Random Access to Grammar Compressed Strings

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Random Access to Compressed Strings

text
DNA
XML
Random Access to Compressed Strings

- What is the $i$th character?
- What is the substring at $[i,j]$?
- Does pattern $P$ appear in text? (perhaps with $k$ errors?)
Random Access to Grammar Compressed Strings

Grammar based compression captures many popular compression schemes with no or little blowup in space [Charikar et al. 2002, Rytter 2003].

Lempel-Ziv family, Sequitur, Run-Length Encoding, Re-Pair, ...

AGTAGTAG

N = 8

X_7 \rightarrow X_6 X_3
X_6 \rightarrow X_5 X_5
X_5 \rightarrow X_3 X_4
X_4 \rightarrow T
X_3 \rightarrow X_1 X_2
X_2 \rightarrow G
X_1 \rightarrow A

\leq n

A \quad G \quad T \quad A \quad G \quad T \quad A \quad G

1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8

N
Tradeoffs and Results

- What is the $i$th character?  
  - O(N) space  
  - O(n) space  
  - O(n) space  
  - O(1) query  
  - O(n) query  
  - O(log N) query

- What is the substring at $[i,j]$?  
  - O(n) space  
  - O(log N + j - i) query
Application: Black-Box Compressed String Matching

• Does “AGGA” appear in the text (perhaps with k errors)?
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- Total time $O(n \cdot (\log N + m + \text{Blackbox}(m)))$.

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Extension: Compressed Trees

- Linear space in compressed tree.
- Fast navigation operations (select, access, parent, depth, height, subtree_size, first_child, next_sibling, level_ancestor, nca).
Heavy Path Decomposition
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Random Access Query

- The path from root to $i$ goes through $O(\log N)$ heavy paths
- Query: Binary search all heavy paths on the way
  
  $O(\log n) \cdot O(\log N)$
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Interval Biased Search Tree

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$O(\log n) \cdot O(\log N)$

$O(\log N/x)$ Telescopes to $O(\log N)$

- Space: Each IBSTs uses linear space => total $O(n^2)$ space for all heavy paths.
O(n) Representation of Heavy Paths

- Search for \( i \) on heavy path = lowest ancestor of distance \( i \).
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- A heavy path decomposition of heavy path representation.
- In-path: $O(\log N/x)$ time, total $O(n)$ space.
O(n) Representation of Heavy Paths

• Search for i on heavy path = lowest ancestor of distance i.

• A heavy path decomposition of heavy path representation.

• In-path: O(log N/x) time, total O(n) space.

• Between-paths: O(log N/x) time, total O(n log n) space.
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Between-paths: \( O(\log N/x) \) time, total \( O(n \log n) \) space.

The number of leaves is \( n/\log n \). Nodes are \( \log n \) each.
Summary

• Random access and substring decompression.
  • O(n) space and O(log N + length of substring) time.
• Black compressed (approximate) string matching.
• Random access in compressed trees.