02157 Functional Programming

Sequences

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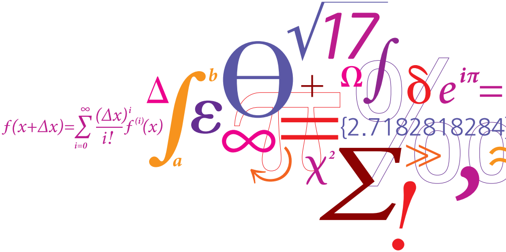

DTU Informatics
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Sequences (or Lazy Lists)

- *lazy evaluation* or *delayed evaluation* is the technique of delaying a computation until the result of the computation is needed.

Default in lazy languages like Haskell

It is occasionally efficient to be lazy.

A special form of this is *Sequences*, where the elements are not evaluated until their values are required by the rest of the program.

- a *sequence* may be infinite

just a finite part of a it is used in computations

Example:

- Consider the sequence of all prime numbers: $2, 3, 5, 7, 11, 13, 17, 19, 23, \ldots$
- the first 5 are $2, 3, 5, 7, 11$

Sieve of Eratosthenes

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Delayed computations

The computation of the value of *e* can be delayed by ”packing” it into a function (a closure):

```
fun () -> e
```

Example:

```
fun () -> 3+4;;
val it : unit -> int = <fun:clo@10-2>

it();;
val it : int = 7
```

The addition is deferred until the closure is applied.

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Example continued

One can make it visible when computations are performed by use of side effects:

```
let idWithPrint i = let _ = printfn "%d" i
                    i;;
val idWithPrint : int -> int

idWithPrint 3;;
3
val it : int = 3
```

The value is printed before it is returned.

```
fun () -> (idWithPrint 3) + (idWithPrint 4);;
val it : unit -> int = <fun:clo@14-3>
```

Nothing is printed yet.

```
it();;
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4
val it : int = 7
```

Example continued

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Sequences in F#

A lazy list or *sequence* in F# is a possibly infinite, ordered collection of elements, where the elements are computed by demand only.

A natural number sequence $0, 1, 2, \ldots$ is created as follows:

```
let nat = Seq.initInfinite (fun i -> i);;
val nat : seq<int>
```

A `nat` element is computed by demand only:

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let nat = Seq.initInfinite idWithPrint;;
val nat : seq<int>

Seq.nth 4 nat;;
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val it : int = 4
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Further examples

A sequence of even natural numbers is easily obtained:

```
let even = Seq.filter (fun n -> n%2=0) nat;;
val even : seq<int>

Seq.toList(Seq.take 4 even);;
0
1
2
3
4
5
6
val it : int list = [0; 2; 4; 6]
```

Demanding the first 4 even numbers demands a computation of the first 7 natural numbers.

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Sieve of Eratosthenes

Greek mathematician (194 – 176 BC)

Computation of prime numbers

- start with the sequence $2, 3, 4, 5, 6, \ldots$
  select head (2), and remove multiples of 2 from the sequence

  2
- next sequence $3, 5, 7, 9, 11, \ldots$
  select head (3), and remove multiples of 3 from the sequence

  $2, 3$
- next sequence $5, 7, 11, 13, 17, \ldots$
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  $2, 3, 5$
- $\vdots$

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- $\vdots$

Sieve of Eratosthenes in F# (I)

Remove multiples of *a* from sequence *sq*:

```
let sift a sq = Seq.filter (fun n -> n % a <> 0) sq;;
val sift : int -> seq<int> -> seq<int>
```

Select head and remove multiples of head from the tail – recursively:

```
let rec sieve sq =
   Seq.delay (fun () ->
                  let p = Seq.nth 0 sq
                  Seq.append
                    (Seq.singleton p)
                    (sieve(sift p (Seq.skip 1 sq))));;
val sieve : seq<int> -> seq<int>
```

- Delay is needed to avoid infinite recursion
- Seq.append is the sequence sibling to @
- Seq.nth 0 sq gives the head of sq
- Seq.skip 1 sq gives the tail of sq

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Examples

The sequence of prime numbers and the *n*’th prime number:

```
let primes = sieve(Seq.initInfinite (fun n -> n+2));;
val primes : seq<int>

let nthPrime n = Seq.nth n primes;;
val nthPrime : int -> int

nthPrime 100;;
val it : int = 547
```

Re-computation can be avoided by using cached sequences:

```
let primesCached = Seq.cache primes;;

let nthPrime’ n = Seq.nth n primesCached;;
val nthPrime’ : int -> int
```

Computing the 700’th prime number takes about 8s; a subsequent computation of the 705’th is fast since that computation starts from the 700 prime number

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Sieve of Eratosthenes using Sequence Expressions

Sequence expressions can be used for defining step-by-step generation of sequences.

The sieve of Erastothenes:

```
let rec sieve sq =
   seq { let p = Seq.nth 0 sq
         yield p
         yield! sieve(sift p (Seq.skip 1 sq)) };;
val sieve : seq<int> -> seq<int>
```

- By construction lazy – no explicit `Seq.delay` is needed
- `yield` *x* adds the element *x* to the generated sequence
- `yield!` *sq* adds the sequence *sq* to the generated sequence
- seqexp_1
  seqexp_2 appends the sequence of seqexp_1 to that of seqexp_2

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Example: Catalogue search (I)

Extract (recursively) the sequence of all files in a directory:

```
open System.IO ;;

let rec allFiles dir =
  seq {yield! Directory.GetFiles dir
       yield! Seq.collect allFiles (Directory.GetDirectories dir)}
val allFiles : string -> seq<string>
```

where

```
Seq.collect: ('a -> seq<'c>) -> seq<'a> -> seq<'c>
```

combines a ’map’ and ’concatenate’ functionality.

```
Directory.SetCurrentDirectory @"C:\mrh\Forskning\Cambridge\";;
let files = allFiles ".";;
val files : seq<string>

Seq.nth 100 files;;
val it : string = ".\BOOK\Satisfiability.fs"
```

Nothing is computed beyond element 100.

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Example: Catalogue search (II)

We want to search for files with certain extensions, e.g. as follows:

```
let funFiles=Seq.cache (searchFiles (allFiles ".") ["fs";"fsi"]);;
val funFiles : seq<string * string * string>

Seq.nth 0 funFiles;;
val it: string * string * string= (".\", "CatalogueSearch", "fs")

Seq.nth 6 funFiles;;
val it : string * string * string = (".\BOOK\", "Curve", "fsi")

Seq.nth 11 funFiles;;
val it : string * string * string
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- a sequence in chosen so that the search is terminated when the wanted file is found
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Example: Catalogue search (III)

The search function ca be declared using regular expressions:

```
open System.Text.RegularExpressions ;;

let rec searchFiles files exts =
    let reExts = List.foldBack (fun ext re -> ext+"|"+re) exts ""
    let re = Regex (@"\G(\S*\\)([^\\]+)\.(" + reExts + ")$")
    seq {for fn in files do
            let m = re.Match fn
            if m.Success
            then let path = captureSingle m 1
                 let name = captureSingle m 2
                 let ext  = captureSingle m 3
                 yield (path, name, ext)      };;
val searchFiles : seq<string> -> string list
                  -> seq<string * string * string>
```

- `reExts` is a regular expression matching the extensions
- The path matches the regular expression `\S*\\`
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- The function `captureSingle` can extract captured strings

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Summary

- Anonymous functions `fun () ->` *e* can be used to delay the computation of *e*.
- Possibly infinite sequences provide natural and useful abstractions
- The computation by demand only is convenient in many applications

It is occasionally efficient to be lazy.

The type `seq<'a>` is a synonym for the .NET type `IEnumerable<'a>`.

Any .NET type that implements this interface can be used as a sequence.

- Lists, arrays and databases, for example.

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