

02424 Week 3

The main purpose of this collection of small exercises is to illustrate various important aspects in relation to estimation techniques in Chapter 2 of the lecture notes and in particular to the likelihood methods. Most of the problems are simplified in order to illustrate the concepts using rather simple analytical techniques, however, in practice numerical methods are often required.

Some of the illustrated concepts are the Minimum Variance Unbiased Estimator (MVUE), Likelihood functions for iid samples, maximum likelihood estimate (MLE), Fisher Information Matrix, Score Function, Cramer-Rao lower bound (CRLB).

Exercise 1

Question 1 and 2 from Exercise 2.1 in the textbook, but with the important change in Question 2 that T_w should be defined as:

$$T_w = wY_1 + (1 - w)10Y_2$$

Exercise 2

Consider a sample consisting of n i.i.d. observations from a Gamma distribution, ie. $Y_i \sim G(\alpha, \beta)$.

Question 1

Explain how the χ^2 distribution is obtained as special cases of the Gamma distribution.

Question 2

Write down the log-likelihood function.

Question 3

Assume now that α is known. Write down the score function.

Question 4

Find the maximum likelihood estimate (MLE) for β .

Exercise 3

Suppose y_1, \dots, y_n are an iid sample from the exponential distribution with density

$$g(y) = \beta^{-1} \exp(-y/\beta) = \lambda \exp(-\lambda y)$$

where λ is the rate – or the inverse scale.

Question 1

Derive the MLE.

Exercise 4

Assume y_1, \dots, y_n are an iid sample from $N(\mu, \sigma^2)$, where μ is assumed known.

Question 1

Write down the log-likelihood function.

Question 2

Find the score function for estimating σ .

Question 3

Find the observed information.

Question 4

Find the expected information.

Question 5

Find the Cramer-Rao lower bound (CRLB) for estimating σ^2 .

Question 6

Do you know any estimator/statistic which has the variance equal to the CRLB?