Combining Logics

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Combining reasoning!

- Do we really reason using propositional, quantified, epistemic, alethic, doxastic, temporal, many-valued, fuzzy, intuitionistic, paraconsistent $\cdots$ logics?
- Or we do combine everything, and perhaps more?
- How is really the reasoning in domains like legal reasoning, computer systems, economic reasoning, etc, expressed in terms of elementary concepts?

Project ConsRel - CLE at Campinas

ConsRel: Logical Consequence and Combinations of Logics

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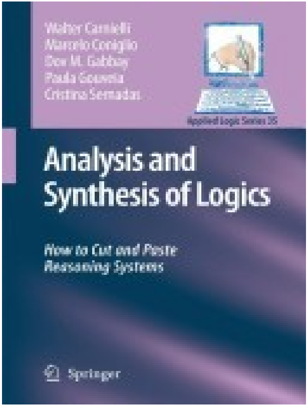
Combining $\times$ decomposing logics

- If we can combine reasoning, or at least combine logics, why not decompose them?
- If a logic is decomposed into “elementary” sublogics, is it possible to recover it by combining such fragments?
- What kind of properties of logics (like completeness, decidability, interpolation properties, axiomatizability, computable efficiency, etc.) can be transferred to their combinations?

A consequence:

General methods for combining logics, lots of examples and some suggested applications.



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Humes's warning from 18th Century

"For as this ought, or ought not, expresses some new relation or affirmation, it is necessary that it should be observed and explained; and at the same time that a reason should be given, for what seems altogether inconceivable, how this new relation can be a deduction from others, which are entirely different from it. But as authors do not commonly use this precaution..."

- Hume was asking for something we may call "bridge principle", without which we seem to be unable to handle combined reasoning...

Hume's 'is'- 'ought' problem

- expressed in "A Treatise of Human Nature" (Book 3, Part 1, Section 1, paragraph 27)...
- ... generated a controversy about the legitimacy of statements that bind factualities to norms
- ... and inaugurated the idea of "bridge principles" as necessary principles for mixed reasoning.

Emergent phenomena?

- spontaneous or hidden bridge principles pose intriguing questions to combined logics;
- bridge principles may spontaneously arise in the operation of combining logics ...
- they may have however, desirable or undesirable consequences for combined reasoning;
- moreover, we also find collapsing and anti-collapsing problems.

Hume's law and combined logics

- In order to perform such a jump from 'is' to 'ought', one might appeal to an explicit "bridge principle", which specifically connects 'is' and 'ought';
- $\alpha \to \bigcirc\alpha$ is a simple bridge principle representing 'is-ought';
- Are bridge principles *necessary*?

Where is the problem?

- Bridge principles may not be necessarily analytical, in the sense that they might not be true because of the meaning of their symbols alone;
- Yet, bridge principles in a broad sense may appear spontaneously when combining logics;
- How can something non-analytical appear analytically?

Bridge principles: strict and wide

Definition

G. Schurz: An axiom schema *A* is a bridge principle iff *A* contains at least one schematic letter which has at least one occurrence within the scope of a deontic "obligation" operator $\bigcirc$, and at least one occurrence outside the scope of any $\bigcirc$.

- 'Ought-implies-can': $\bigcirc\alpha \rightarrow \Diamond\alpha$;
- But this can be widely extended beyond modal logics.

Splicing (combining) versus splitting (decomposing) logics

- Most relevant methods: fusion, product of modal logics and fibring.
- Paradigmatic splicing method: algebraic fribring.
- Used in computer science and knowledge representation; used less by logicians, and very timidly by philosophers.
- Integrating several reasoning modules: temporal, epistemic, alethic. and more...
- Paradigmatic splitting method: possible-translations semantics.

Conceptual issues

- Is there a unique “correct” logic (monism), or many (pluralism), or none (instrumentalism)?
- Does composition of logics restore the unity from “fragments”, or create more specimens, expanding the “pluralism”?
- Philosophers of logic should take combined logics into account!

Fusion of normal modal logics

- Introduced by R. Thomason in 1984 (but anticipated, in examples of fusing alethic and deontic modalities, by M. Fitting in 1968);
- The fusion of $\mathcal{L}_1$ with $\mathcal{L}_2$ is the bimodal logic $\mathcal{L}$, defined over a language with two boxes. The rest of the connectives are assumed to be classical, and so they are shared by $\mathcal{L}_1$ and $\mathcal{L}_2$.

Fusion, semantic level

Definition

1. Semantic fusion of $\mathcal{L}_1$ (with $\Box_1$) and $\mathcal{L}_2$ (with $\Box_2$): bimodal $\mathcal{L}$ with $\Box_1$ and $\Box_2$ characterized by general Kripke frames $\langle W, R_1, R_2 \rangle$ with a set of worlds W and two relations R_1 and R_2 over W.

2. $\langle W, R_1, R_2 \rangle$ is such that $\langle W, R_1 \rangle$ and $\langle W, R_2 \rangle$ are Kripke frames for $\mathcal{L}_1$ and $\mathcal{L}_2$.

- The Hilbert calculi of $\mathcal{L}$ is the merging of the axioms and rules of both logics (but in $\mathcal{L}$ they can be instantiated with mixed formulas).

Products of modal logics

- Introduced by K. Segerberg in 1973 and by V. Shehtman in 1978 (in two papers with the same title...).

Definition

Product of $\mathcal{L}_1$ (with $\Box_1$) and $\mathcal{L}_2$ (with $\Box_2$) is also a bimodal logic $\mathcal{L}$ with $\Box_1$ and $\Box_2$ is $\mathcal{L}$, characterized by all Kripke models $\langle W_1 \times W_2, \overline{R}_1, \overline{R}_2, V_1 \times V_2 \rangle$

Products, semantic level

Definition

1. $\overline{R}_i \subseteq (W_1 \times W_2) \times (W_1 \times W_2)$ is defined from R_i as:
   - $(w_1, w_2)\overline{R}_1(u_1, u_2)$ iff $w_1 R_1 u_1$ and $w_2 = u_2$;
   - $(w_1, w_2)\overline{R}_2(u_1, u_2)$ iff $w_2 R_2 u_2$ and $w_1 = u_1$.
2. $V_1 \times V_2 : \mathbb{P} \longrightarrow \wp(W_1 \times W_2)$ is the mapping $(V_1 \times V_2)(p) = V_1(p) \times V_2(p)$, such that $V_i : \mathbb{P} \longrightarrow \wp(W_i)$ is a valuation in $\langle W_i, R_i, V_i \rangle$

Fibring modal logics: putting yourself in somebody else's shoes

- D. Gabbay, 1996; also generates bi-modal logics.

Definition

1. Given $\mathcal{L}_1$ and $\mathcal{L}_2$ and their Kripke models, take transfer maps: h_1 from worlds of models $\mathcal{M}_1$ of $\mathcal{L}_1$ into models $\mathcal{M}_2$ of $\mathcal{L}_2$, and h_2 vice-versa.
2. A Kripke model of $\mathcal{L}_1$ evaluates $\Box_2\varphi$ at the actual world w_1 by transferring the validity checking to checking $\Box_2\varphi$ within the Kripke model $h_1(w_1)$ at its actual world.
3. Vice-versa for $\Box_1\varphi$ within a Kripke model for $\mathcal{L}_2$.

Categorial fibring

- Designed to overcome the limitations of fusion, product and fribing (all them for modal logics only).
- Proposed by A. Sernadas, C. Sernadas and C. Caleiro in 1999.

Definition

1. The categorial fibring of $\mathcal{L}_1$ and $\mathcal{L}_2$ is the *least* logic $\mathcal{L}$ over the combined language which extends $\mathcal{L}_1$ and $\mathcal{L}_2$.
2. It is the *coproduct* of $\mathcal{L}_1$ and $\mathcal{L}_2$ in the category of logics and their morphisms.

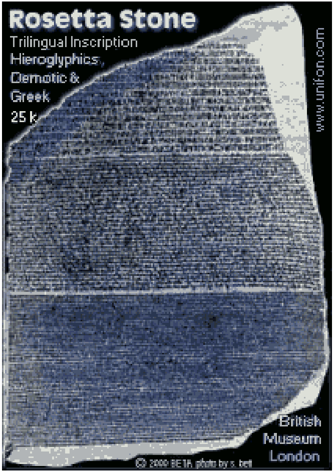
Metafibring

- Categorial fibring is universal in the sense of category theory, and generalize fusion and fibring;
- Metafibring, a restriction proposed by M. Coniglio in 2005, a categorial construction where morphisms preserve meta-properties of the logics.
- Metafibring permits a logic to be recovered from its fragments (that is, from logics defined over sub-languages).

Possible-translations semantics

- Idealized for paraconsistent logics, specially for Logics of Formal Inconsistency (LFIs).
- However, they are applicable in several other cases.
- They constitute a most general method for decomposing (splitting) logics.

Rosetta stone: how translations work

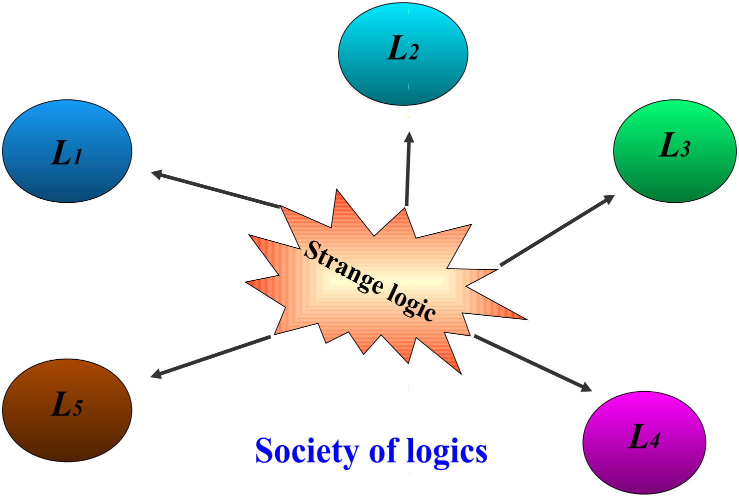


Hieroglyphics

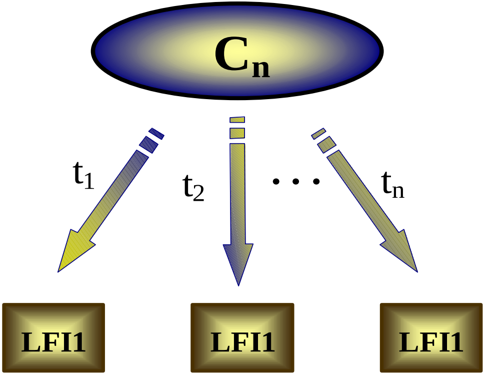
Demotic

Greek

Translations acting together



Possible-translations semantics for LFIs



Self-generated bridge principles

- "Bridge principles" wide sense: interactions (i.e., derivations) among distinct logic operators which are not instances of valid derivations in the logics being combined.
- So, e.g., in the logic $\mathcal{L}$ obtained by combining $\wedge$ with $\vee$ via metafibring:
- $p \wedge r \vdash (p \wedge r) \vee q$ is not a bridge principle, as it is derived by substitution from $p \vdash p \vee q$, valid in the logic of $\vee$.

Self-generated bridge principles

- However, $p \wedge (q \vee r) \vdash (p \wedge q) \vee r$ (distributivity of $\wedge$ over $\vee$) is not obtained in the logic of $\wedge$, nor in the logic of $\vee$, but appears spontaneously in the combination! (Béziau & Coniglio).
- Another case of spontaneous emergence of a bridge principle: in the metafibring of the logic of classical $\neg$ and the logic of classical $\vee$, the law of excluded-middle $p \vee \neg p$ emerges unavoidably in the combined logic (Coniglio).

What is the meaning of spontaneous laws?

- Also, in the metafibring of the logics of classical $\neg$ and classical $\rightarrow$, the Principle of Pseudo-Scotus $p \rightarrow (\neg p \rightarrow q)$ emerges unavoidably (Coniglio).
- The first case obtains the full $\vee$-$\wedge$fragment of **PC**.
- The second and third cases obtain full **PC**.
- In all cases, the bridge principles arise spontaneously due to the nature of the combination process.
- Are they expected? No, from the point of view of intuitionists or paraconsistentists!

Self-generated, but undesirable

- Self-generated bridge principles in the product of two normal modal logics $\mathcal{L}_1$ and $\mathcal{L}_2$ (whose languages have, respectively, $\Box_1$ and $\Diamond_1$, and $\Box_2$ and $\Diamond_2$) (Gabbay):
  - $(\Box_1\Box_2\alpha \leftrightarrow \Box_2\Box_1\alpha)$ $\Box$-commutativity;
  - $(\Diamond_1\Diamond_2\alpha \leftrightarrow \Diamond_2\Diamond_1\alpha)$ $\Diamond$-commutativity;
  - $(\Diamond_1\Box_2\alpha \rightarrow \Box_2\Diamond_1\alpha)$ $(1, 2)$-Church-Rosser;
  - $(\Diamond_2\Box_1\alpha \rightarrow \Box_1\Diamond_2\alpha)$ $(2, 1)$-Church-Rosser.

And more...

- Consequently, other bridge principles will be also derivable:
  - $(\Diamond_1^k \Box_2^m \alpha \rightarrow \Box_2^m \Diamond_1^k \alpha)$ $(1^k, 2^m)$-Church-Rosser property;
  - $(\Diamond_2^k \Box_1^m \alpha \rightarrow \Box_1^m \Diamond_2^k \alpha)$ $(2^k, 1^m)$-Church-Rosser property.

...but undesirable for epistemic logic

- A form of $(1, 2)$-Church-Rosser property for "knowledge" K: $\Diamond K\alpha \to K\Diamond\alpha$ will emerge spontaneously in the product of any normal modal logics.
- We may inform a previously ignorant person about the following fact p: "there exists an egg-laying mammal" (namely, the platypus).
- So, $\Diamond Kp$ is true (as we may inform her), but $K\Diamond p$ is false (indeed, her ignorance excludes *a priori* the possibility of her knowing about the existence of such an animal).

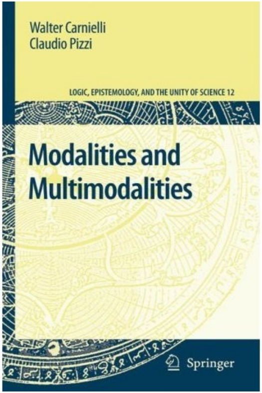
Transgenic logics?

- Interactions are only revealed after a careful semantic analysis, and one has no control on which bridge principles might crop up.
- A side effect: it is not possible to obtain *a priori* a complete Hilbert calculus for products of modal logics.
- Additional bridge principles might have to be explicitly added to ensure completeness.
- Within multimodalities a profusion of bridge principles naturally appears.

Creating monsters

- The *collapsing problem* (D. Gabbay, and independently L. Fariñas del Cerro, A. Herzig, 1996).
- By freely combining **PC** and intuitionistic propositional logic the resulting logic collapses to classical logic: intuitionistic implication becomes classic.
- The collapsing phenomenon–a spontaneous and undesirable bridge principles: $\alpha_1 \to_c \alpha_2 \vdash \alpha_1 \to_i \alpha_2$ and $\alpha_1 \to_i \alpha_2 \vdash \alpha_1 \to_c \alpha_2$ (where $\to_c$ and $\to_i$ are, respectively, **PC** and **HI** implication).

Modalities and Multimodalities a newborn consequence



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