Train Composition and Decomposition: Domain and Requirements

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Abstract

The problem is to be tackled is as follows: There is a railway net. Trains travel from station to station. When leaving an intermediate station on a train journey, a train may have its direction reversed. Trains are composed from assemblies. An assembly is a sequence of carriages. At stations trains may have assemblies added (composed) to, or removed (decomposed) from the train. Station tracks may restrict train additions and removals to occur only at either the front, or at the back of a train. Given requirements for trains to provide suitable load (for example passenger) capacity along a journey with varying such demands, the problem is now to plan that trains, during their journeys, have suitable assemblies added to or removed from the train.

We present a standard informal narrative and a formal model of train composition and decomposition, of their planning and effectuation.

We relate this model to the rough sketch description provided by Dr. Leo Kroon of NLS Reisigers.

Contents

1 Domain

T:1, F:1

1.1 Brief Outline of the Formal Model

The end result of our model at this stage is schedule-generating function. The schedules this function generates are such, so that the desired rolling stock changes can be derived from the schedule itself. This function is producing a set of all allowed schedules which satisfy a given passenger statistics:

gen_Sched: Stat1 × RulReg × RS → Sched-**set**
gen_Sched(st,rr,rs) ≡ {sch | sch: Sched • fitting(reform_stat(st),sch)}

F:2

The satisfiability of a given statistics by a schedule is defined by a fitting function:

fitting: Stat1 × Sched → **Bool**

The statistics as entered into the fitting function are in a well formed format. In this format, only the number of passengers travelling between consecutive stations at given times is given. A general non-well-formed statistics may be transformed into a well-formed one through a function made for that purpose:

reform_stat: Stat1′ → Stat1

F:3

Crucial for the definition of this reformation function is the ability to sum over all possible 'trips' within a statistics of which a 'trip' between two given consecutive stations is a part. For that purpose we use a recursive summing function:

sum_of_nums: Stat1′ × Trip-**set** → Stat_Number

F:4

As well as a function that returns the desired set of trips within a statistics for a given trip, such that the given trip is part of every trip in that set:

super_trip_set: Stat1′ × Trip → Trip-**set**

F:5

Necessary for the definition of such a function is, among others, a function generating the set of stations that form the shortest path between two given stations:

connecting_set: Sta × Sta → Sta-**set**

F:6 The connecting set function is based on, among others, a path existence function, which, for two given stations, returns a true value when there exists a path between them within a network:

exists_path: Sta × Sta → **Bool**

The path existence function is defined recursively, while its basis is an observer function, which gives the set of neighbouring stations for a given station:

obs_SSS: Sta → Sta-**set**

It is axiomatically required that a station belongs to the unique same network as its neighbours. After this short excursion we may proceed to the presentation of the whole
F:7 model.

1.2 Introduction of the Formal Model

1.2.1 Railway Topology

We take as base concept for the railway net topology that of nets. From a railway net we can observe lines and stations. There are at least two stations and one line in a net. From a line we can observe the exactly two distinct stations it connects. From a station we can observe the set of one or more tracks (on which trains may halt). From a station we can observe the set of those lines from which the station can be reached. From a station we can observe the set of lines that can be reached from that station. From a track of a station we can observe F:8
the lines from which the track can be reached. From a track of a station we can observe those lines that can be reached from that track. Given a track and a pair of incoming, respectively outgoing lines, we can observe whether a train, in order to pass from the incoming to the outgoing line will be reverse or not. From a route we can observe the ordered list of stations that it contains, including at least two stations. Given a route and a station, we can observe whether a train following this route will undergo a reversal at this station or not.

type

Net, Lin, Sta, Tra,
Route = Sta×Tra×(Lin×Sta×Tra)*

value

obs_Stas: Net → Sta-**set**,
obs_Lins: Net → Lin-**set**,

obs_SS: Lin → Sta-**set**,
obs_LS: Sta → Lin-**set**,
obs_SSS: Sta → Sta-**set**,

obs_Tras: Sta → Tra-**set**,

obs_in_Lins: Sta×Tra → Lin-**set**,
obs_out_Lins: Sta×Tra → Lin-**set**,

is_Line_Reversal: Lin × Tra × Lin → **Bool**,

obs_RStas: Route → Sta*,

is_RReversal: Route × Sta → **Bool**,

exists_path: Sta × Sta → **Bool**
exists_path(s1,s2) ≡
(s2 ∈ obs_SSS(s1)
  ∨
  (∃ s3: Sta •
    s3 ∈ obs_SSS(s1) ∧


```
        exists_path(s3,s2) )
  ),

  next_station: Route × Sta → Sta
  next_station((FS,FT,LSTlist),s) as s′
   post
     ( ∃ idx,idx′: Nat, l,l′: Lin, t,t′: Tra •
         idx′ = idx + 1 ∧
         (l,s,t) = LSTlist(idx) ∧
         (l′,s′,t′) = LSTlist(idx′)
     )
   pre
     (s = FS) ∨
     ( ∃ idx: Nat, l: Lin, t: Tra •
         (l,s,t) = LSTlist(idx)
     )
```

axiom

```
  ∀ n : Net • card obs_Stas(n) ≥ 2 ∧
                          card obs_Lins(n) ≥ 1,

  ∀ s: Sta • ∃! n: Net • s ∈ obs_Stas(n),
  ∀ l: Lin • ∃! n: Net • l ∈ obs_Lins(n),
  ∀ t: Tra • ∃! s: Sta • t ∈ obs_Tras(s),

  ∀ s: Sta, l: Lin •
   ( l ∈ obs_LS(s) ⇒
     ( ∃! n: Net •
      ( l ∈ obs_Lins(n) ∧ s ∈ obs_Stas(n) ))),

  ∀ ℓ: Lin •
   obs_SS(ℓ) =
     { s | s: Sta • ℓ∈ obs_LS(s) }
   ∧
   card obs_SS(ℓ) = 2,

  ∀ s: Sta •
   obs_SSS(s) =
     { s′ | s′: Sta •
         ∃ ℓ: Lin •
           {s,s′} ⊆ obs_SS(ℓ) },

  ∀ s : Sta • obs_Tras(s) ≠ {},
```

∀ s : Sta •
∃ t: Tra •
obs_in_Lins(s,t) ≠ {} ∧
(∀ l : Lin •
l ∈ obs_in_Lins(s,t) ⇒
s ∈ obs_SS(l)),

∀ s : Sta •
∃ t: Tra •
obs_out_Lins(s,t) ≠{} ∧
(∀ l : Lin •
l ∈ obs_out_Lins(s,t) ⇒
s ∈ obs_SS(l)),

∀ t : Tra, s : Sta, ℓ: Lin •
ℓ∈ obs_in_Lins(s,t) ⇒
t ∈ obs_Tras(s),

∀ t : Tra, s : Sta, ℓ: Lin •
ℓ∈ obs_out_Lins(s,t) ⇒
t ∈ obs_Tras(s),

∀ r : Route • **len** obs_RStas(r) > 1

F:9

1.2.2 Trains, Assemblies, Carriages, and Rolling Stock

From a train we can observe the ordered list of assemblies that it contains, including at least one assembly. From an assembly we can observe the ordered list of carriages that it contains, including at least one carriage. Given two trains, we can observe whether they constitute a reversal of each other, in case they are comparable. Given a route, we can observe the next station within it. We define equivalence and identity relationships between trains.

type

Train, Assem, Carrg

value

obs_Assems: Train → Assem*,
obs_Carrgs: Assem → Carrg*,

is_TReversal: Train × Train $\xrightarrow{\sim}$ **Bool**,
next_state_in_route: Route × Sta × Train → Train × Sta,

equivalent_trains: Train × Train → **Bool**

equivalent_trains(t1,t2) ≡
(∀ c: Carrg •
  (∃ asm: Assem •
      asm ∈ obs_Assems(t1) ∧
      c ∈ obs_Carrgs(asm)
)
  ≡
  (∃ asm′: Assem •
      asm′ ∈ obs_Assems(t2) ∧
      c ∈ obs_Carrgs(asm′)
)
),

identical_trains: Train × Train → **Bool**
identical_trains(t1,t2) ≡
(∀ c: Carrg, idx1,idx2: **Nat** •
  (∃ asm: Assem •
      asm = obs_Assems(t1)(idx1) ∧
      c = obs_Carrgs(asm)(idx2)
)
  ≡
  (∃ asm′: Assem •
      asm′ = obs_Assems(t2)(idx1) ∧
      c = obs_Carrgs(asm′)(idx2)
)
)

axiom

∀ t : Train • **len** obs_Assems(t) > 0,
∀ a : Assem • **len** obs_Carrgs(a) > 0,

∀ r: Route, s: Sta, t: Train •
 let
   (t′,s′) = next_state_in_route(r,s,t)
 in
   is_RReversal(r,s,t) ⇒ is_TReversal(t,t′)
 end,

∀ r: Route, s: Sta, t: Train •
 let
   (t′,s′) = next_state_in_route(r,s,t)
 in
   s′ = next_station(r,s)
 end

F:10

1.2.3 Journeys, Time Tables and Schedules

From a schedule we can observe get the set of journeys described in it. From a schedule and a train number we can observe the journey the train with this number makes according to this schedule. From a journey we can observe the list of stations visited in it, including at least two stations. Given a schedule, a train number and a station, we can observe the set of pairs of arrival and departure times for the train with this number respectively to and from that station according to this time table. Given a schedule, a train number, a station and a time, we can observe the characteristics of the train that appears in that station bearing this train number at that time by (according to) this schedule. We formulate well-formedness conditions for journeys and schedules. An assembly map is a pairing of assembly types to their numbers within a train configuration. For mathematical reasons that will be apparent in the following section, we define a well-formed assembly map so that its domain includes all existing assembly types, with possible zero values as numbers of assemblies. We define a function that gives the expected travelling time between two stations.

type

TrainNum, TrainChars, Platform, AssemType, OClock,
TimeDur == null|__,
TravelTime = TimeDur,
StopTime = TimeDur,
Interval = TimeDur,
FirstTime = OClock,
LastTime = OClock,
Num_of_Assems = **Nat**,
Num_of_Passengers = **Nat**,

SV = Sta×OClock×OClock×Platform×TrainChars,
Journ′ = SV*,
Journ = {|j: Journ′ • wf_journ(j) |},

TrSer = TrainNum*,

Sched = TrainNum $\xrightarrow[m]{}$ Journ,

AssemMap′ = AssemType $\xrightarrow[m]{}$ Num_of_Assems,
AssemMap = {|am: AssemMap′ • wf_assem_map(am) |}

value

obs_Assemblies: TrainChars → AssemMap,
obs_Num_of_Passengers: AssemType → Num_of_Passengers,

journs_in_sched: Sched → Journ-**set**
journs_in_sched(sch) ≡ **rng** sch,

journ_of_num: Sched × TrainNum → Journ
journ_of_num(sch,tn) ≡ sch(tn)
pre
tn ∈ **dom** sch,

journ_stas: Journ → Sta-**set**
journ_stas(j) **as** station_set
post
(∀ s: Sta •
  (∃ idx: **Nat**, dt,at: OClock,
    p: Platform, tc: TrainChars •
    j(idx) = (s,dt,at,p,tc)
)
  ≡
  s ∈ station_set
),

times: Sched × TrainNum × Sta → (OClock × OClock)-**set**
times(sch,tn,s) **as** times_set
post
(∀ at,dt: OClock •
  (at,dt) ∈ times_set
  ≡
  (∃ idx: **Nat**, p: Platform, tc: TrainChars •
    (s,at,dt,p,tc) = sch(tn)(idx)
)
)
pre
tn ∈ **dom** sch ∧
s ∈ journ_stas(sch(tn)),

trainchars: Sched × TrainNum × Sta × OClock → TrainChars
trainchars(sch,tn,s,t) **as** tc
post
(∃ idx: **Nat**, dt: OClock, p: Platform •
  (s,t,dt,p,tc) = sch(tn)(idx)
)
pre

tn ∈ **dom** sch ∧
s ∈ obs_JStas(sch(tn)),

obs_travelling_time: Sta × Sta → TravelTime,

>: OClock × OClock → **Bool**,
>: TimeDur × TimeDur → **Bool**,

≥: OClock × OClock → **Bool**,
≥: TimeDur × TimeDur → **Bool**,

∗: **Nat** × Interval → Interval,
+: OClock × TimeDur → OClock,
+: TimeDur × TimeDur → TimeDur,
−: OClock × OClock → TimeDur,

wf_journ: Journ → **Bool**
wf_journ(j) ≡
(∀ idx1,idx2: **Nat** •
  idx1 < **len** j ∧ idx2 < **len** j ⇒
  let
    (s1,at1,dt1,p1,tc1) = j(idx1),
    (s2,at2,dt2,p2,tc2) = j(idx2)
  in
    s2 ∈ obs_SSS(s1) ∧
    dt2 > at2 ∧ at2 > dt1 ∧ dt1 > at1 ∧
    at2 − dt1 ≥ obs_travelling_time(s1,s2)
  end
),

wf_schedule: Sched → **Bool**
wf_schedule(sch) ≡
(∀ j: Journ •
  j ∈ **rng** sch ⇒ wf_journ(j)
),

wf_assem_map: AssemMap → **Bool**
wf_assem_map(am) ≡
(∀ at: AssemType •
  at ∈ **dom** am
)

axiom

∀ sch : Sched • journs_in_sched(sch) ≠ {},

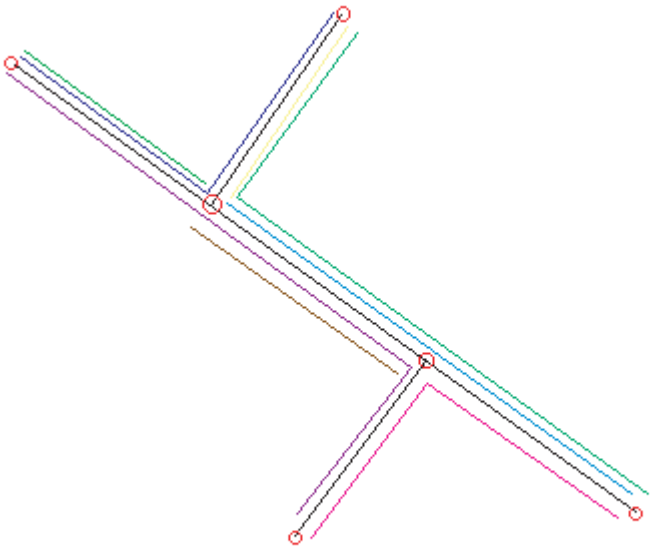
∀ j : Journ • **card** journ_stas(j) > 1,

∀ s: Sta • obs_travelling_time(s,s) = null

F:11

1.2.4 Passenger Statistics

A Statistics is a mapping from pairs of time values to maps of couples of Stations mapped to the Number of passangers commuting between those two stations in the time interval between those two time values. We overload the addition and subtraction operators so that these may be used for the composition and decomposition of trains, respectively, expressed as adding and subtracting of assemply maps. Given a journey and a station within this journey, we may derive the rolling stock to be added and to be taken out of the train making that journey in that station. A real world statistics given in a general form, in terms of numbers of commuters between pairs of stations and departure and arrival times, is transformed into a specific statistics as needed in our model, in terms of number of commuters between consecutive stations only.



type

Stat_Number = **Nat**,
Trip = (OClock × OClock) × (Sta × Sta),

Stat1′ = (OClock×OClock) $\overrightarrow{m}$ ((Sta×Sta) $\overrightarrow{m}$ Number),
Stat1 = {|s: Stat1 • consecutive(s) |}

value

+: AssemMap × AssemMap → AssemMap
am1 + am2 **as** am3
post
(∀ at: AssemType •
am3(at) = am1(at) + am2(at)
),

−: AssemMap × AssemMap → AssemMap
am1 − am2 **as** am3
post
(∀ at: AssemType •
am3(at) = am1(at) − am2(at)
)
pre
(∀ at: AssemType •
am1(at) ≥ am2(at)
),

orthogonal: AssemMap × AssemMap → **Bool**
orthogonal(am1,am2) ≡
(∀ at: AssemType •
am1(at)∗am2(at) = 0
),

train_change: Journ × Sta → TrainChars × TrainChars
train_change(j,s) **as** (tc0,tc1)
post
(∃ idx: **Nat**, s0: Sta, at,at0,dt,dt0: OClock,
p,p0: Platform •
(s,at,dt,p,tc1) = j(idx) ∧
(s0,at0,dt0,p0,tc0) = j(idx − 1)
)
pre
s ∈ journ_stas(j),

implement_change: TrainChars × TrainChars → AssemMap × AssemMap
implement_change(tc1,tc2) **as** (out_assem_map,in_assem_map)


```
post
  let
   ante_assem_map = obs_Assemblies(tc1),
   post_assem_map = obs_Assemblies(tc2)
  in
   post_assem_map = ante_assem_map + in_assem_map − out_assem_map
   ∧
   orthogonal(in_assem_map,out_assem_map)
  end,
```

```
comp_decomp: Journ × Sta → AssemMap × AssemMap
comp_decomp(j,s) ≡
 implement_change(train_change(j,s)),
```

The following function is checking whether a given statistic possessed the property of consecutivity, i.e. whether the pairs of stations and times in it are compatible to the properties of the network they are applied on.

```
consecutive: Stat1′ → Bool
consecutive(st) ≡
( ∀ dst,ast: Sta, num: Stat_Number, dt,at: OClock •
    (dt,at) ∈ dom st ∧
    (dst,ast) ∈ dom st(dt,at) ∧
    num = st(dt,at)(dst,ast)
    ⇒
    ast ∈ obs_SSS(dst) ∧ at − dt ≥ obs_travelling_time(dst,ast)
),
```

The following function produces the set of all stations the lie in the optimal path between two given stations.

```
connecting_set: Sta × Sta → Sta-set
connecting_set(s1,s2) as connecting_set
post
 ( ∀ s: Sta •
      exists_path(s1,s) ∧ exists_path(s,s2) ∧
      obs_travelling_time(s1,s) + obs_travelling_time(s,s2) =
      obs_travelling_time(s1,s2)
      ≡
      s ∈ connecting_set
 )
pre
```

exists_path(s1,s2),

The following function produces the set of all trips within a given statistics which include another given trip.

```
super_trip_set: Stat1′ × Trip → Trip-set
super_trip_set(st,((dt,at),(dst,ast))) as super_trip_set
post
 ( ∀ dst′,ast′: Sta, dt′,at′: OClock •
       (dt′,at′) ∈ dom st ∧
       (dst′,ast′) ∈ dom st(dt′,at′) ∧
       {dst,ast} ⊆ connecting_set(dst′,ast′) ∧
       dt ≥ dt′ + obs_travelling_time(dst,dst′) ∧
       at′ ≥ at + obs_travelling_time(ast′,ast)
       ≡
       ((dt′,at′),(dst′,ast′)) ∈ super_trip_set
 ),
```

The following function returns the sum of the numbers of commuters in a given set of trips.

```
sum_of_nums: Stat1′ ×  Trip-set → Stat_Number
sum_of_nums(st,super_trip_set) ≡
 case card super_trip_set of
   0 → 0,
   _ →
    let
      dst: Sta, ast: Sta, dt: OClock, at: OClock •
      ((dt,at),(dst,ast)) ∈ super_trip_set
    in
      st(dt,at)(dst,ast) +
      sum_of_nums(st,super_trip_set\{((dt,at),(dst,ast))})
    end
 end,
```

The following function reforms a given statisics into one which has the property of consecutivity.

```
reform_stat: Stat1′ → Stat1
reform_stat(st′) as st
post
```


(∀ dst,ast: Sta, dt,at: OClock •
  (dt,at) ∈ **dom** st ∧
  (dst,ast) ∈ **dom** st(dt,at)
  ⇒
  st(dt,at)(dst,ast) =
  sum_of_nums(st′,super_trip_set(st′,((dt,at),(dst,ast))))
)
pre
(∀ dst,ast: Sta, dt,at: OClock •
  (dt,at) ∈ **dom** st′ ∧
  (dst,ast) ∈ **dom** st′(dt,at)
  ⇒
  at − dt ≥ obs_travelling_time(dst,ast)
)

F:12

1.2.5 Planning

Given a statistics and a set of rules and regulations we can generate a set of scedules for the given statistics under the given rules and regulations. The generated schedules satisfy the given statistics. A recursive function computes the sum of passenger load that a given configuration of train characteristics may take.

type

RulReg

value

```
passengers_sum: AssemMap → Nat
passengers_sum(assem_map) ≡
 case card dom assem_map of
   0 → 0,
   _ →
    let
      assem_type: AssemType •
      assem_type ∈ dom assem_map
    in
      assem_map(assem_type)∗obs_Num_of_Passengers(assem_type) +
      passengers_sum(assem_map\{assem_type})
    end
 end,

fitrules: RulReg × Sched → Bool,

fitting: Stat1 × RulReg-set × Sched → Bool
fitting(st,rrs,sch) ≡
 ( ∀ rr: RulReg •
      rr ∈ rrs ⇒ fitrules(rr,sched)
 )
 ∧
 ( ∀ dt,at: OClock, dst,ast: Sta, num: Stat_Number •
    (dt,at) ∈ dom st ∧
    (dst,ast) ∈ dom st(dt,at) ∧
    num = st(dt,at)(dst,ast)
    ⇒
    ( ∃ j: Journ •
       j ∈ rng sch ∧
       (∃ idx1,idx2: Nat, at1,dt2: OClock,
                  p1,p2: Platform, tc1,tc2: TrainChars •
          idx1 ≤ len j ∧ idx2 ≤ len j ∧
```



```
            (dst,at1,dt,p1,tc1) = j(idx1) ∧
            (ast,at,dt2,p2,tc2) = j(idx2) ∧
            passengers_sum(obs_Assemblies(tc1)) ≥ num
        )
    )
),
```

gen_Sched: Stat1 × RulReg-**set** → Sched-**set**
gen_Sched(st,rrs) ≡ {sch | sch: Sched • fitting(reform_stat(st),rrs,sch)}

F:13

2 Requirements

T:2, F:14

2.1 Domain Requirements

2.1.1 Projection

We should first ask for which parts of the domain the client wishes computing support. In our case the part of the domain where computing support is needed is the Planning part outlined above.

2.1.2 Determination

We consider that the projected parts of our model do not contain any undesired looseness or non-determinism. However, we do need to insert more content in the rules and regulations that are inserted in the planning function.

2.1.3 Extension

Below we specify how the developed domain may be extended to include the selection of the optimal schedules from within a generated set of schedules. The set of schedules generated by the function provided in the previous section is redundant in a number of ways: Although all the generated schedules fitting the same given statistics, they do not have to do so by implementing the same set of trips. At this stage our purpose is to reduce those schedules who implement the same set of trips as other schedules in the same set, without being optimal in comparison to them. Firstly, two or more schedules in it may implement exactly the same set of trips, albeit in different combinations of journeys and allocation of trips to journeys. Secondly, two or more schedules in the generated set may implement exactly the same set of trips, and even through the same set of journeys, but the allocation of rolling stock to trains may be different between them, i.e. one of them may allocate more rolling stock than necessary. In order to eliminate these redundancies, we follow the approach described below. The main idea is given in the function optimizing a schedule set. The rest of the following functions are auxiliary to that main function. Each of the following functions is accompanied by comments that explain its role in the model.

value

The following two functions are responsible for deciding whether a given trip belongs to a certain schedule, i.e. whether the given schedule includes the given trip. In order for a trip to belong to a schedule, it has to belong to a journey belonging to that schedule.

```
belongs_journ: Trip × Journ → Bool
belongs_journ(((dt,at),(dst,ast)),j) ≡
 ( ∃ idx1,idx2: Nat, dt,at: OClock, dst,ast: Sta •
       (dst,_,dt,_,_) = j(idx1) ∧
       (ast,at,_,_,_) = j(idx2)
 ),

belongs_sched: Trip × Sched → Bool
belongs_sched(trip,sch) ≡
 ( ∃ j: Journ •
       j ∈ rng sch ∧
       belongs_journ(trip,j)
 ),
```

The following two functions are responsible for deciding whether two given schedules are equivalent to each other. Equivalence of schedules is defined as the property that a pair of schedules has when they include the same set of trips.

```
sched_trip_set: Sched → Trip-set
sched_trip_set(sch) as tripset
post
 ( ∀ trip: Trip •
       belongs_sched(trip,sch)
       ≡
       trip ∈ tripset
 ),

equiv_Sched: Sched × Sched → Bool
equiv_Sched(sch1,sch2) ≡
 ( sched_trip_set(sch1) = sched_trip_set(sch2) ),
```

The following group of functions is responsible for the computation of a schedule's total cost. This total cost is computed as the sum of the costs of the journeys in the schedule. In its own turn, the cost of a journey is computed as the sum of the costs of the station visits that compose this journey. Finally, a station visit's cost is computed as the cost for the train making that station visit, which is a function of this train's characteristics. This is allowed to

be so, since all the other factors of interest (time duration) are equal under the circumstances in which we are interested to compare schedules' costs: we compare the costs of equivalent schedules, i.e. schedules which include exactly the same sets of trips.

```
train_cost: TrainChars → Nat,

sv_cost: SV → Nat
sv_cost(sv) ≡
 let
   tc: TrainChars •
    (_,_,_,_,tc) = sv
 in
   cost(tc)
 end,

journ_cost: Journ → Nat
journ_cost(j) ≡
 case len j of
   0 → 0,
   _ →
    let
      sv: SV •
       sv = hd j
    in
      sv_cost(sv) +
      journ_cost(tl j)
    end
 end,

sched_cost: Sched → Nat
sched_cost(sch) ≡
 case card rng sch of
   0 → 0,
   _ →
    let
      j: Journ, tn: TrainNum •
       tn ∈ dom sch ∧
       j = sch(tn)
    in
      journ_cost(j) +
      sched_cost(sch\{tn})
    end
 end,
```

The following function returns the optimal schedule of a set of equivalent schedules, i.e. a member of the set for which no other member of the set costs less than it.

```
opt_Sched: Sched-set → Sched
opt_Sched(sch_set) as opt_sch
 post
   ( opt_sch ∈ sch_set
     ∧
     ( ∀ sch: Sched •
         sch ∈ sch_set
         ⇒
         sched_cost(sch) ≥ sched_cost(opt_sch)
     )
   ),
```

The following function returns an optimized set of schedules given a general set of schedules. The optimization is performed by eliminating those schedules which are equivalent to a schedule already included in the set, which is optimal compared to them. In other words, those and only those schedules of the original set are included in the returned set, which are such that an equivalent to and better schedule than them does not exist in the original set.

```
optimize_Sched_set: Sched-set → Sched-set
optimize_Sched_set(sch_set) as opt_sch_set
 post
   ( opt_sch_set ⊆ sch_set )
   ∧
   ( ∀ sch: Sched •
       sch ∈ opt_sch_set
       ≡
       ( ∀ sch′: Sched •
           sch′ ∈ sch_set
           ∧
           equiv_Sched(sch,sch′)
           ⇒
           sch = opt_Sched({sch,sch′})
       )
   )
```

2.1.4 Initialization

In dscribing a domain, such as we have done for the domain of railway nets, we have designated the space of all its components. Sooner or later, one has to initialize the computing system to

reflect all these many entities. Hence we need to establish requirements for how to initialize the computing system, how to maintain and update it, etc. In the present case initialization includes the acquisition of statistical information as well as of the data structures that describe the net and the data on the available types of assemblies.

2.1.5 Fitting

The developed software system does not need to fit some existing product.

2.2 Interface Requirements

2.2.1 Computer-Human Interface

In the present case the computer-human interface needs to be sufficiently robust to be able to handle a large amount of statistical and network data. There are no special needs concerning the user-friendliness of the system, as this is intended to be used only by the railway company.

2.3 Machine Requirements

A software system has been already developed at NS Reizigers, the passenger division of the "old" Nederlandse Spoorwegen (Dutch Railways, NS), that implements a solution for the problem described here.

3 Conclusion

T:3, F:15

The present paper has outlined an a posteriori attempt to shed light onto the formal aspects of the presented problem. In a more formal-oriented approach, we would have liked a software system to have been developed based on the requirements outlined in this paper.