.

Output: "yes" iff there is an epistemic protocol $\mathrm{Prot} \subseteq (\mathcal{M}, w)\mathcal{S}^*$ such that $\mathrm{Prot} \models \varphi$.

Theorem

The epistemic protocol synthesis problem is decidable. If the nesting depth of the goal formulas is bounded by k, then the problem is in $\max(2, k)$-EXPTIME.

Outline

Summary: three planning problems with DEL

Input:

1. $(\mathcal{M}, w)$ pointed epistemic model
2. φ formula
3. set of pointed event models.

Output:

- **One-shot epistemic plan**: decidable, exponential in $deg(\varphi)$.
- **Epistemic plan**: undecidable in general, decidable with propositional preconditions.
- **Epistemic protocol**: decidable with propositional preconditions, in $max(2, depth(\varphi))$-EXPTIME.

(An epistemic protocol is a set of epistemic plans.)

Perspectives: distributing an epistemic protocol

A pointed event model $(\mathcal{E}, e)$ represents the **external** (global) perception of the event e by all the agents.

If we assume that each event is performed by an agent, then the problem of **distributing the epistemic protocol** boils down to answer the following questions:

- How to **decompose** a pointed event model to obtain the **internal** perception of the event by each agent ?
- How to **distribute** an epistemic protocol to obtain an **internal** epistemic protocol for each agent ?
- Do we need a '**scheduler**' to '**orchestrate**' the internal epistemic protocols of the agents ?

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Thank you

Thank you for your attention !