Modelling the fMRI response using smooth FIR filters

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Introduction

We describe a flexible semi-parametric linear time-invariant convolution model for the fMRI response. It extends the FIR model [1] with a Gaussian process prior over the parameters (the FIR coefficients). Linear models with Poisson, Gamma and Gaussian kernels have few parameters and are not very flexible, e.g., they are not able to model post-activation undershoot. A flexible model such as the FIR filter is able to model more complex linear responses but suffers from unstable (i.e., large variance) estimation in the case where long filters are necessary due to short TRs. Ridge regularization for the FIR filter estimation does not help sufficiently and principal component regression introduces biases that are dependent on the input function (the paradigm). The smooth FIR overcomes the disadvantages of the FIR filter by introducing a regularization that ties the parameters together, enabling a more stable estimation. This is done by modelling the smoothness of the filter by a correlation between the parameters and use that information as a prior in a Bayesian setting.

Method

For a fMRI signal $y(t)$ corresponding to activation $\mathbf{x}$, and a standard assumption of additive Gaussian noise, the linear filter of parameters $\mathbf{w}$ models the fMRI signal as $y(t) = \mathbf{w}^\mathsf{T}\mathbf{x} + \epsilon$. Using the Gaussian process prior on the parameters $\mathbf{w}$, $p(\mathbf{w}) \propto \exp\left[-\mathbf{w}^\mathsf{T}\mathbf{\Sigma}^{-1}\mathbf{w}/\mathbf{2}\right]$, the posterior probability distribution is tractable and the maximum a posteriori (MAP) estimate becomes:

$$\mathbf{w}_{\text{MAP}} = \left[\mathbf{X}^\mathsf{T}\mathbf{X} + \sigma^2\mathbf{\Sigma}^{-1}\right]\mathbf{X}^\mathsf{T}\mathbf{y} \qquad \text{with } \Sigma_{ij} = \nu \exp\left(-\frac{1}{2l^2}(i-j)^2\right) \tag{1}$$

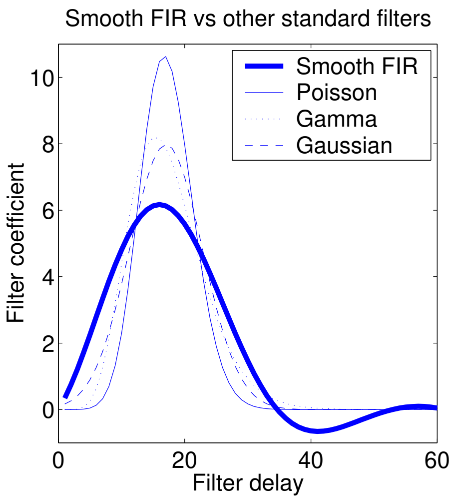
where we model the correlation by a decreasing Gaussian function. If we are only concerned about the MAP estimate, only two hyperparameters need to be set: The ratio σ^2/ν determines the amount of regularization and is equivalent to the ridge parameter in ridge regression; l corresponds to a "characteristic length", i.e. the typical length in which the filter varies. When l goes to zero, we recover ridge regression, while large values of l give a constant filter.

There are several options to set the hyperparameters. Strong prior knowledge can be used in some cases, e.g., setting the characteristic length to a value corresponding to 7sec. It is also possible to use a data oriented adaptive method, such as optimization of the likelihood or posterior of the hyperparameters, the evidence framework [2] or integrate over hyperparameters using Markov Chain Monte Carlo sampling [3]. Finally, the significance of activation can be assessed by calculating the probability that $\mathbf{w} = 0$ lies in a credible region: the "highest posterior density" (HPD) region [4, 5]. Our full approach is described in more detail in [6].

Result and Discussion

We exemplify our model on a fast sampled fMRI data set: TR = 333ms, 8Hz checker board, 128×128 pixels, 10 runs of 150 images.

The figure shows the smooth FIR kernel together with other standard kernels fitted on a single voxel: A more complex linear smooth response can be modelled and the resulting kernel can have features that standard kernels do not exhibit. The algorithm is fast for simple MAP estimation of the parameters, however, integration and to a certain extent, optimization of the hyperparameters are impractical for a whole slice if performed on voxels individually. HPD provides a concept for assessing the significance of activation in this model generating pseudo-p-values that are comparable to the output of traditional statistical tests.



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