02157 Functional Programming
Lecture 1: Introduction and Getting Started

Michael R. Hansen

DTU Informatics
Department of Informatics and Mathematical Modelling
WELCOME to
02157 Functional Programming

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Both at DTU Informatics

Homepage:  www.imm.dtu.dk/courses/02157
Today: Friday, September 7.

- Introduction to functional programming and F#  
  (341.23 — here)
- about 9:15 – lecture notes can be bought here.
- Make your first programs in the databar  
  (341 Rooms: 015 and 019 — E-databar)
- Introduction to lists in F#  
  (341.23 — here)
- Computations with polynomials in F#  
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By noon you have solved a non-trivial problem using F#
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Practical Matters


  Can be bought at the reception of DTU Informatics. Price 100 kr.

  Published by Cambridge University Press the coming winter.

- F# is an open-source functional language integrated in the Visual Studio development platform and with access to all features in the .NET program library. The language is also supported on Linux and MAC systems using the Mono platform.

- We use F# on the Windows platform in the E-databar.

- Look at homepage concerning installations for your own PC (Windows, Linux or Mac).
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Imperative models

- **Imperative models of computations** are expressed in terms of states and sequences of state-changing operations

Example:

```plaintext
i := 0;
s := 0;
while i < length(A)
do  s := s + A[i];
   i := i + 1
od
```

An imperative model describes *how* a solution is obtained
Imperative models

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An imperative model describes *how* a solution is obtained
Object-oriented models

- An object is characterized by a state and an interface specifying a collection of state-changing operations.
- Object-oriented models of computations are expressed in terms of a collection of objects which exchange messages by using interface operations.

Object-oriented models add structure to imperative models.

An object-oriented model describes how a solution is obtained.
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Object-oriented models add structure to imperative models

An object-oriented model describes how a solution is obtained
In declarative models focus is on *what* a solution is.

- **Logical programming** (02156 Logical Systems and Logical Programming)
  - Programs are (typically) expressed in a fragment of first-order logic. The formulas have a standard meaning, as well as a *procedural interpretation* based on logical inferences.

- **Functional programming**
  - A program is expressed as a mathematical function
    
    \[ f : A \rightarrow B \]

    and function applications guide computations.

Some advantages

- fast prototyping based on abstract concepts
- more advanced applications are within reach
- Supplement modelling and problem solving techniques
- Execute in parallel on multi-core platforms

F# is as efficient as C#
Declarative models

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Some functional programming background

In functional programming, the model of computation is the application of functions to arguments. 

- Introduction of \( \lambda \)-calculus around 1930 by Church and Kleene when investigating function definition, function application, recursion and computable functions. For example, \( f(x) = x + 2 \) is represented by \( \lambda x . x + 2 \).

- Introduction of the type-less functional-like programming language LISP was developed by McCarthy in the late 1950s.

- Introduction of the "variable-free" programming language FP (Backus 1977), by providing a rich collection of functionals (combining forms for functions).

- Introduction of functional languages with a strong type system like ML (by Milner) and Miranda (by Turner) in the 1970s.
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Some background of the “SML-family”

- **Standard Meta Language** (SML) was originally designed for theorem proving
  Logic for Computable Functions (Edinburgh LCF)
  Gordon, Milner, Wadsworth (1977)

- High quality compilers, e.g. Standard ML of New Jersey and Moscow ML, based on a *formal semantics*
  Milner, Tofte, Harper, MacQueen 1990 & 1997

- SML-like systems (SML, OCAML, F#, ...) have now applications far away from its origins
  Compilers, Artificial Intelligence, Web-applications, Financial sector, ...

- F# is now integrated in the .net environment

- Declarative aspects are sneaking into more "main stream languages"

- Often used to teach high-level programming concepts
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Overview of the course

- Functional programming concepts and techniques
- A model-based programming approach using a functional language with a strong type system.
- Program correctness, including structural induction and well-founded induction

Fun with a variety of applications, such as
- a library for piecewise linear curves – with applications
- a Sudoku solver
- an interpreter for a simple programming language
- a lambda-calculus interpreter
- a model checker for CTL – a temporal logic
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A major goal

Teach **abstraction** (not a concrete programming language)

- Modelling
- Design
- Programming

Why?

More complex problems can be solved in an succinct, elegant and understandable manner

How?

Solving a broad class of problems showing the applicability of the theory, concepts, techniques and tools.

Functional programming techniques once mastered are useful for the design of programs in other programming paradigms as well.
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F# supports

- Functions as first class citizens
  - Structured values like lists, trees, ...
  - Strong and flexible type discipline, including type inference and polymorphism
  - Imperative and object-oriented programming assignments, loops, arrays, objects, Input/Output, etc.

Programming as a modelling discipline
- High-level programming, declarative programming, executable declarative specifications B, Z, VDM, RAISE
- Fast time-to-market
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Course context

**Prerequisites for 02157**: Programming in an imperative/object-oriented language, discrete mathematics, algorithms and data structure, as obtained, for example, from the bachelor programme in Software Technology.

Successor course of 02157:

**02257 Applied functional programming**  
3-weeks period January

An extension of 02157 that aims at an effective use of functional programming in connection with courses and projects at the M.Sc. programme in computer science and engineering, and in industrial applications.

- Computer science applications. Interpreter for a programming language, for example.
- “Practical applications”. Involving a database, for example.
- Functional pearls. Monadic parsing, for example.
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Overview of Getting Started

Main functional ingredients of F#:

- The interactive environment
- Values, expressions, types, patterns
- Declarations of values and recursive functions
- Binding, environment and evaluation
- Type inference

**GOAL:** By the end of this first part you have constructed succinct, elegant and understandable F# programs, e.g. for

- \( \text{sum}(m, n) = \sum_{i=m}^{n} i \)
- Fibonacci numbers \( F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2} \)
- Binomial coefficients \( \binom{n}{k} \)
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The Interactive Environment

\[ 2 \times 3 + 4;; \]
\[ \text{val it : int = 10} \]

\[ \begin{array}{c}
\text{Input to the F# system} \\
\text{Answer from the F# system}
\end{array} \]

- The keyword `val` indicates a value is computed
- The integer 10 is the computed value
- `int` is the type of the computed value
- The identifier `it` names the (last) computed value

The notion binding explains which entities are named by identifiers.

\[ \text{it} \mapsto 10 \]

reads: “it is bound to 10”
The Interactive Environment

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⇐ Input to the F# system
⇐ Answer from the F# system

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Functional Programming

Michael R. Hansen

The Interactive Environment

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\[ \text{it} \mapsto 10 \]
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The Interactive Environment

\[2 \times 3 + 4;;\]
\[
\text{val } \textit{it} : \text{int } = 10
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\[\leftarrow \text{ Answer from the F# system}\]

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- The \textit{integer} 10 is the computed value
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- The \textit{identifier} \texttt{it} names the (last) computed value

The notion \textit{binding} explains which entities are named by identifiers.

\[\texttt{it} \mapsto 10 \quad \text{reads: } \texttt{it} \text{ is bound to 10}\]
Value Declarations

A value declaration has the form: \( \text{let } \text{identifier } = \text{expression} \)

\[
\begin{align*}
\text{let } \text{price } &= 25 \times 5; \\
\text{val } \text{price : int } &= 125
\end{align*}
\]

\( \Rightarrow \) A declaration as input

\( \Rightarrow \) Answer from the F# system

The effect of a declaration is a binding: \( \text{price } \mapsto 125 \)

Bound identifiers can be used in expressions and declarations, e.g.

\[
\begin{align*}
\text{let } \text{newPrice } &= 2 \times \text{price}; \\
\text{val } \text{newPrice : int } &= 250
\end{align*}
\]

\[
\begin{align*}
\text{newPrice } &> 500; \\
\text{val } \text{it : bool } &= \text{false}
\end{align*}
\]

A collection of bindings

\[
\begin{bmatrix}
\text{price } & \mapsto & 125 \\
\text{newPrice } & \mapsto & 250 \\
\text{it } & \mapsto & \text{false}
\end{bmatrix}
\]

is called an environment
Value Declarations

A value declaration has the form: \texttt{let identifier = expression}

\begin{verbatim}
let price = 25 * 5;;
val price : int = 125
\end{verbatim}

\textit{⇐ A declaration as input}

\textit{⇐ Answer from the F# system}

The effect of a declaration is a binding: \texttt{price} $\mapsto$ 125

Bound identifiers can be used in expressions and declarations, e.g.

\begin{verbatim}
let newPrice = 2*price;;
val newPrice : int = 250

newPrice > 500;;
val it : bool = false
\end{verbatim}

A collection of bindings

\begin{verbatim}
[ price $\mapsto$ 125 \\
  newPrice $\mapsto$ 250 \\
  it $\mapsto$ false ]
\end{verbatim}

is called an environment
Value Declarations

A value declaration has the form: \texttt{let identifier = expression}

\begin{verbatim}
let price = 25 * 5;;  \quad \Leftarrow \text{A declaration as input}
val price : int = 125  \quad \Leftarrow \text{Answer from the F\# system}
\end{verbatim}

The effect of a declaration is a binding: \texttt{price} \mapsto 125

Bound identifiers can be used in expressions and declarations, e.g.

\begin{verbatim}
let newPrice = 2*price;;  \quad \text{A collection of bindings}
val newPrice : int = 250

newPrice > 500;;
val it : bool = false
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which is called an environment
Function Declarations 1: \( \text{let } f \, x = e \)

Declaration of the circle area function:

\[
\text{let } \text{circleArea } r = \text{System.Math.PI } \times r \times r;;
\]

- \text{System.Math} is a program library
- \text{PI} is an identifier (with type \text{float}) for \( \pi \) in \text{System.Math}

The type is \text{automatically inferred} in the answer:

\[
\text{val circleArea : float } \rightarrow \text{float}
\]

Applications of the function:

\[
\text{circleArea } 1.0;; \quad (* \, \text{this is a comment } *)
\]
\[
\text{val it : float } = 3.141592654
\]

\[
\text{circleArea}(3.2) ;; \quad // \, \text{A comment: optional brackets}
\]
\[
\text{val it : float } = 32.16990877
\]
Function Declarations 1: $\text{let } f \ x = e$

Declaration of the circle area function:

```plaintext
let circleArea r = System.Math.PI * r * r;;
```

- `System.Math` is a program library
- `PI` is an identifier (with type `float`) for $\pi$ in `System.Math`

The type is **automatically inferred** in the answer:

```plaintext
val circleArea : float -> float
```

Applications of the function:

```plaintext
circleArea 1.0;; (* this is a comment *)
val it : float = 3.141592654

circleArea(3.2);; // A comment: optional brackets
val it : float = 32.16990877
```
Function Declarations 1:  \( \text{let } f \ x = e \)

Declaration of the circle area function:

\[
\text{let circleArea } r = \text{System.Math.PI } \ast r \ast r;
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- \text{System.Math} is a program library
- \text{PI} is an identifier (with type \text{float}) for \( \pi \) in \text{System.Math}

The type is \text{automatically inferred} in the answer:

\[
\text{val circleArea : float } \rightarrow \text{float}
\]

Applications of the function:

\[
\text{circleArea 1.0;; (\ast \text{this is a comment } \ast)}
\]
\[
\text{val it : float } = 3.141592654
\]

\[
\text{circleArea(3.2);};; \text{\ // A comment: optional brackets}
\]
\[
\text{val it : float } = 32.16990877
\]
Anonymous functions: by example

An anonymous function computing the number of days in a month:

```ocaml
function
| 1  -> 31  // January
| 2  -> 28  // February // not a leap year
| 3  -> 31  // March
| 4  -> 30  // April
| 5  -> 31  // May
| 6  -> 30  // June
| 7  -> 31  // July
| 8  -> 31  // August
| 9  -> 30  // September
|10  -> 31  // October
|11  -> 30  // November
|12  -> 31;; // December

... warning ... Incomplete pattern matches ... 
```

```ocaml
val it : int -> int = <fun:clo@17-2>
```

```ocaml
it 2;;
val it : int = 28
```

A function expression with a pattern for every month
Anonymous functions: by example

An **anonymous function** computing the number of days in a month:

```haskell
function
| 1 -> 31 // January
| 2 -> 28 // February // not a leap year
| 3 -> 31 // March
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| 6 -> 30 // June
| 7 -> 31 // July
| 8 -> 31 // August
| 9 -> 30 // September
| 10 -> 31 // October
| 11 -> 30 // November
| 12 -> 31;; // December
... warning ... Incomplete pattern matches ...
val it : int -> int = <fun:clo@17-2>

it 2;;
val it : int = 28
```

A function expression with a pattern for every month
Anonymous functions: by example (1)

An **anonymous function** computing the number of days in a month:

```ml
function
| 1 -> 31 // January
| 2 -> 28 // February // not a leap year
| 3 -> 31 // March
| 4 -> 30 // April
| 5 -> 31 // May
| 6 -> 30 // June
| 7 -> 31 // July
| 8 -> 31 // August
| 9 -> 30 // September
| 10 -> 31 // October
| 11 -> 30 // November
| 12 -> 31;;// December

... warning ... Incomplete pattern matches ...
```

```ml
val it : int -> int = <fun:clo@17-2>
```

```ml
it 2;;
val it : int = 28
```

A function expression with a pattern for every month
Anonymous functions: by example (2)

One wildcard pattern can cover many similar cases:

function
| 2  -> 28 // February
| 4  -> 30 // April
| 6  -> 30 // June
| 9  -> 30 // September
| 11 -> 30 // November
| _  -> 31;; // All other months

An even more succinct definition can be given using an or-pattern:

function
| 2  -> 28 // February
| 4|6|9|11 -> 30 // April, June, September, November
| _  -> 31 // All other months
;;
Anonymous functions: by example  

One *wildcard pattern* can cover many similar cases:

```plaintext
function
  | 2  -> 28  // February
  | 4  -> 30  // April
  | 6  -> 30  // June
  | 9  -> 30  // September
  | 11 -> 30  // November
  | _  -> 31;  // All other months
```

An even more succinct definition can be given using an *or-pattern*:

```plaintext
function
  | 2  -> 28  // February
  | 4|6|9|11 -> 30  // April, June, September, November
  | _  -> 31  // All other months
```

;;
Recursion. Example \( n! = 1 \cdot 2 \cdot \ldots \cdot n, \ n \geq 0 \)

Mathematical definition: 

\[
\begin{align*}
0! & = 1 \\
n! & = n \cdot (n - 1)!, \quad \text{for } n > 0
\end{align*}
\]

recursion formula

Computation:

\[
\begin{align*}
3! & = 3 \cdot (3 - 1)! \quad (ii) \\
& = 3 \cdot 2 \cdot (2 - 1)! \quad (ii) \\
& = 3 \cdot 2 \cdot 1 \cdot (1 - 1)! \quad (ii) \\
& = 3 \cdot 2 \cdot 1 \cdot 1 \quad (i) \\
& = 6
\end{align*}
\]
Recursion. Example $n! = 1 \cdot 2 \cdot \ldots \cdot n$, $n \geq 0$

Mathematical definition:

\begin{align*}
0! &= 1 \\
(\text{recursion formula}) \\
(\text{ii}) \\
n! &= n \cdot (n - 1)!, \quad \text{for } n > 0
\end{align*}

Computation:

\begin{align*}
3! &= 3 \cdot (3 - 1)! \\
&= 3 \cdot 2 \cdot (2 - 1)! \\
&= 3 \cdot 2 \cdot 1 \cdot (1 - 1)! \\
&= 3 \cdot 2 \cdot 1 \cdot 1 \\
&= 6
\end{align*}
Recursive declaration. Example $n!$

Function declaration:

```
let rec fact = function
    | 0  -> 1       (* i *)
    | n  -> n * fact(n-1);;

val fact : int -> int
```

Evaluation:

```
fact(3)
⇝ 3 * fact(3 - 1)  (ii) [n ↦→ 3]
⇝ 3 * 2 * fact(2 - 1) (ii) [n ↦→ 2]
⇝ 3 * 2 * 1 * fact(1 - 1) (ii) [n ↦→ 1]
⇝ 3 * 2 * 1 * 1   (i) [n ↦→ 0]
⇝ 6
```

$e_1 \Rightarrow e_2$ reads: $e_1$ evaluates to $e_2$
Recursive declaration. Example $n!$

Function declaration:

```plaintext
let rec fact = function
| 0 -> 1  (* i *)
| n -> n * fact(n-1);; (* ii *)
val fact : int -> int
```

Evaluation:

```
fact(3)
\[\Rightarrow 3 \times fact(3-1) \quad (ii) \quad [n \mapsto 3]
\]
\[\Rightarrow 3 \times 2 \times fact(2-1) \quad (ii) \quad [n \mapsto 2]
\]
\[\Rightarrow 3 \times 2 \times 1 \times fact(1-1) \quad (ii) \quad [n \mapsto 1]
\]
\[\Rightarrow 3 \times 2 \times 1 \times 1 \quad (i) \quad [n \mapsto 0]
\]
\[\Rightarrow 6\]
```

\[e_1 \Rightarrow e_2\] reads: $e_1$ evaluates to $e_2$
Recursive declaration. Example $n!$

Function declaration:

```ml
let rec fact = function
| 0 -> 1
| n -> n * fact(n-1);;

val fact : int -> int
```

Evaluation:

\[
\begin{align*}
\text{fact}(3) & \Rightarrow 3 \cdot \text{fact}(2) \\
& \Rightarrow 3 \cdot 2 \cdot \text{fact}(1) \\
& \Rightarrow 3 \cdot 2 \cdot 1 \cdot \text{fact}(0) \\
& \Rightarrow 3 \cdot 2 \cdot 1 \cdot 1 \\
& \Rightarrow 6
\end{align*}
\]

$e_1 \Rightarrow e_2$ reads: $e_1$ evaluates to $e_2$
Recursive declaration. Example $n!$

Function declaration:

```plaintext
let rec fact = function
    | 0 -> 1               (* i *)
    | n -> n * fact(n-1);; (* ii *)
val fact : int -> int
```

Evaluation:

\[
\begin{align*}
\text{fact}(3) & \rightarrow 3 \cdot \text{fact}(2) \\
               & \rightarrow 3 \cdot 2 \cdot \text{fact}(1) \\
               & \rightarrow 3 \cdot 2 \cdot 1 \cdot \text{fact}(0) \\
               & \rightarrow 6
\end{align*}
\]

$e_1 \rightarrow e_2$ reads: $e_1$ evaluates to $e_2$
Recursive declaration. Example \( n! \)

Function declaration:

\[
\text{let rec fact = function}
\begin{align*}
| & 0 \to 1 \quad (* \ i \ *) \\
| & n \to n \ast \text{fact}(n-1);; \quad (* \ ii \ *)
\end{align*}
\]

\text{val fact : int -> int}

Evaluation:

\[
\text{fact}(3)
\begin{align*}
\leadsto & \ 3 \ast \text{fact}(3-1) \quad (ii) \quad [n \mapsto 3] \\
\leadsto & \ 3 \ast 2 \ast \text{fact}(2-1) \quad (ii) \quad [n \mapsto 2] \\
\leadsto & \ 3 \ast 2 \ast 1 \ast \text{fact}(1-1) \quad (ii) \quad [n \mapsto 1] \\
\leadsto & \ 3 \ast 2 \ast 1 \ast 1 \quad (i) \quad [n \mapsto 0] \\
\leadsto & \ 6
\end{align*}
\]

\( e_1 \leadsto e_2 \) reads: \( e_1 \) evaluates to \( e_2 \)
Recursive declaration. Example $n!$

Function declaration:

```ocaml
let rec fact = function
| 0 -> 1
| n -> n * fact(n-1);

val fact : int -> int
```

Evaluation:

```
fact(3)    ⇝  3 * fact(3 - 1)  (ii) [n ↦ 3]
            ⇝  3 * 2 * fact(2 - 1) (ii) [n ↦ 2]
            ⇝  3 * 2 * 1 * fact(1 - 1) (ii) [n ↦ 1]
            ⇝  3 * 2 * 1 * 1  (i)  [n ↦ 0]
            ⇝  6
```

$e_1 \leadsto e_2$ reads: $e_1$ evaluates to $e_2$
Recursion. Example $x^n = x \cdot \ldots \cdot x$, $n$ occurrences of $x$

Mathematical definition:  

\[
\begin{align*}
x^0 &= 1 \\
x^n &= x \cdot x^{n-1}, & \text{for } n > 0
\end{align*}
\]

Function declaration:

```plaintext
let rec power = function
  | (_,0) -> 1.0
  | (x,n) -> x * power(x,n-1)
```

Patterns:

- \((_,0)\) matches any pair of the form \((x,0)\).
- The wildcard pattern \(_\) matches any value.
- \((x,n)\) matches any pair \((u,i)\) yielding the bindings \(x \mapsto u, n \mapsto i\).
Recursion. Example $x^n = x \cdot \ldots \cdot x$, $n$ occurrences of $x$

Mathematical definition:

\[
\begin{align*}
  x^0 &= 1 \\
  x^n &= x \cdot x^{n-1}, \quad \text{for } n > 0
\end{align*}
\]

Function declaration:

```plaintext
let rec power = function
  | (_, 0) -> 1.0  (* 1 *)
  | (x, n) -> x * power(x, n-1)  (* 2 *)
```

Patterns:

- $(\_, 0)$ matches any pair of the form $(x, 0)$.
- The wildcard pattern $\_\_$ matches any value.
- $(x, n)$ matches any pair $(u, i)$ yielding the bindings
  \[
  x \mapsto u, \ n \mapsto i
  \]
Recursion. Example $x^n = x \cdot \ldots \cdot x$, $n$ occurrences of $x$

Mathematical definition: 

\begin{align*}
  x^0 &= 1 \\
  x^n &= x \cdot x^{n-1}, \text{ for } n > 0
\end{align*} 

Function declaration:

\begin{verbatim}
let rec power = function
  | (_,0) -> 1.0
  | (x,n) -> x * power(x,n-1)
\end{verbatim}

Patterns:

\begin{itemize}
  \item $(\_, 0)$ matches any pair of the form $(x, 0)$. The \texttt{wildcard} pattern \_ matches any value.
  \item $(x, n)$ matches any pair $(u, i)$ \texttt{yielding} the bindings $x \mapsto u$, $n \mapsto i$
\end{itemize}
Evaluation. Example: \( \text{power}(4.0, 2) \)

**Function declaration:**

```ml
let rec power = function
    | (_,0) -> 1.0
    | (x,n) -> x * power(x,n-1)
```

**Evaluation:**

\[
\begin{align*}
\text{power}(4.0,2) & \Rightarrow 4.0 \ast \text{power}(4.0,2-1) & \text{Clause 2, } [x \mapsto 4.0, n \mapsto 2] \\
\Rightarrow 4.0 \ast \text{power}(4.0,1) & \Rightarrow 4.0 \ast (4.0 \ast \text{power}(4.0,1-1)) & \text{Clause 2, } [x \mapsto 4.0, n \mapsto 1] \\
\Rightarrow 4.0 \ast (4.0 \ast \text{power}(4.0,0)) & \Rightarrow 4.0 \ast (4.0 \ast 1) & \text{Clause 1} \\
\Rightarrow 16.0 & 
\end{align*}
\]
If-then-else expressions

Form:

\[
\text{if } b \text{ then } e_1 \text{ else } e_2
\]

Evaluation rules:

\[
\begin{align*}
\text{if true then } e_1 \text{ else } e_2 & \Rightarrow e_1 \\
\text{if false then } e_1 \text{ else } e_2 & \Rightarrow e_2
\end{align*}
\]

Alternative declarations:

\[
\begin{align*}
\text{let rec } \text{fact} \ n & = \text{if } n=0 \text{ then } 1 \\
& \quad \text{else } n \times \text{fact}(n-1); \\
\text{let rec } \text{power}(x,n) & = \text{if } n=0 \text{ then } 1.0 \\
& \quad \text{else } x \times \text{power}(x,n-1);
\end{align*}
\]

Use of patterns usually gives more understandable programs
If-then-else expressions

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Alternative declarations:

\[
\begin{align*}
\text{let rec fact } n & = \text{ if } n=0 \text{ then } 1 \\
& \quad \text{else } n \times \text{ fact} (n-1) ; \\
\text{let rec power}(x, n) & = \text{ if } n=0 \text{ then } 1.0 \\
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\]

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\]

Use of patterns usually gives more understandable programs
Booleans

Type name `bool`

Values `false, true`

<table>
<thead>
<tr>
<th>Operator</th>
<th>Type</th>
<th>Description</th>
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<tbody>
<tr>
<td><code>not</code></td>
<td><code>bool -&gt; bool</code></td>
<td>negation</td>
</tr>
</tbody>
</table>

- `not true = false`
- `not false = true`

Expressions

- `e_1 && e_2` — conjunction `e_1 \land e_2`
- `e_1 || e_2` — disjunction `e_1 \lor e_2`

— are lazily evaluated, e.g.

- `1 < 2 || 5 / 0 = 1 → true`

Precedence: `&&` has higher than `||`
Booleans

Type name: bool
Values: false, true

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Expressions:

\[ e_1 \land e_2 \quad \text{“conjunction } e_1 \land e_2 \text{”} \]
\[ e_1 \lor e_2 \quad \text{“disjunction } e_1 \lor e_2 \text{”} \]

— are lazily evaluated, e.g.
\[ 1 < 2 \lor 5 / 0 = 1 \rightarrow \text{true} \]

Precedence: \&\& has higher than \lor
Booleans

Type name `bool`

Values `false`, `true`

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Expressions

\[ e_1 \&\& e_2 \]  
\[ e_1 \|\| e_2 \]  

“conjunction \( e_1 \land e_2 \)”

“disjunction \( e_1 \lor e_2 \)”

— are lazily evaluated, e.g.

\[ 1<2 \|\| 5/0 = 1 \Rightarrow true \]

Precedence: `&&` has higher than `||`
Strings

**Type name** `string`  

**Values** "abcd", " ", "", "123\"321"  
(escape sequence for ")

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**Examples**

- "auto" < "car";
  > val it = true : bool
- "abc"+"de";
  > val it = "abcde": string
- String.length("abc"^"def");
  > val it = 6 : int
- string(6+18);
  > val it = "24": string
Strings

Type name string

Values "abcd", " ", "", "123" \"321" (escape sequence for "")

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Examples

- "auto" < "car";
  > val it = true : bool

- "abc" + "de";
  > val it = "abcde" : string

- String.length("abc"^"def");
  > val it = 6 : int

- string(6+18);
  > val it = "24" : string
Strings

Type name `string`

Values "abcd", " ", "", "123\"321" (escape sequence for " )

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Types — every expression has a type $e : \tau$

Basic types:

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Pairs:
If $e_1 : \tau_1$ and $e_2 : \tau_2$
then $(e_1, e_2) : \tau_1 \times \tau_2$    pair (tuple) type constructor

Functions:
if $f : \tau_1 \rightarrow \tau_2$ and $a : \tau_1$
then $f(a) : \tau_2$

Examples:

$(4.0, 2) : \text{float} \times \text{int}$
power: \text{float} \times \text{int} \rightarrow \text{float} * has higher precedence than $\rightarrow$
power(4.0, 2) : \text{float}
Types — every expression has a type \( e : \tau \)

### Basic types:

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### Pairs:

If \( e_1 : \tau_1 \) and \( e_2 : \tau_2 \) then \((e_1, e_2) : \tau_1 \ast \tau_2\) pair (tuple) type constructor

### Functions:

If \( f : \tau_1 \rightarrow \tau_2 \) and \( a : \tau_1 \) then \( f(a) : \tau_2 \) function type constructor

### Examples:

\[(4.0, 2) : \text{float} \ast \text{int}\]
\[\text{power} : \text{float} \ast \text{int} \rightarrow \text{float}\]
\[\text{power}(4.0, 2) : \text{float}\]
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### Functions:

if \( f : \tau_1 \rightarrow \tau_2 \) and \( a : \tau_1 \) then \( f(a) : \tau_2 \)

### Examples:

\( (4.0, 2) : \text{float*int} \)

\( \text{power} : \text{float*int} \rightarrow \text{float} \)

\( \text{power}(4.0, 2) : \text{float} \)

\* has higher precedence than ->
Types — every expression has a type \( e : \tau \)

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Pairs:
If \( e_1 : \tau_1 \) and \( e_2 : \tau_2 \) then \( (e_1, e_2) : \tau_1 \ast \tau_2 \) pair (tuple) type constructor

Functions:
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Examples:

\( (4.0, 2) : \text{float} \ast \text{int} \)

\( \text{power} : \text{float} \ast \text{int} \rightarrow \text{float} \) * has higher precedence that \( \rightarrow \)

\( \text{power}(4.0, 2) : \text{float} \)
Type inference: power

```fsharp
let rec power = function
    | (_,0) -> 1.0          (* 1 *)
    | (x,n) -> x * power(x,n-1) (* 2 *)
```

- The type of the function must have the form: $\tau_1 \times \tau_2 \rightarrow \tau_3$, because argument is a pair.
- $\tau_3 = \text{float}$ because $1.0: \text{float}$ (Clause 1, function value.)
- $\tau_2 = \text{int}$ because $0: \text{int}$.
- $x \times \text{power}(x, n-1): \text{float}$, because $\tau_3 = \text{float}$.
- Multiplication can have $\text{int} \times \text{int} \rightarrow \text{int}$ or $\text{float} \times \text{float} \rightarrow \text{float}$ as types, but no “mixture” of $\text{int}$ and $\text{float}$.
- Therefore $x: \text{float}$ and $\tau_1 = \text{float}$.

The F# system determines the type $\text{float} \times \text{int} \rightarrow \text{float}$.
Type inference: `power`

```fsharp
let rec power = function
    | (_,0) -> 1.0 (* 1 *)
    | (x,n) -> x * power(x,n-1) (* 2 *)
```

- The type of the function must have the form: $\tau_1 \times \tau_2 \rightarrow \tau_3$, because argument is a pair.
  - $\tau_3 = \text{float}$ because $1.0 : \text{float}$ (Clause 1, function value.)
  - $\tau_2 = \text{int}$ because $0 : \text{int}$.
  - $x \times \text{power}(x,n-1) : \text{float}$, because $\tau_3 = \text{float}$.
  - Multiplication can have
    - int*int -> int or float*float -> float
    as types, but no “mixture” of int and float
  - Therefore $x : \text{float}$ and $\tau_1 = \text{float}$.

The F# system determines the type `float*int -> float`
Type inference: \texttt{power}

\begin{verbatim}
let rec power = function
    | (_,0) -> 1.0 (* 1 *)
    | (x,n) -> x * power(x,n-1) (* 2 *)
\end{verbatim}

- The type of the function must have the form: $\tau_1 \times \tau_2 \rightarrow \tau_3$, because argument is a pair.
- $\tau_3 = \texttt{float}$ because $1.0:\texttt{float}$ (Clause 1, function value.)
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- $x \times \text{power}(x,n-1):\texttt{float}$, because $\tau_3 = \texttt{float}$.
- Multiplication can have
  \[ \text{int} \times \text{int} \rightarrow \text{int} \text{ or } \texttt{float} \times \texttt{float} \rightarrow \texttt{float} \]
  as types, but no “mixture” of \texttt{int} and \texttt{float}
- Therefore $x:\texttt{float}$ and $\tau_1=\texttt{float}$.

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Type inference: \texttt{power}

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- \( x*\text{power}(x,n-1):\texttt{float} \), because \( \tau_3 = \texttt{float} \).
- Multiplication can have \( \texttt{int*int} \rightarrow \texttt{int} \) or \( \texttt{float*float} \rightarrow \texttt{float} \) as types, but no “mixture” of \texttt{int} and \texttt{float}.
- Therefore \( x:\texttt{float} \) and \( \tau_1=\texttt{float} \).

The F\# system determines the type \( \texttt{float*int} \rightarrow \texttt{float} \)
Type inference: `power`

```
let rec power = function
    | (_,0) -> 1.0               (* 1 *)
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- Therefore \( x: \text{float} \) and \( \tau_1 = \text{float} \).

The F# system determines the type \text{float} \times \text{int} \rightarrow \text{float}.
Type inference: power

\[
\text{let rec power = function}
\begin{align*}
\text{    | (_,0) -> 1.0} & \quad \text{(\# 1 \#)} \\
\text{    | (x,n) -> x * power(x,n-1)} & \quad \text{(\# 2 \#)}
\end{align*}
\]

- The type of the function must have the form: \( \tau_1 \times \tau_2 \rightarrow \tau_3 \), because argument is a pair.
- \( \tau_3 = \text{float} \) because \( 1.0: \text{float} \) (Clause 1, function value.)
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- Therefore \( x: \text{float} \) and \( \tau_1 = \text{float} \).

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- Therefore $x: \text{float}$ and $\tau_1 = \text{float}$.

The F# system determines the type $\text{float} \times \text{int} \rightarrow \text{float}$
Summary

- The interactive environment
- Values, expressions, types, patterns
- Declarations of values and recursive functions
- Binding, environment and evaluation
- Type inference

Breath first round through many concepts aiming at program construction from the first day.

We will go deeper into each of the concepts later in the course.
Summary

- The interactive environment
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- Declarations of values and recursive functions
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Overview

- Lists: values and constructors
- Recursions following the structure of lists

The purpose of this lecture is to give you an (as short as possible) introduction to lists, so that you can solve a problem which can illustrate some of F#’s high-level features.

This part is not intended as a comprehensive presentation on lists, and we will return to the topic again later.
Overview

- Lists: values and constructors
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The purpose of this lecture is to give you an (as short as possible) introduction to lists, so that you can solve a problem which can illustrate some of F#'s high-level features.

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Lists

A list is a finite sequence of elements having the same type:

\[ [v_1; \ldots; v_n] \quad ([]) \text{ is called the empty list} \]

[2;3;6];;
val it : int list = [2; 3; 6]

["a"; "ab"; "abc"; "] ;;
val it : string list = ["a"; "ab"; "abc"; "]

[sin; cos];;
val it : (float->float) list = [<fun:...>; <fun:...>]

[(1,true); (3,true)];;
val it : (int * bool) list = [(1, true); (3, true)]

[[]; [1]; [1;2]];;
val it : int list list = [[]; [1]; [1; 2]]
Lists

A list is a finite sequence of elements having the same type:

\[ v_1; \ldots; v_n \]  

([ ] is called the empty list)

\[
\begin{align*}
\text{[2;3;6];} \\
\text{val it : int list = [2; 3; 6]} \\
\text{["a"; "ab"; "abc"; "]]} \\
\text{val it : string list = ["a"; "ab"; "abc"; "]]} \\
\text{[sin; cos];} \\
\text{val it : (float->float) list = [<fun:...>; <fun:...>]} \\
\text{[(1,true); (3,true)];} \\
\text{val it : (int * bool) list = [(1, true); (3, true)]} \\
\text{[[]; [1]; [1;2]];} \\
\text{val it : int list list = [[]; [1]; [1; 2]]}
\end{align*}
\]
Lists

A list is a finite sequence of elements having the same type:

\[ v_1; \ldots; v_n \]  \hspace{1cm} ([] is called the empty list)

\[ [2; 3; 6]; \]
\texttt{val it : int list = [2; 3; 6]}

\[ ["a"; "ab"; "abc"; "]\];
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\[ [\text{sin}; \cos]; \]
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\[ [(1,\text{true}); (3,\text{true})]; \]
\texttt{val it : (int * bool) list = [(1, true); (3, true)]}

\[ [[]; [1]; [1; 2]]; \]
\texttt{val it : int list list = [[]; [1]; [1; 2]]}
A list is a finite sequence of elements having the same type:

$$[v_1; \ldots; v_n]$$  \[[\ ]\] is called the empty list

\[
\begin{align*}
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\end{align*}
\]

\[
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\]

\[
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\end{align*}
\]

\[
\begin{align*}
[(1, true); (3, true)];; \\
val it : (int \times bool) list = [(1, true); (3, true)]
\end{align*}
\]

\[
\begin{align*}
[[]]; [1]; [1; 2];; \\
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\[\text{[[]; [1]; [1;2]];; val it : int list list = [[]; [1]; [1; 2]]}\]
Trees for lists

A non-empty list \([x_1, x_2, \ldots, x_n]\), \(n \geq 1\), consists of

- a \textit{head} \(x_1\) and
- a \textit{tail} \([x_2, \ldots, x_n]\)

Graph for \([2, 3, 2]\)

Graph for \([2]\)
Trees for lists

A non-empty list \([x_1, x_2, \ldots, x_n]\), \(n \geq 1\), consists of

- a *head* \(x_1\) and
- a *tail* \([x_2, \ldots, x_n]\)

Graph for \([2, 3, 2]\)  
Graph for \([2]\)
List constructors: [ ] and ::

Lists are generated as follows:

- the empty list is a list, designated [ ]
- if \( x \) is an element and \( xs \) is a list, then so is \( x :: xs \) (type consistency)

:: associate to the right, i.e. \( x_1 :: x_2 :: xs \) means \( x_1 :: (x_2 :: xs) \)

Graph for \( x_1 :: x_2 :: xs \)
List constructors: \([\ ]\) and \(::\)

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Graph for \(x_1 :: x_2 :: xs\)
Recursion on lists – a simple example

\[
\text{suml} \ [x_1, x_2, \ldots, x_n] = \sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n = x_1 + \sum_{i=2}^{n} x_i
\]

Constructors are used in list patterns

```ocaml
let rec suml = function
| []    -> 0
| x::xs -> x + suml xs;;
> val suml : int list -> int
```

\[
\text{suml} \ [1;2]
\]

\[
\leadsto 1 + \text{suml} \ [2] \quad (x \mapsto 1 \text{ and } xs \mapsto [2])
\]

\[
\leadsto 1 + (2 + \text{suml} \ []) \quad (x \mapsto 2 \text{ and } xs \mapsto [])
\]

\[
\leadsto 1 + (2 + 0) \quad (\text{the pattern } [] \text{ matches the value } [])
\]

\[
\leadsto 1 + 2
\]

\[
\leadsto 3
\]

Recursion follows the structure of lists
Recursion on lists – a simple example

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\text{suml } [1;2] \\
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\begin{align*}
\text{suml} \ [1;2] & \Rightarrow 1 + \text{suml} \ [2] \\
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& \Rightarrow 1 + 2 \\
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\end{align*}
\]

Recursion follows the structure of lists
Infix functions

It is possible to declare infix functions in F#, i.e. the function symbol is between the arguments.

The prefix function on lists is declared as follows:

```fsharp
let rec (<=.) xs ys =
    match (xs,ys) with
    | ([],_) -> true
    | (_,[]) -> false
    | (x::xs’,y::ys’) -> x=y && xs’ <=. ys’;;
```

```fsharp
[1;2;3] <=. [1;2];;
val it : bool = false
```

- The special way of declaring the function (<=.) xs ys makes <=. an infix operator
- The match (xs,ys) construct allows for branching out on patterns for (xs,ys)

Suitable use of infix functions can increase readability significantly
Infix functions

It is possible to declare *infix functions* in F#, i.e. the function symbol is *between* the arguments.

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**Suitable use of infix functions can increase readability significantly**
Exercises

- `length xs`: the length of the list `xs` (is a predefined function).
- `remove(x, ys)`: removes all occurrences of `x` in the list `ys`

Have fun with your first non-trivial functional program: polynomials represented as lists.
Exercises

- `length xs` : the length of the list `xs` (is a predefined function).
- `remove(x, ys)` : removes all occurrences of `x` in the list `ys`

Have fun with your first non-trivial functional program: polynomials represented as lists