Compact Multi-frame Blind Deconvolution

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Motivation

Big Data Imaging Problems

- Multi-Frame Blind Deconvolution (MFBD) where multi = very large
- MFBD combined with 3D (shape) or 4D (shape and color) reconstructions.

Requirements

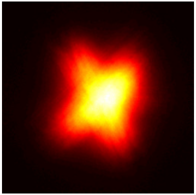
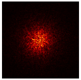
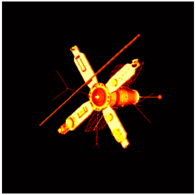
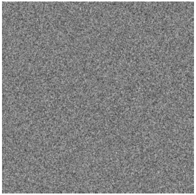
- Powerful computers
- More efficient algorithms
    - More efficient use of current algorithms
    - Smarter ways to process massive data sets
- Collaborative/synergistic teams

    e.g., physics, math, computer science, engineering

Outline

Convolution

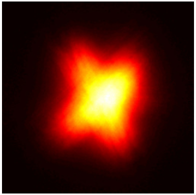
Consider the convolution image formation model:

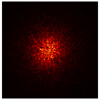
 =  *  + 

Deconvolution

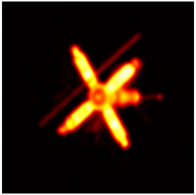
Deconvolution: Given

- Blurred image, and
- Point spread function (convolution kernel).





$\Rightarrow$ compute $\Rightarrow$

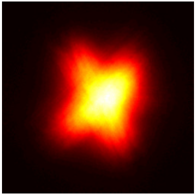


- Compute estimate of true image.

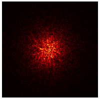
Blind Deconvolution

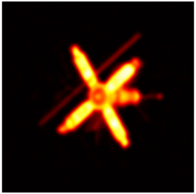
Blind Deconvolution: Given

- Blurred image.



$\Rightarrow$ compute $\Rightarrow$



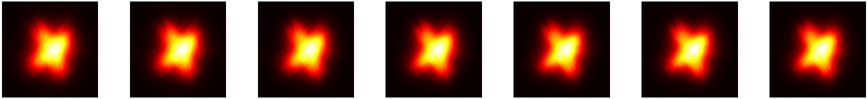


- Compute estimate of true image, and
- Compute estimate of PSF.

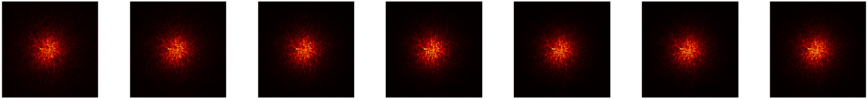
Multi-Frame Blind Deconvolution

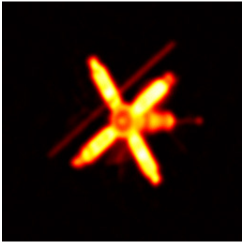
Multi-Frame Blind Deconvolution (MFBD):

- Given multiple frames of blurred images:



- Reconstruct PSFs and object:



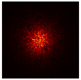
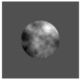


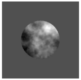
Single Frame Blind Deconvolution (SFBD) Model

Parameterize point spread function

- Using convolution model: $\mathbf{b} = \mathbf{psf}(\mathbf{y}) * \mathbf{x} + \boldsymbol{\eta}$
- Or, using matrix notation: $\mathbf{b} = \mathbf{A}(\mathbf{y})\mathbf{x} + \boldsymbol{\eta}$

Example parameterizations:

- PSF pixels **psf** ()
- Wavefront phase: **psf** () $= |\mathcal{F}^{-1}(Pe^{i\mathbf{y}})|^2$
- Wavefront phase with Zernikes:

psf () $= |\mathcal{F}^{-1}(Pe^{i(y_1\mathbf{z}_1 + \cdots y_m\mathbf{z}_m})|^2$

General Mathematical Model

General mathematical model for image formation:

$$\mathbf{b} = \mathbf{A}(\mathbf{y})\,\mathbf{x} + \boldsymbol{\eta}$$

where

- $\mathbf{b}$ = vector representing observed image
- $\mathbf{x}$ = vector representing true image
- $\mathbf{A}(\mathbf{y})$ = matrix defining blurring operation

  For example,

  - Convolution with imposed boundary conditions
  - Spatially variant blurs
- $\mathbf{y}$ = vector of parameters defining blurring operation

Goal: Given $\mathbf{b}$, jointly compute approximations of $\mathbf{y}$ and $\mathbf{x}$.

Multi-Frame Blind Deconvolution (MFBD)

The MFBD problem is:

$$\begin{aligned} \mathbf{b}_1 &= \mathbf{A}(\mathbf{y}_1)\mathbf{x} + \boldsymbol{\eta}_1 \\ \mathbf{b}_2 &= \mathbf{A}(\mathbf{y}_2)\mathbf{x} + \boldsymbol{\eta}_2 \\ &\vdots \\ \mathbf{b}_m &= \mathbf{A}(\mathbf{y}_m)\mathbf{x} + \boldsymbol{\eta}_m \end{aligned}$$

To solve, could consider least squares best fit objective:

$$\left\| \begin{bmatrix} \mathbf{b}_1 - \mathbf{A}(\mathbf{y}_1)\mathbf{x} \\ \vdots \\ \mathbf{b}_m - \mathbf{A}(\mathbf{y}_m)\mathbf{x} \end{bmatrix} \right\|_2^2 = \left\| \begin{bmatrix} \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_m \end{bmatrix} - \begin{bmatrix} \mathbf{A}(\mathbf{y}_1) \\ \vdots \\ \mathbf{A}(\mathbf{y}_m) \end{bmatrix} \mathbf{x} \right\|_2^2 = \|\mathbf{b} - \mathbf{A}(\mathbf{y})\mathbf{x}\|_2^2$$

Also need regularization, but we omit that complication for now.

Compact Multi-Frame Blind Deconvolution

Processing a large number of frames is computationally intensive.

Compact MFBD (CMFBD)

D. Hope, S. Jefferies, Optics Letters, 36 (2011), pp. 867–869.

- Identify a small set of *control frames* that contain most independent information.
- Reduce full set of data to small set of control frames, without losing any important information.

CMFBD: Identifying Control Frames

- Suppose $\mathbf{A}_j \equiv \mathbf{A}(\mathbf{y}_j)$ are simultaneously diagonalizable (e.g. Fourier transforms for circulant matrices)

$$\mathbf{A}_j = \mathbf{F}^* \mathbf{\Lambda}_j \mathbf{F}$$

- Consider noise free data, and the j-th frame:

$$\begin{aligned} \mathbf{A}_j \mathbf{x} = \mathbf{b}_j \quad &\Rightarrow \quad \mathbf{\Lambda}_j \hat{\mathbf{x}} = \hat{\mathbf{b}}_j \\ &\Rightarrow \quad \mathbf{\Lambda}_j \operatorname{diag}(\hat{\mathbf{x}}) = \operatorname{diag}(\hat{\mathbf{b}}_j) \\ &\Rightarrow \quad \operatorname{diag}(\hat{\mathbf{b}}_j)^\dagger = \operatorname{diag}(\hat{\mathbf{x}})^\dagger \mathbf{\Lambda}_j^\dagger \end{aligned}$$

where $\hat{\mathbf{x}} = \mathbf{F}\mathbf{x}$ and $\hat{\mathbf{b}}_j = \mathbf{F}\mathbf{b}_j$ are unitary Fourier transforms.

CMFBD: Identifying Control Frames

- Assume there is a uniformly "best" conditioned matrix $\mathbf{A}_k$. That is, there is a $\boldsymbol{\Lambda}_k$ such that

$$[\,|\boldsymbol{\Lambda}_k|\,]_{ii} \geq \tau \quad \text{if there exists } j \text{ with} \quad [\,|\boldsymbol{\Lambda}_j\,|]_{ii} \geq \tau$$

  where $\tau > 0$ is a tolerance.
- In this case, where there is a single control frame, observe:

$$\text{diag}(\hat{\mathbf{b}}_j) = \boldsymbol{\Lambda}_j \text{diag}(\hat{\mathbf{x}}) \quad \text{and} \quad \text{diag}(\hat{\mathbf{b}}_k)^\dagger = \text{diag}(\hat{\mathbf{x}})^\dagger \boldsymbol{\Lambda}_k^\dagger$$

- This allows to compute *spectral ratios*

$$\underbrace{\text{diag}(\hat{\mathbf{b}}_j)\,\text{diag}(\hat{\mathbf{b}}_k)^\dagger}_{\text{known}} = \underbrace{\boldsymbol{\Lambda}_j\,\boldsymbol{\Lambda}_k^\dagger}_{\text{unknown}}$$

CMFBD: Exploiting Control Frames

- WLOG, assume the control frame is $k = 1$, and observe:

$$\left\| \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \mathbf{A}_m \end{bmatrix} \mathbf{x} - \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_m \end{bmatrix} \right\|_2^2 = \left\| \begin{bmatrix} \mathbf{\Lambda}_1 \\ \mathbf{\Lambda}_2 \\ \vdots \\ \mathbf{\Lambda}_m \end{bmatrix} \hat{\mathbf{x}} - \begin{bmatrix} \hat{\mathbf{b}}_1 \\ \hat{\mathbf{b}}_2 \\ \vdots \\ \hat{\mathbf{b}}_m \end{bmatrix} \right\|_2^2$$

$$= \left\| \begin{bmatrix} \mathbf{\Lambda}_1 \mathbf{\Lambda}_1^\dagger \\ \mathbf{\Lambda}_2 \mathbf{\Lambda}_1^\dagger \\ \vdots \\ \mathbf{\Lambda}_m \mathbf{\Lambda}_1^\dagger \end{bmatrix} \mathbf{\Lambda}_1 \hat{\mathbf{x}} - \begin{bmatrix} \hat{\mathbf{b}}_1 \\ \hat{\mathbf{b}}_2 \\ \vdots \\ \hat{\mathbf{b}}_m \end{bmatrix} \right\|_2^2$$

CMFBD: Exploiting Control Frames

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$$= \left\| \begin{bmatrix} \mathbf{I} \\ \mathbf{D}_2 \\ \vdots \\ \mathbf{D}_m \end{bmatrix} \boldsymbol{\Lambda}_1 \hat{\mathbf{x}} - \begin{bmatrix} \hat{\mathbf{b}}_1 \\ \hat{\mathbf{b}}_2 \\ \vdots \\ \hat{\mathbf{b}}_m \end{bmatrix} \right\|_2^2$$

where $\mathbf{D}_j = \mathrm{diag}(\hat{\mathbf{b}}_j)\,\mathrm{diag}(\hat{\mathbf{b}}_1)^\dagger$

CMFBD Observations

- The initial MFBD problem has unknowns:

$$\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_m, \mathbf{x} \quad \text{or, equivalently} \quad \mathbf{\Lambda}_1, \mathbf{\Lambda}_2, \ldots, \mathbf{\Lambda}_m, \hat{\mathbf{x}}$$

- After identifying a control frame, significantly fewer unknowns:

$$\mathbf{A}_1, \mathbf{x} \quad \text{or, equivalently} \quad \mathbf{\Lambda}_1, \hat{\mathbf{x}}$$

- More control frames may be needed to capture all $[\,|\mathbf{\Lambda}_j\,|]_{ii} \geq \tau$.
- For noisy data, algebra relating known and unknown information holds only approximately.
- Frame Selection: Based on heuristics
  - "Best" conditioned $\mathbf{A}_k \Leftrightarrow$ least blurred image
  - Many techniques can be used – we use a Fourier based power spectrum approach.

CMFBD Practical Details

- **Reduction of Single Frame Problem:** Use Givens rotations

$$\left\| Q^* \left(\begin{bmatrix} \mathbf{I} \\ \mathbf{D}_2 \\ \vdots \\ \mathbf{D}_m \end{bmatrix} \boldsymbol{\Lambda}_1 \hat{\mathbf{x}} - \begin{bmatrix} \hat{\mathbf{b}}_1 \\ \hat{\mathbf{b}}_2 \\ \vdots \\ \hat{\mathbf{b}}_m \end{bmatrix} \right) \right\|_2^2 = \left\| \begin{bmatrix} \mathbf{D} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \boldsymbol{\Lambda}_1 \hat{\mathbf{x}} - \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \\ \vdots \\ \mathbf{d}_m \end{bmatrix} \right\|_2^2$$

Therefore, we need only consider

$$\| \mathbf{D} \boldsymbol{\Lambda}_1 \hat{\mathbf{x}} - \mathbf{d}_1 \|_2^2 = \| \mathbf{D} \mathbf{F} \mathbf{A}_1 \mathbf{x} - \mathbf{d}_1 \|_2^2$$

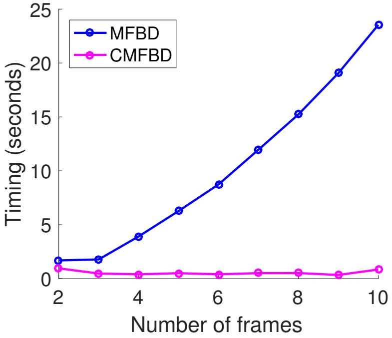
Thus, the MFBD problem

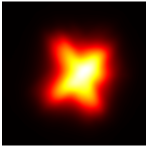
$$\min_{\mathbf{y}_j,\, \mathbf{x}} \sum_{j=1}^{m} \| \mathbf{A}(\mathbf{y}_j)\, \mathbf{x} - \mathbf{b}_j \|_2^2$$

reduces to the CSFBD problem

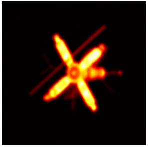
$$\min_{\mathbf{y}_1,\, \mathbf{x}} \| \mathbf{W} \mathbf{A}(\mathbf{y}_1)\, \mathbf{x} - \mathbf{d}_1 \|_2^2 , \quad \mathbf{W} = \mathbf{D}\mathbf{F}$$

Numerical Illustration of Time Savings

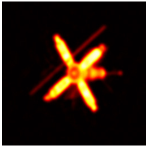




Mean data frame



MFBD



CMFBD

Global Variable Consensus

The MFBD problem can be written as:

$$\min_{\mathbf{y}_i, \mathbf{x}} \sum_{i=1}^{m} \|\mathbf{b}_i - \mathbf{A}(\mathbf{y}_i)\mathbf{x}\|_2^2 + g(\mathbf{x})$$

where here we include an object regularization term, $g(\mathbf{x})$.

Remarks:

- Regularization $g(\mathbf{x})$ can be used to enforce nonnegativity, sparsity, etc.
- The unknown $\mathbf{x}$ couples the objective terms $i = 1, \ldots, m$
- We can get a partial decoupling by reformulating as:

$$\min_{\mathbf{y}_i, \mathbf{x}_i} \sum_{i=1}^{m} \|\mathbf{b}_i - \mathbf{A}(\mathbf{y}_i)\mathbf{x}_i\|_2^2 + g(\mathbf{z}) \quad \text{subject to } \mathbf{x}_i = \mathbf{z}, \; i = 1, \ldots, m$$

Global Variable Consensus

Using an augmented Lagrangian approach, and the Alternating Direction Method of Multipliers (ADMM)[1], the optimization decouples:

for k = 1, 2, ...

$$\left[\mathbf{y}_i^{(k+1)}, \mathbf{x}_i^{(k+1)}\right] = \operatorname*{argmin}_{\mathbf{y}_i, \mathbf{x}_i} \|\mathbf{b}_i - \mathbf{A}(\mathbf{y}_i)\mathbf{x}_i\|_2^2 + \frac{\beta}{2}\|\mathbf{x}_i - \mathbf{z}^{(k)} + \mathbf{u}_i^{(k)}\|_2^2$$

$$\bar{\mathbf{x}}^{(k+1)} = \frac{1}{m}\sum_{i=1}^{m} \mathbf{x}_i^{(k+1)}$$

$$\bar{\mathbf{u}}^{(k)} = \frac{1}{m}\sum_{i=1}^{m} \mathbf{u}_i^{(k)}$$

$$\mathbf{z}^{(k+1)} = \operatorname*{argmin}_{\mathbf{z}} \left\{ g(\mathbf{z}) + \frac{m\beta}{2}\|\mathbf{z} - \bar{\mathbf{x}}^{(k+1)} - \bar{\mathbf{u}}^{(k)}\|_2^2 \right\}$$

$$\mathbf{u}_i^{(k+1)} = \mathbf{u}_i^{(k)} + \mathbf{x}_i^{(k+1)} - \mathbf{z}^{(k+1)}$$

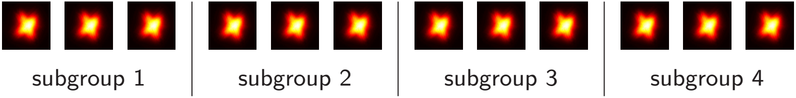
end

[1] Good ADMM references: Wahlberg, Boyd, Annergren, Wang, *Proc. 16th IFAC Symposium on System Identification*, 2012, and Boyd, et. al., *Foundations and Trends in Machine Learning*, 2010.

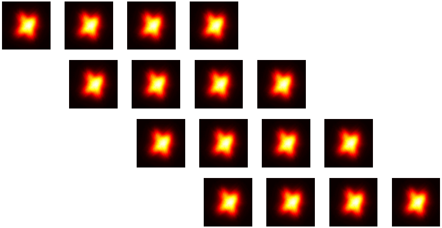
Global Variable Consensus

Advantages:

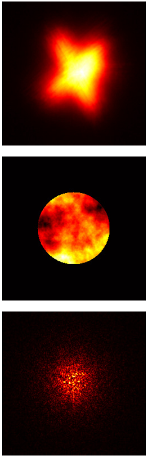
- Decoupling allows for easy parallel processing of groups of frames



  - Can either use standard MFBD on subgroups of frames, or
  - Use CMFBD on subgroups of frames
- Regularization term is also decoupled, allowing users to plug in many options, and it simplifies the computation.
- Sliding window approach might be possible:

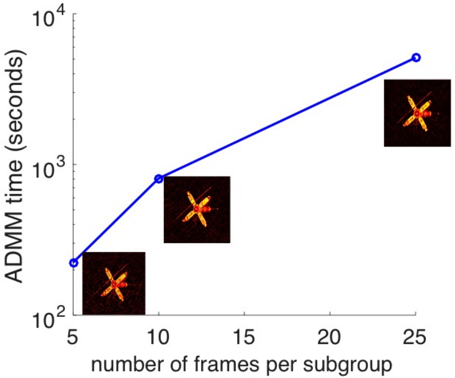


Global Variable Consensus: Numerical Illustration[2]



50 total frames, split in three different ways

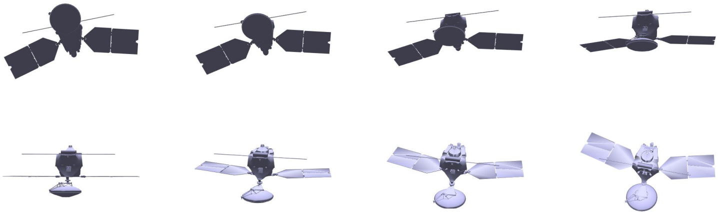
10 subgroups (5 frames each)	5 subgroups (10 frames each)	2 subgroups (25 frames each)

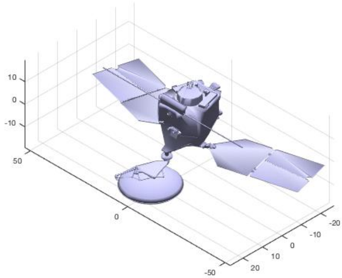


[2] J. D. Schmidt, Numerical Simulation of Optical Wave Propagation, SPIE Press Monograph Vol. PM199, 2010

Higher Dimensional Image Reconstruction

Three dimensional reconstruction from two dimensional measurements:





Higher Dimensional Image Reconstruction

Some computational challenges:

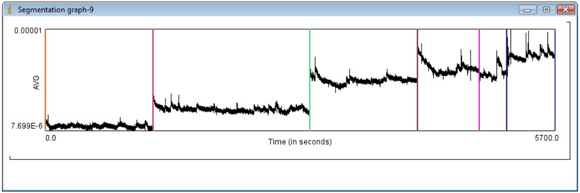
- Requires processing many, many frames of data.
- Mathematical model is similar to MFBD, but
  - Number of unknowns for object significantly increases.
  - Additional unknowns associated with parameters defining object orientation.
- Some related work has been done for molecular structure determination, e.g. in Cryo-EM and x-ray crystallography[3]

[3] J. Chung, P. Sternberg and C. Yang, *High Performance 3-D Image Reconstruction for Molecular Structure Determination*, International Journal of High Performance Computing Applications, 24 (2010), pp. 117–135.

Higher Dimensional Image Reconstruction

What further information can be used?

- Possibly assume blocks of data have constant orientation parameters
  Idea like this was used in PET brain image reconstruction[4]



- Can use consensus ADMM type approach on blocks of data.
- Use other information (e.g., a frozen flow assumption), or technologies (e.g., laser guide stars).

[4]P. Wendykier, J. Nagy, *Parallel Colt: High Preformance Java Library for Scientific Computing and Image Processing*, ACM Transactions on Mathematical Software, 37 (2010), pp. 31:1–31:22

Summary

- Big data, multi-frame image processing requires not only powerful computers, but also new approaches to process massive data sets.

  This is especially true for 3D/4D image reconstructions.

- Goal should be to extract as much information as possible from collected data, but to also do it quickly.

- Important to have synergistic collaborations with various expertise.