

Domain Modelling

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ACM Reference Format:

Dines Bjørner. 2024. Domain Modelling. 1, 1 (March 2024), 27 pages. <https://doi.org/10.1145/nnnnnnn.nnnnnnn>

The Triptych Dogma

In order to *specify* *Software*, we must understand its *Requirements*.

In order to *prescribe* *Requirements* we must understand the *Domain*.

So we must study, analyze and describe *Domains*.

$D, S \models R$:

In proofs of *Software* correctness,

with respect to *Requirements*,

assumptions are made with respect to the *Domain*.

We present a systematic *method*, its *principles*, *procedures*, *techniques* and *tools*, for efficiently *analyzing* & *describing* domains. This paper is based on [12–14]. It simplifies the methodology of these – as well as introduces some novel presentation and description language concepts.

1 DOMAINS

We start by delineating the informal concept of domain,¹

1.1 What are They?

What do we mean by ‘domain’?

CHARACTERIZATION 1. *Domain:* domain By a *domain* we shall understand a *rationaly describable* segment of a *discrete dynamics* fragment of a *human assisted* reality: the world that we daily observe – in which we work and act, a reality made significant by human-created entities. The domain embody *endurants* and *perdurants* •

EXAMPLE 1. *Some Domain Examples:* A few, more-or-less self-explanatory examples:

- **Rivers** – with their natural sources, deltas, tributaries, waterfalls, etc., and their man-made dams, harbours, locks, etc. – and their conveyage of materials (ships etc.) [17, *Chapter B*].
- **Road nets** – with street segments and intersections, traffic lights and automobiles – and the flow of these [17, *Chapter E*].

¹Our use of the term ‘domain’ should not be confused with that of Dana Scott’s Domain Theory: https://en.wikipedia.org/wiki/Scott_domain.

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ACM XXXX-XXXX/2024/3-ART

<https://doi.org/10.1145/nnnnnnn.nnnnnnn>

- **Pipelines** – with their liquids (oil, or gas, or water), wells, pipes, valves, pumps, forks, joins and wells and the flow of fluids [17, *Chapter I*].
- **Container terminals** – with their container vessels, containers, cranes, trucks, etc. – and the movement of all of these [17, *Chapter K*].

Characterization 1 relies on the understanding of the terms ‘*rationaly describable*’, ‘*discrete dynamics*’, ‘*human assisted*’, ‘*solid*’ and ‘*fluid*’. The last two will be explained later. By ***rationaly describable*** we mean that what is described can be understood, including reasoned about, in a rational, that is, logical manner – in other words ***logically tractable***.² By ***discrete dynamics*** we imply that we shall basically rule out such domain phenomena which have properties which are continuous with respect to their time-wise, i.e., dynamic, behaviour. By ***human-assisted*** we mean that the domains – that we are interested in modelling – have, as an important property, that they possess man-made entities.

1.2 Some Introductory Remarks

1.2.1 A Discussion of Our Characterization of a Concept of Domain. Characterization 1 is our attempt to delineate the subject area. That is, “our” concept of ‘*domain*’ is ‘*novel*’: *new and not resembling something formerly known or used*. As such it may be unfamiliar to most readers. So it takes time to digest that characterization. So the reader may have to return to the page, Page 1, to be reminded of the definition.

1.2.2 Formal Methods and Description Language. The reader is assumed to have a reasonable grasp of formal methods – such as espoused in [7; 20; 21; 44].

The descriptions evolving from the modelling approach of this paper are in the abstract, model-oriented specification language RSL [25] of the **Raise³ Specification Language**. But other abstract specification languages could be used: VDM [20; 21], Z [44], Alloy [32], CafeOBJ [24], etc. We have chosen RSL since it embodies a variant of CSP [30] – being used to express domain behaviours.

1.2.3 Programming Languages versus Domain Semantics. From around the late 1960s, spurred on by the works of John McCarthy, Peter Landin, Christopher Strachey, Dana Scott and others, it was not unusual to see publications of entire formal definitions of programming language semantics. Widespread technical reports were [1; 2, 1969, 1974] Notably so was [37, 1976]. There was the 1978 publication [20, *Chapter 5, Algol 60, 1978*]. Others were [21, *Chapters 6–7, Algol 60 and Pascal, 1982*] As late as into the 1980s there were such publications [3, 1980].

Formal descriptions of domains, such as we shall unravel a method for their study, analysis and description, likewise amount to semantics for the terms of the professional languages spoken by stakeholders of domains. So perhaps it is time to take the topic serious.

1.2.4 A New Universe. The concept of domain – such as we shall delineate and treat it – is novel. That is: new and not treated in this way before. Its presentation, therefore, necessarily involves the introduction of a new universe of concepts. Not the neat, well-defined concepts of neither “classical” computer science nor software engineering. It may take some concentration on the part of the reader to get used to this!

You will therefore be introduced to quite a universe of new concepts. You will find these concepts named in most display lines⁴ and in Figs. 1 and 2.

2 ENDURANTS AND PERDURANTS, I

The above characterization hinges on the characterizations of endurants and perdurants.

²Another, “upside-down” – after the fact – [perhaps ‘cheating’] way of defining ‘describable’ is: is it describable in terms of the method of this paper!

³**RAISE** stands for **R**igorous **A**pproach to **I**ndustrial **S**oftware **E**ngineering [26].

⁴– that is, section, subsection, sub-subsection, paragraph and sub-paragraph lines

CHARACTERIZATION 2. *Endurants*: Endurants are those quantities of domains that we can observe (see and touch), in *space*, as “complete” entities at no matter which point in *time* – “material” entities that persists, endures – capable of enduring adversity, severity, or hardship [Merriam Webster] •

Endurants are either *natural* [“God-given”] or *artefactual* [“man-made”]. Endurants may be either solid (discrete) or fluid, and solid endurants, called parts, may be considered *atomic* or *compound* parts; or, as in this paper solid endurants may be further unanalysed *living species: plants and animals* – including *humans*.

CHARACTERIZATION 3. *Perdurants*: Perdurants are those quantities of domains for which only a fragment exists, in *space*, if we look at or touch them at any given snapshot in *time* [Merriam Webster] •

Perdurants are here considered to be *actions, events and behaviours*.

• • •

We exclude, from our treatment of domains, issues of ethics, biology and psychology.

3 A DOMAIN ANALYSIS & DESCRIPTION ONTOLOGY

3.1 The Chosen Ontology

Figure 1 expresses an ontology⁵ for our analysis of domains. Not a taxonomy⁶ for any one specific domain. The idea of Fig. 1 is the following:

- It presents a recipe for how to **analyze** a domain.
- You, the *domain analyzer cum describer*, are ‘confronted’⁷ with, or by a domain.
- You have Fig. 1 in front of you, on a piece of paper, or in Your mind, or both.
- You are then asked, by the domain **analysis** & description method of this paper, to “start” at the uppermost •, just below and between the ‘r’ and the first ‘s’ in the main title, Phenomena of Natural and Artefactual **Universes** of Discourse.
- The **analysis** & description ontology of Fig. 1 then *directs* You to inquire as to whether the phenomenon – whichever You are “looking at/reading about/...” – is either *rationally describable*, i.e., is an *entity* (is_entity) or is *indescribable*.
- That is, You are, in general, “positioned” at a bullet, •, labeled α , “below” which there may be two alternative bullets, one, β , to the right and one to the left, γ .
- It is Your decision whether the answer to the “query” that each such situation warrants, is yes, is_ β , or no, is_ γ .
- The characterizations of the concepts whose names, α, β, γ etc., are attached to the •s of Fig. 1 are given in the following sections.
- Whether they are precise enough to guide You in Your obtaining reasonable answers, “yes” or “no”, to the •ed queries is, of course, a problem. I hope they are.
- If Your answer is “yes”, then Your **analysis** is to proceed “down the tree”, usually indicated by “yes” or “no” answers.
- If one, or the other is a “leaf” of the ontology tree, then You have finished examining the phenomena You set out to **analyze**.
- If it is not a leaf, then further **analysis** is required.

⁵An ontology is the philosophical study of being. It investigates what types of entities exist, how they are grouped into categories, and how they are related to one another on the most fundamental level (and whether there even is a fundamental level) [Wikipedia].

⁶A taxonomy (or taxonomic classification) is a scheme of classification, especially a hierarchical classification, in which things are organized into groups or types [Wikipedia].

⁷By ‘confronted’ we mean: You are reading about it, in papers, in books, in postings on the Internet, visiting it, talking with domain stakeholders: professional people working “in” the domain; You may, yourself, “be an entity” of that domain!

philosophy. In paragraphs, in the rest of this paper, marked **Ontological Choice**, we shall relate Sørlander’s philosophy’s “inevitability” to the ontology for studying domains.

4 THE NAME, TYPE AND VALUE CONCEPTS

Domain *modelling*, as well as *programming*, depends, in their *specification*, on *separation of concerns*: which kind of *values* are subjectable to which kinds of *operations*, etc., in order to achieve ease of *understanding* a model or a program, ease of *proving properties* of a model, or *correctness* of a program.

4.1 Names

We name things in order to refer to them in our speech, models and programs. Names of types and values in models and programs are usually not so-called “first-citizens”, i.e., values that can be arguments in functions, etc. The “science of names” is interesting.⁹ In botanicalsociety.org.za/the-science-of-names-an-introduction-to-plant-taxonomy the authors actually speak of a “science of names” in connection with plant taxonomy: the “art” of choosing such names that reflect some possible classification of what they name.

4.2 Types

The type concept is crucial to programming and modelling.

CHARACTERIZATION 4. *Type*: A *type* is a class of values (“of the same kind”) •

We name types.

EXAMPLE 2. *Type Names*: Some examples of type names are:

- RT – the class of all road transport instances: the *Metropolitan London Road Transport*, the *US Federal Freeway System*, etc.
- RN – the class of all road net instances (within a road transport).
- SA – the class of all automobiles (within a road transport) •

You, the domain describer, choose type names. Choosing type names is a “serious affair”. It must be done carefully. You can choose short (as above) or long names: *Road_Transport*, *Road_Net*, etc. We prefer short, but not cryptic names, like X, Y, Z, Names that are easy to *memorize*, i.e., *mnemonics*.

4.3 Values

Values are what programming and modelling, in a sense, is all about”. In programming, values are the *data* “upon” which the program code specifies computations. In modelling values are, for example, what we observe: the entities in front of our eyes.

5 PHENOMENA AND ENTITIES

CHARACTERIZATION 5. *Phenomena*: By a *phenomenon* we shall understand a fact that is observed to exist or happen •

Some phenomena are rationally describable – to some degree¹⁰ – others are not.

⁹The study of names is called *onomastics* or *onomatology*. *Onomastics* covers the naming of all things, including place names (toponyms) and personal names (*anthroponyms*).

¹⁰That is: It is up to the domain analyzer cum describer to decide as to how many rationally describable phenomena to select for analysis & description. Also in this sense one practices abstraction by “abstracting away” [the analysis & description of] phenomena that are irrelevant for the “current” (!) domain description.

CHARACTERIZATION 6. *Entities*: By an entity By an *entity* we shall understand a more-or-less rationally describable phenomenon •

We introduce the informal presentation language predicate `is_entity`. It holds for phenomena ϕ if ϕ is describable.

EXAMPLE 3. *Phenomena and Entities*: Some, but not necessarily all aspects of a river can be rationally described, hence can be still be considered entities. Similarly, many aspects of a road net can be rationally described, hence will be considered entities •

If You are not happy with this ‘characterization’, then substitute “rationally describable” with: *describable in terms of the endurants and perdurants brought forward in this paper: their external and internal qualities, unique identifiers, mereologies and attributes, channels and behaviours!*

Ontological Choice: We choose to “initialize” our ontological “search” to a question of whether a phenomenon is rationally describable – based on the tenet of Kai Sørlander’s philosophy, namely that “whatever” we postulate is either *true* or *false* and that a *principle of contradiction* holds: *whatever we so express can not both hold and not hold* •

Kai Sørlander then develops his inquiry – *as to what is absolutely necessary in any description of the world* – into the rationality of such descriptions necessarily be based on time and space and, from there, by a series of transcendental deductions, into a base in *Newton’s* physics. We shall, in a sense, stop there. That is, in the domain concept, such as we have delineated it, we shall not need to go into *Einsteinian* physics.

6 ENDURANTS AND PERDURANTS, II

We repeat our characterizations of endurants and perdurants.

6.1 Endurants

We repeat characterization 2.

CHARACTERIZATION 7. *Endurant*: Endurants are those quantities of domains that we can observe (see and touch), in *space*, as “complete” entities at no matter which point in *time* – “material” entities that persists, endures – capable of enduring adversity, severity, or hardship [Merriam Webster] •

EXAMPLE 4. *Endurants*: Examples of endurants are: a street segment [link], a street intersection [hub], an automobile •

We introduce the informal presentation language predicate `is_endurant` to hold for entity e if `is_endurant(e)` holds.

6.2 Perdurants

We repeat characterization 3.

CHARACTERIZATION 8. *Perdurant*: Perdurants are those quantities of domains for which only a fragment exists, in *space*, if we look at or touch them at any given snapshot in *time* [Merriam Webster] •

EXAMPLE 5. *Perdurant*: A moving automobile is an example of a perdurant •

We introduce the informal presentation language predicate `is_perdurant` to hold for entity e if `is_perdurant(e)` holds.

6.3 Ontological Choice

The **ontological choice** of entities being “viewed” as either endurants or perdurants is motivated as follows: The concept of endurants can be justified in terms of Newton’s physics without going into kinematics, i.e., without including time considerations. The concept of perdurants can then, on one hand, be justified in terms of Newton’s physics now taking time into consideration, hence kinematics, and from there causality, etc.; and, on the other hand, and as we shall see, by transcendently deducing perdurants from solid endurants •

7 EXTERNAL AND INTERNAL ENDURANT QUALITIES

The main contribution of this section is that of a calculus of domain analysis and description prompts. Two facets are being presented. Aspects of a domain science: of how we suggest domains can, and should, be viewed – ontologically. And aspects of a domain engineering: of how we suggest domains can, and should, be analyzed and described.

We begin by characterizing the two concepts: external and internal qualities.

CHARACTERIZATION 9. *External Qualities:* External qualities of endurants of a manifest domain are, in a simplifying sense, those we can see, touch and have spatial extent. They, so to speak, take form.

CHARACTERIZATION 10. *Internal Qualities:* Internal qualities are those properties [of endurants] that do not occupy *space* but can be measured or spoken about •

Perhaps we should instead label these two qualities tangible and intangible qualities.

Ontological Choice: The rational, analytic philosophy issues of the inevitability of these qualities is this: (i) can they be justified as inevitable, and (ii) can they be suitably “separated”, i.e., both disjoint and exhaustive? Or are they merely of empirical nature?

The choice here is also that we separate our inquiry into examining *both external and internal qualities* of endurants [not ‘either or’] •

7.1 External Qualities – Tangibles

EXAMPLE 6. *External Qualities:* An example of external qualities of a domains is: the Cartesian¹¹ of sets of solid atomic street intersections, and of sets of solid atomic street segments, and of sets of solid automobiles of a road transport system where *Cartesian, sets, atomicity, and solidity* reflect external qualities •

7.1.1 *The Universe of Discourse.* The most immediate external quality of a domain is the “entire” domain – “itself”! So any domain analysis starts by identifying that “entire” domain! By giving it a name, say UoD, for *universe of discourse*, Then describing it, in *narrative* form, that is, in natural language containing terms of professional/technical nature, the domain. And, finally, *formalizing* just the name: giving the name “status” of being a type name, that is, of the type of a class of domains whose further properties will be described subsequently.

Narration:

The name, and hence the type, of the domain] is UoD

The UoD domain can be briefly characterized by ...

Formalization:

type UoD

¹¹Cartesian after the French philosopher, mathematician, scientist René Descartes (1596–1650)

7.1.2 *Solid and Fluid Endurants.* Given then that there are endurants we now postulate that they are either [mutually exclusive] *solid* (i.e., discrete) or *fluid*.

Ontological Choice: Here we [seem to] make a practical choice, not one based on a philosophical argument, one of logical necessity, but one based on empirical evidence. It is possible for endurants to either be solid or fluid; and here we shall not consider the case where solid [fluid] endurants, due to being heated [cooled], enters a fluid state [or vice versa] •

7.1.2.1 *Solid [or Discrete] Endurants.*

CHARACTERIZATION 11. *Discrete or Solid Endurants:* By a *solid* [or *discrete*] endurant we shall understand an endurant which is separate, individual or distinct in form or concept, or, rephrasing: have ‘body’ [or magnitude] of three-dimensions: length, breadth and depth [36, *OED*, Vol. II, pg. 2046] •

EXAMPLE 7. *Solid Endurants:* Pipeline system examples of solid endurants are *wells, pipes, valves, pumps, forks, joins* and *sinks* of pipelines. (These units may, however, and usually will, contain fluids, e.g., oil, gas or water.) •

We introduce the informal presentation language predicate `is_solid` to hold for endurant `e` if `is_solid(e)` holds.

7.1.2.2 *Fluids.*

CHARACTERIZATION 12. *Fluid Endurants:* By a *fluid endurant* we shall understand an endurant which is prolonged, without interruption, in an unbroken series or pattern; or, rephrasing: a substance (liquid, gas or plasma) having the property of flowing, consisting of particles that move among themselves [36, *OED*, Vol. I, pg. 774] •

EXAMPLE 8. *Fluid Endurants:* Examples of fluid endurants are: *water, oil, gas, compressed air, smoke* •

Fluids are otherwise liquid, or gaseous, or plasmatic, or granular¹², or plant products, i.e., chopped sugar cane, threshed, or otherwise¹³, et cetera. Fluid endurants will be analyzed and described in relation to solid endurants, viz. their “containers”.

We introduce the informal presentation language predicate `is_fluid` to hold for endurant `e` if `is_fluid(e)` holds.

7.1.3 *Parts and Living Species Endurants.* Given then that there are solid endurants we now postulate that they are either [mutually exclusive] *parts* or *living species*.

Ontological Choice: With Sørlander, [42, Sect. 5.7.1, pages 71–72] we reason that one can distinguish between parts and living species •

7.1.3.1 *Parts.*

CHARACTERIZATION 13. *Parts:* The non-living species solids are what we shall call parts •

Parts are the “work-horses” of man-made domains. That is, we shall mostly be concerned with the analysis and description of endurants into parts.

EXAMPLE 9. *Parts:* Example 7, of solids, is an example of parts •

¹² This is a purely pragmatic decision. “Of course” sand, gravel, soil, etc., are not fluids, but for our modelling purposes it is convenient to “compartmentalise” them as fluids!

¹³ See footnote 12.

We introduce the informal presentation language predicate `is_part` to hold for solid endurants `e` if `is_part(e)` holds.

We distinguish between atomic and compound parts.

Ontological Choice: It is an empirical fact that parts can be composed from parts. That possibility exists. Hence we can [philosophy-wise] reason likewise •

7.1.3.1.1 Atomic Parts.

CHARACTERIZATION 14. *Atomic Part:* By an *atomic part* we shall understand a part which the domain analyzer considers to be indivisible in the sense of not meaningfully consist of sub-parts •

EXAMPLE 10. *Atomic Parts:* Examples of atomic parts are: hubs, `H`, i.e., street intersections; links, `L`, i.e., the stretches of roads between two neighbouring hubs; and automobiles, `A`:

type `H, L, A` •

We introduce the informal presentation language predicate `is_atomic` to hold for parts `p` if `is_atomic(p)` holds.

7.1.3.1.2 Compound Parts.

CHARACTERIZATION 15. *Compound Part:* Compound parts are those which are observed to [potentially] consist of several parts •

EXAMPLE 11. *Compound Parts:* An example of a compound parts is: a road net consisting of a set of hubs, i.e., street intersections or “end-of-streets”, and a set of links, i.e., street segments (with no contained hubs), is a Cartesian compound; and the sets of hubs and the sets of links are part set compounds •

We introduce the informal presentation language predicate `is_compound` to hold for parts `p` if `is_compound(p)` holds.

We, pragmatically, distinguish between Cartesian product- and set-oriented parts.

Ontological Choice: The Cartesian versus set parts is an empirical choice. It is not justified in terms of philosophy, but in terms of mathematics – of mathematical expediency! •

7.1.3.1.3 *Cartesians.* Cartesians are product-like types – and are named after the French philosopher, scientist and mathematician René Descartes (1596–1640) [Wikipedia].

CHARACTERIZATION 16. *Cartesians:* Cartesian parts are those compound parts which are observed to consist of two or more distinctly sort-named endurants (solids or fluids) •

EXAMPLE 12. *Cartesians: Road Transport:* A road transport, `rt:RT`, is observed to consist of an aggregate of a road net, `rn:RN`, and a set of automobiles, `SA`, where the road net is observed, i.e., abstracted, as a Cartesian of a set of hubs, `ah:AH`, i.e., street intersections (or specifically designated points segmenting an otherwise “straight” street into two such), and a set of links, `al:AL`, i.e., street segments between two “neighbouring” hubs.

type

`RT, RN, SA, AH = H-set, AL = L-set`

value

`obs_RN: RT → RN, obs_SA: RT → SA, obs_AH: RN → AH, obs_AL: RN → AL` •

We introduce the informal presentation language predicate `is_Cartesian` to hold for compound parts `p` if `is_Cartesian(p)` holds.

Once a part, say $p:P$, has been analyzed into a Cartesian, we inquire as to the type names of the endurants¹⁴ of which it consists. The inquiry: $\text{record_Cartesian_part_type_names}(p:P)$, we decide, then yields the type of the constituent endurants.

SCHEMA 1. *record-Cartesian-part-type-names*

value

$\text{record_Cartesian_part_type_names}: P \rightarrow \mathbb{T}\text{-set}$
 $\text{record_Cartesian_part_type_names}(p) \text{ as } \{\eta E_1, \eta E_2, \dots, \eta E_n\} \bullet$

Here \mathbb{T} is the **name** of the type of all type names, and ηE_i is the **name** of type E_i .

Please note the novel introduction of type names as values. Where a type identifier, say T , stands for, denotes, a class of values of that type, ηT denotes the name of type T .

Please also note that $\text{record_Cartesian_part_type_names}$ is not a description language construct. It is an analysis language, i.e., an informal natural language, here English, construct. As such it is being used by the domain analyzer cum describer who “applies” it to an observed endurant and notes down, in her mind or jots it on a scratch of paper, her decision as to appropriate [new] type names.

EXAMPLE 13. *Cartesian Parts*: The Cartesian parts of a road transport, $rt:RT$, is thus observed to consists of

- an aggregate of a road net, $rn:RN$, and
- an aggregate set of automobiles, $sa:SA$:

that is:

- $\text{record_Cartesian_part_type_names}(rt:RT) = \{\eta RN, \eta SA\}$

where the type name ηRT was – and the type names ηRN and ηSA are – coined, i.e., more-or-less freely chosen, by the domain analyzer cum describer •

7.1.3.1.4 Part Sets.

CHARACTERIZATION 17. *Part Sets*: Part sets are those compound parts which are observed to consist of an indefinite number of zero, one or more parts •

We introduce the informal presentation language predicate is_part_set to hold for compound parts e if $\text{is_part_set}(e)$ holds.

Once a part, say $e:E$, has been analyzed into a part set we inquire as to the set of parts and their type of which it consists. The inquiry: $\text{record_part_set_part_type_names}$, we decide, then yields the (single) type of the constituent parts.

SCHEMA 2. *record-part-set-part-type-names*

value

$\text{record_part_set_part_type_names}: E \rightarrow \mathbb{T}Ps \times \mathbb{T}P$
 $\text{record_part_set_part_type_names}(e:E) \text{ as } (\eta Ps, \eta P) \bullet$

Here the name of the value, e , and the type names ηPs and ηP are coined, i.e., more-or-less freely chosen, by the domain analyzer cum describer •

Please also note that $\text{record_part_set_part_type_names}$ is not a description language construct. It is an analysis language, i.e., an informal natural language, here English, construct. As such it is being used by the domain analyzer cum describer who “applies” in to an observed endurant and notes down, in her mind or jots it on a scratch of paper, her decision as to appropriate [new] type names.

¹⁴We emphasize that the observed elements of a Cartesian part may be both solids, at least one, and fluids.

EXAMPLE 14. *Part Sets: Road Transport*: The road transport contains a set of automobiles. The part set type name has been chosen to be SA. It is then determined (i.e., analyzed) that SA is a set of Automobile of type A

- $\text{record_part_set_part_type_names}(sa:SA) = (\eta As, \eta A)$ •

7.1.3.1.5 *Compound Observers*. Once the domain analyzer cum describer has decided upon the names of atomic and compound parts, **obs_erver** functions can be applied to Cartesian and part set, $e:E$, parts:

SCHEMA 3. *Describe-Cartesians-and-Part-Set-Parts*

```

value
  let  $\{\eta P1, \eta P2, \dots, \eta Pn\} = \text{record\_Cartesian\_part\_type\_names}(e:E)$  in
  “type
    P1, P2, ..., Pn;
  value
    obs_P1:  $E \rightarrow P1$ , obs_P2:  $E \rightarrow P2, \dots, n$  obs_Pn:  $E \rightarrow Pn$  ”
    [respectively:]
  let  $(\eta Ps, \eta P) = \text{record\_part\_set\_part\_type\_names}(e:E)$  in
  “type
    P, Ps = P-set,
  value
    obs_Ps:  $E \rightarrow Ps$  ”
  end end •

```

The “...” texts are the RSL texts “generated”, i.e., written down, by the domain describer. They are *domain model specification units*. The “surrounding” RSL-like texts are not written down as phrases, elements, of the domain description. They are elements of the domain describers’ “notice board”, and, as such, elements of the development of domain models. We have introduced a core domain modelling tool the **obs_...** observer function, one to be “applied” mentally by the domain describer, and one that appears in (RSL) domain descriptions The **obs_...** observer function is “applied” by the domain describer, it is not a computable function.

Please also note that Describe-Cartesians-and-Part-Set-Parts schema, 3, is not a description language construct. It is an analysis language, i.e., an informal natural language, here English, construct. As such it is being used by the domain analyzer cum describer who “applies” in to an observed enduring and notes down, but now in a final form, elements, that is *domain description units*.

• • •

A major step of the development of domain models has now been presented: that of the analysis & description of the external qualities of domains.

Schema 3 is the first manifestation of the domain analysis & description method leading to actual domain description elements.

From unveiling a *science of domains* we have “arrived” at an *engineering of domain descriptions*.

7.1.4 *States*.

CHARACTERIZATION 18. *States*: By a *state* we shall mean any subset of the parts of a domain •

EXAMPLE 15. *Road Transport State*:

```

variable
   $hs:AH := \text{obs\_AH}(\text{obs\_RN}(rt))$ ,
   $ls:AL := \text{obs\_AL}(\text{obs\_RN}(rt))$ ,
   $as:SA := \text{obs\_SA}(rt)$ ,

```

$$\sigma:(H|L|A)\text{-set} := hs \cup ls \cup as \bullet$$

We have chosen to model domain states as **variables** rather than as **values**. The reason for this is that the values of monitorable, including biddable part attributes¹⁵ can change, and that domains are often extended and “shrunk” by the addition, respectively removal of parts:

EXAMPLE 16. *Road Transport Development*: adding or removing hubs, links and automobiles •

We omit coverage of the aspect of bidding changes to monitorable part attributes.

7.1.5 *Validity of Endurant Observations*. We remind the reader that the **obs_**erver functions, as all later such functions: **uid_**-, **mereo_**- and **attr_**-functions, are applied by humans and that the outcome of these “applications” is the result of human choices, and possibly biased by inexperience, taste, preference, bias, etc. How do we know whether a domain analyzer & describer’s description of domain parts is valid? Whether relevantly identified parts are modeled reasonably wrt. being atomic, Cartesians or part sets Whether all relevant endurants have been identified? Etc. The short answer is: we never know. Our models are conjectures and may be refuted [38]. A social process of peer reviews, by domain stakeholders and other domain modelers is needed – as may a process of verifying¹⁶ properties of the domain description held up against claimed properties of the (real) domain.

7.1.6 *Summary of Analysis Predicates*. Characterizations 6–17 imply the following analysis predicates (Defn.: δ , page π):

- **Endurant Ontology:**
- is_entity, $\delta 6 \pi 6$
- is_entity, $\delta 6 \pi 6$
- is_entity, $\delta 6 \pi 6$
- is_endurant, $\delta 7 \pi 6$
- is_perdurant, $\delta 8 \pi 6$
- is_solid, $\delta 11 \pi 8$
- is_fluid, $\delta 12 \pi 8$
- is_part, $\delta 13 \pi 8$
- is_atomic, $\delta 14 \pi 9$
- is_compound, $\delta 15 \pi 9$
- is_Cartesian, $\delta 16 \pi 9$
- is_part_set, $\delta 17 \pi 10$

We remind the reader that the above predicates represent “formulas” in the presentation, **not** the description, language. They are not RSL clauses. They are in the mind of the domain analyzers cum describers. They are “executed” by such persons. Their result, whether **true**, **false** or **chaos**¹⁷, are noted by these persons and determine their next step of domain analysis.

7.1.7 *“Trees are Not Recursive”*. A ‘fact’, that seems to surprise many, is that parts are not “recursive”. Yes, in all our domain modelling experiments, [17], we have not come across the need for recursively observing compound parts. Trees, for example, are not recursive in this sense. Trees have roots. Sub-trees not. Banyan trees¹⁸ have several “intertwined trees”. But it would be ‘twisting’ the modelling to try fit a description of such trees to a ‘recursion wim’! Instead, trees are defined as nets, such as are road nets, where these nets then satisfy certain constraints [17, Chapter B].

¹⁵The concepts of monitorable, including biddable part attributes is treated in Sect. 7.2.3.2.

¹⁶testing, model checking and theorem proving

¹⁷The outcome of applying an analysis predicate of the prescribed kind may be **chaos** if the prerequisites for its application does not hold.

¹⁸<https://www.britannica.com/plant/banyan>

7.2 Internal Qualities – Intangibles

The previous section has unveiled an ontology of the external qualities of endurants. The unveiling consisted of two elements: a set of analysis predicates, predicates 6–17, and analysis functions, schemas 1–2, and a pair of description functions, schema 3.

The application of description functions result in RSL text.

That text conveys certain properties of domains: that they consists of such-and-such endurants, notably parts, and that these endurants “derive” from other endurants. But the RSL description texts do not “*give flesh & blood*” to these endurants. Questions like: ‘*what are their spatial extents?*’, ‘*how much do they weigh?*’, ‘*what colour do they have?*’, et cetera, are left unanswered. In the present section we shall address such issues. We call them *internal qualities*.

CHARACTERIZATION 19. *Internal Qualities*: Internal qualities are those properties [of endurants] that do not occupy *space* but can be measured or spoken about •

EXAMPLE 17. *Internal qualities*: Examples of internal qualities are the *unique identity* of a part, the *mereological relation* of parts to other parts, and the endurant *attributes* such as temperature, length, colour, etc. •

This section therefore introduces a number of domain description tools:

- **uid_**: the unique identifier observer of parts;
- **mereo_**: the mereology observer of parts;
- **attr_**: (zero,) one or more attribute observers of endurants; and
- **attributes_**: the attribute query of endurants.

7.2.1 *Unique Identity. Ontological Choice*: We postulate that separately discernible parts have unique identify. The issue, really, is a philosophical one. We refer to [14, *Sects. 2.2.2.3–2.2.2.4, pages 14–15*] for a discussion of the existence and uniqueness of entities •

CHARACTERIZATION 20. *Unique Identity*: A unique identity is an immaterial property that distinguishes any two *spatially* distinct solids¹⁹ •

The unique identity of a part p of type P is obtained by the postulated observer **uid_P**:

SCHEMA 4. *Describe-Unique-Identity-Part-Observer*

“**type**
P,PI
value
uid_P: $P \rightarrow PI$ ” •

Here PI is the type of the unique identifiers of parts of type P .

EXAMPLE 18. *Unique Road Transport Identifiers*: The unique identifiers of a road transport, $rt:RT$, consists of the unique identifiers of the

- | | |
|--------------------------------------|-------------------------------|
| • road transport – $rti:RTI$, | • (set of) hubs, $hai:AHl$, |
| • (Cartesian) road net – $rni:RNI$, | • (set of) links, $lai:LAI$, |
| • (set of) automobiles – $sa:SAI$, | • hub, $hi:HI$, and |
| • automobile, $ai:AI$, | • link, $li:LI$, |

¹⁹For pragmatic reasons we do not have to speculate as to whether “bodies” of fluids can be ascribed unique identity. The pragmatics is that we, in our extensive modelling experiments have not found a need for such ascription!

where the type names are all coined, i.e., more-or-less freely chosen, by the domain analyzer cum describer – though, as You can see, these names were here formed by “suffixing” ls to relevant part names •

We have thus introduced a core domain modelling tool the **uid**_... observer function, one to be “applied” mentally by the domain describer, and one that appears in (RSL) domain descriptions The **uid**_... observer function is “applied” by the domain describer, it is not a computable function.

7.2.1.1 *Uniqueness of Parts.* No two parts have the same unique identifier.

EXAMPLE 19. *Road Transport Uniqueness:*

variable

$$\begin{aligned} hs_{uids}:Hl\text{-set} &:= \{ \mathbf{uid_H}(h) \mid h:H \bullet u \in \sigma \} \\ ls_{uids}:Ll\text{-set} &:= \{ \mathbf{uid_L}(l) \mid l:L \bullet u \in \sigma \} \\ as_{uids}:Al\text{-set} &:= \{ \mathbf{uid_A}(a) \mid a:A \bullet u \in \sigma \} \\ \sigma_{uids}:(H|L|A)\text{-set} &:= \{ \mathbf{uid_}(H|L|A)(u) \mid u:(H|L|A) \bullet u \in \sigma \} \end{aligned}$$

axiom

$$\square \mathbf{card} \sigma = \mathbf{card} \sigma_{uids} \quad \bullet \text{ For } \sigma \text{ see Sect. 7.1.4.}$$

We have chosen, for the same reason as given in Sect. 7.1.4, to model a unique identifier state. The \square [*always*] prefix in the **axiom** then expresses that changes of parts or addition of parts to and deletions of parts from the domain shall maintain their uniqueness over time (i.e., always).

7.2.2 *Mereology.* The concept of mereology is due to the Polish mathematician, logician and philosopher Stanisław Leśniewski (1886–1939) [10; 43].

CHARACTERIZATION 21. *Mereology:* Mereology is a theory of [endurant] part-hood relations: of the relations of an [endurant] parts to a whole and the relations of [endurant] parts to [endurant] parts within that whole •

Ontological Choice: Stanisław Leśniewski was not satisfied with Bertrand Russell’s “repair” of Gottlob Frege’s axiom systems for set theory. Instead he put forward his axiom system for, as he called it, mereology. Both as a mathematical theory and as a philosophical reasoning •

EXAMPLE 20. *Mereology:* Examples of mereologies are that a link is topologically *connected* to exactly one or, usually, two specific hubs, that hubs are *connected* to zero, one or more specific links, and that links and hubs are *open* to the traffic of specific subsets of automobiles •

Mereologies can be expressed in terms of unique identifiers.

EXAMPLE 21. *Mereology Representation:* For our ‘running road transport example’ the mereologies of links, hubs and automobiles can thus be expressed as follows:

- **mereo_L**(l) = { hi', hi'' } where hi, hi', hi'' are the unique identifiers of the hubs that the link connects, i.e., are in hs_{uids} ;
- **mereo_H**(h) = { li_1, li_2, \dots, li_n } where li_1, li_2, \dots, li_n are the unique identifiers of the links that are imminent upon (i.e., emanates from) the hub, i.e., are in ls_{uids} ; and
- **mereo_A**(a) = { ri_1, ri_2, \dots, ri_m } where ri_1, ri_2, \dots, ri_m are unique identifiers of the road (hub and link) elements that make up the road net, i.e., are in $hs_{uids} \cup ls_{uids}$ •

Once the unique identifiers of all parts of a domain has been described we can analyses and describe their mereologies. The inquiry: **mereo_P**(p) yields a mereology type (name), say PMer, and its description²⁰:

²⁰Cf. Sect. 7.1.3.1.5

SCHEMA 5. *Describe-Mereology*

“**type**
 $\text{PMer} = \mathcal{M}(\text{PI1}, \text{PI2}, \dots, \text{PI}m)$
value
mereo_P: $P \rightarrow \text{PMer}$
axiom
 $\mathcal{A}(\text{pm}:\text{PMer})” \bullet$

where $\mathcal{M}(\text{PI1}, \text{PI2}, \dots, \text{PI}m)$ is a type expression over unique identifier types of the domain; **mereo_P** is the mereology observer function for parts $p:P$; and $\mathcal{A}(\text{pm}:\text{PMer})$ is an axiom that secures that the unique identifiers of any part are indeed of parts of the domain.

7.2.3 *Attributes*. Attributes are what finally gives “life” to endurants: The external qualities “only” named and gave structure to their atomic or compound types. The internal qualities of uniqueness and mereology are intangible quantities. The internal quality of attributes gives “flesh & blood” to endurants: they let us express endurant properties that we can more easily, i.e., concretely, relate to.

7.2.3.1 *General*.

CHARACTERIZATION 22. *Attributes*: Attributes are properties of endurants that can be measured either physically (by means of length (ruler) and spatial quantity measuring equipment, electronically, chemically, or otherwise) or can be objectively spoken about •

Ontological Choice: First some empirical observation: in reasoning about “the world around us” we express its properties in terms of predicates. These predicates, for example: “*that building’s wall is red*”, *building* refers to an endurant part whereas *wall* and *red* refers to attributes. Now the “rub”: endurant attributes is what give “*flesh & blood*” to domains •²¹

Attributes are of types and, accordingly have values.

We postulate an informal domain analysis function, `record_attribute_type_names`: The domain analyzer, in observing a part, $p:P$, analyzes it into the set of attribute names of parts $p:P$

SCHEMA 6. *record-attribute-type-names*

value
`record_attribute_type_names`: $P \rightarrow \eta\mathbb{T}\text{-set}$
`record_attribute_type_names(p:P)` as $\eta\mathbb{T}\text{-set}$ •

EXAMPLE 22. *Road Net Attributes, I*: Examples of attributes are: hubs have states, $h\sigma:H\Sigma$: the set of pairs of link identifiers, $(f\text{li}, t\text{li})$, of the links *from* and *to* which automobiles may enter, respectively leave the hub; and hubs have state spaces, $h\omega:H\Omega$: the set of hub states “signaling” which states are open/closed, i.e., green/red; links that have lengths, LEN ; and automobiles have road net positions, $A\text{Pos}$, either *at a hub*, atH , or *on a link*, onL , some fraction, f :**Real**, down a link, identified by li , from a hub, identified by fhi , towards a hub, identified by thi . Hubs and links have *histories*: time-stamped, chronologically ordered sequences of automobiles entering and leaving links and hubs, with automobile histories similarly recording hubs and links entered and left.

²¹Editorial remark: I am not yet satisfied with this reasoning. The issue is: to force the concept of attributes to be justified philosophically, as an inevitable element of any world description, and not being forced upon us solely from empirical evidence.

type $H\Sigma = (LI \times LI)\text{-set}$ $H\Omega = H\Sigma\text{-set}$ $LEN = \mathbf{Nat\ m}$ $APos = atH \mid onL$ $atH :: HI$ $onL :: LI \times (fhi:HI \times f:Real \times thi:HI)$ $HHis, LHis = (TIME \times AI)^*$ $AHis = (TIME \times (HI \mid LI))^*$	$attr_H\Sigma: H \rightarrow H\Sigma$ $attr_H\Omega: H \rightarrow H\Omega$ $attr_LEN: L \rightarrow LEN$ $attr_APos: A \rightarrow APos$ $attr_HHis: H \rightarrow HHis$ $attr_LHis: L \rightarrow LHis$ $attr_AHis: A \rightarrow AHis$
value	axiom $\forall (li, (fhi, f, thi)): onL \bullet 0 < f < 1$ $\wedge li \in l_{s_{uids}} \wedge \{fhi, thi\} \subseteq h_{s_{uids}} \wedge \dots \bullet$

SCHEMA 7. *Describe-endurant-attributes(e:E)*

```

let { $\eta A1, \eta A2, \dots, \eta An$ } = record_attribute_type_names(e:E) in
“type
   $A1, A2, \dots, An$ 
value
   $attr\_A1: E \rightarrow A1, attr\_A2: E \rightarrow A2, \dots, attr\_An: E \rightarrow An$ 
axiom
   $\forall a1:A1, a2:A2, \dots, an:An: \mathcal{A}(a1, a2, \dots, an)$ ”
end •

```

7.2.3.2 *Michael A. Jackson’s Attribute Categories.* Michael A. Jackson [33] has suggested a hierarchy of attribute categories: from *static* (is_static^{22}) to *dynamic* ($is_dynamic^{23}$) values – and within the dynamic value category: *inert* values (is_inert^{24}), *reactive* values ($is_reactive^{25}$), *active* values (is_active^{26}) – and within the dynamic active value category: *autonomous* values ($is_autonomous^{27}$), *biddable* values ($is_biddable^{28}$), and *programmable* values ($is_programmable^{29}$). We postulate informal domain analysis predicates, “performed” by the domain analyzer:

value
 $is_static, is_autonomous, is_biddable, is_programmable$ [etc.]: $\eta T \rightarrow \mathbf{Bool}$

We refer to [33] and [14] [*Chapter 5, Sect. 5.4.2.3*] for details. We suggest a minor revision of Michael A. Jackson’s attribute categorization, see left side of Fig. 2. We single out the inert from the ontology of Fig. 2, left side. Inert attributes seem to be “set externally” to the endurant. So we now distinguish between $is_external$ and $is_internal$ dynamic attributes. We summarize Jackson’s attribute and our revised categorization in Fig. 2.

This distinction has [pragmatic] consequences for how we treat arguments of the behaviours of parts, cf. Sect. 8.5.1 (page 21).

²²static: values are constants, cannot change

²³dynamic: values are variable, can change

²⁴inert: values can only change as the result of external stimuli where these stimuli prescribe new values

²⁵reactive: values, if they vary, change in response to external stimuli, where these stimuli either come from outside the domain of interest or from other endurants.

²⁶active: values can change (also) on their own volition

²⁷autonomous: values change only “on their own volition”; the values of an autonomous attributes are a “law unto themselves and their surroundings”.

²⁸biddable: values are prescribed but may fail to be observed as such

²⁹programmable: values can be prescribed

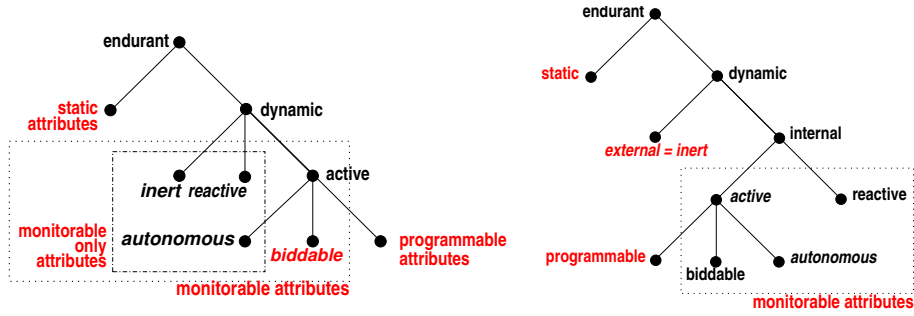


Fig. 2. Michael Jackson's [Revised] Attribute Categories

EXAMPLE 23. *Road Net Attributes, II*: The link length and hub state space attributes are static, hub states and automobile positions programmable. Automobile speed and acceleration attributes, which we do not model, are monitorable •

The attributes categorization determines, in the next major section on perdurants, the treatment of hub, link and automobile behaviours.

7.2.3.3 *Analytic Attribute Extraction Functions*:. For later purpose we need characterize three specific attribute category extraction functions: `static_attributes`, `monitorable_attributes`, and `programmable_attributes`:

value

`p:P`
`tns = record_attribute_type_names(p)`

`static_attributes: $\eta T\text{-set} \rightarrow \eta T\text{-set}$`
`static_attributes(tns) $\equiv \{ \eta tn \mid \eta tn:\eta T \bullet \eta tn \in tns \wedge is_static(tn) \}$`

`inert_attributes: $\eta T\text{-set} \rightarrow \eta T\text{-set}$`
`inert_attributes(tns) $\equiv \{ \eta tn \mid \eta tn:\eta T \bullet \eta tn \in tns \wedge is_inert(tn) \}$`

`monitorable_attributes $\eta T\text{-set} \rightarrow \eta T\text{-set}$`
`monitorable_attributes(tns) $\equiv \{ \eta tn \mid \eta tn:\eta T \bullet \eta tn \in tns \wedge is_monitorable(tn) \}$`

`programmable_attributes $\eta T\text{-set} \rightarrow \eta T\text{-set}$`
`programmable_attributes(tns) $\equiv \{ \eta tn \mid \eta tn:\eta T \bullet \eta tn \in tns \wedge is_programmable(tn) \}$`

`is_monitorable: $T \rightarrow \mathbf{Bool}$`
`is_monitorable(t) $\equiv \sim is_static(t) \wedge \sim is_inert(t) \wedge \sim is_programmable(t)$`

Please be reminded that these functions are informal. They are part of the presentation language. Do not be cofused by their RSL-like appearance.

7.3 Intentional Pull

Ontological Choice: In [40, pages 167–168] Sørlander argues wrt. “*how can entities be the source of forces ?*” and thus reasons for *gravitational pull*. That same kind of reasoning, with proper substitution of terms, leads us to the concept of *intentional pull* •

Two or more parts of different sorts, but with overlapping sets of intents³⁰ may exert an intentional “pull” on one another. This *intentional “pull”* may take many forms. Let $p_x : X$ and $p_y : Y$ be two parts of *different sorts* (X, Y), and with *common intent*, ι . *Manifestations* of these, their common intent must somehow be *subject to constraints*, and these must be *expressed predicatively*.

When a compound artifact models “itself” as put together with a number of other endurants then it does have an intentionality and the components’ individual intentionalities does, i.e., shall relate to that.

EXAMPLE 24. *Road Transport Intentionality:* Automobiles include the *intent* of ‘transport’, and so do *hubs* and *links*. *Manifestations* of “transport” are reflected in *hubs*, *links* and *automobiles* having the *history* attribute. The *intentional “pull”* of these manifestations is this: For every automobile, if it records being in some hub or on some link at time τ , then the designated hub, respectively link, records exactly that automobile; and vice versa: for every hub [link], if it records the visit of some automobile at time τ , then the designated automobile records exactly that hub [link]. We leave the formalization of the above to the reader •

EXAMPLE 25. *Double-entry Bookkeeping:* Another example of intentional “pull” is that of double-entry book-keeping. Here the income/expense ledger must balance the actives/passives ledger •

EXAMPLE 26. *The Henry George Theorem:* The Henry George theorem states that under certain conditions, aggregate spending by government on public goods will increase aggregate rent based on land value (land rent) more than that amount, with the benefit of the last marginal investment equaling its cost •^{31,32}

7.4 Summary of Endurants

We have completed our treatment of endurants. That treatment was based on an ontology for the observable phenomena of domains – such as we have delineated the concept of domains. The treatment was crucially based on an ontology for the structure of domain phenomena, and, in a sense, “alternated” between analysis predicates, analysis functions, and description functions. We have carefully justified this ontology in ‘**Ontological Choice**’ paragraphs

8 PERDURANT CONCEPTS

The main contribution of this section is that of *transcendentally deducing* perdurants from endurant parts, in particular *behaviours* “of” parts.

Major perdurants are those of actions, events and behaviours with behaviours generally being sets of sequences of actions, events and behaviours.

³⁰Intent: purpose; God-given or human-imposed !

³¹Stiglitz, Joseph (1977). “The Theory of Local Public Goods”. In Feldstein, M.S.; Inman, R.P. (eds.). *The Economics of Public Services*. Palgrave Macmillan, London. pp. 274–333. doi:10.1007/978-1-349-02917-4_12. ISBN 978-1-349-02919-8.

³²Henry George (September 2, 1839 – October 29, 1897) was an American political economist and journalist. His writing was immensely popular in 19th-century America and sparked several reform movements of the Progressive Era. He inspired the economic philosophy known as Georgism, the belief that people should own the value they produce themselves, but that the economic value of land (including natural resources) should belong equally to all members of society. George famously argued that a single tax on land values would create a more productive and just society.

8.1 “Morphing” Parts into Behaviours

As already indicated we shall transcendently deduce (perdurant) behaviours from those (endurant) parts which we, as domain analyzers cum describers, have endowed with all three kinds of internal qualities: unique identifiers, mereologies and attributes. We shall use the CSP [30] constructs of RSL (derived from RSL [25]) to model concurrent behaviours.

8.2 Transcendental Deduction

CHARACTERIZATION 23. *Transcendental*: By transcendental we shall understand the philosophical notion: the a priori or intuitive basis of knowledge, independent of experience •

A priori knowledge or intuition is central: By *a priori* we mean that it not only precedes, but also determines rational thought.

CHARACTERIZATION 24. *Transcendental Deduction*: By a transcendental deduction we shall understand the philosophical notion: a transcendental “conversion” of one kind of knowledge into a seemingly different kind of knowledge •

EXAMPLE 27. *Transcendental Deductions – Informal Examples*: We give some intuitive examples of transcendental deductions. They are from the “domain” of programming languages. There is the syntax of a programming language, and there are the programs that supposedly adhere to this syntax. Given that, the following are now transcendental deductions.

The software tool, a *syntax checker*, that takes a program and checks whether it satisfies the syntax, including the statically decidable context conditions, i.e., the *statics semantics* – such a tool is one of several forms of transcendental deductions.

The software tools, an *automatic theorem prover* and a *model checker*, for example SPIN [31], that takes a program and some theorem, respectively a Promela statement, and proves, respectively checks, the program correct with respect the theorem, or the statement.

A *compiler* and an *interpreter* for any programming language.

Yes, indeed, any *abstract interpretation* [22] reflects a transcendental deduction: firstly, these examples show that there are many transcendental deductions; secondly, they show that there is no single-most preferred transcendental deduction •

Ontological Choice: So this, then, is, in a sense, our “final” ontological choice: that of transcendently deduce behaviours from, or of, parts •

8.3 Actors – A Synopsis

This section provides a summary overview.

CHARACTERIZATION 25. *Actors*: An actor is anything that can initiate an **action, event** or **behaviour** •

8.3.1 Action.

CHARACTERIZATION 26. *Actions*: An action is a function that can purposefully change a state •

EXAMPLE 28. *Road Net Actions*: These are some road transport actions: an automobile leaving a hub, entering a link; leaving a link, entering a hubs; entering the road net; and leaving the road net •

8.3.2 Event.

CHARACTERIZATION 27. *Events*: An event is a function that surreptitiously changes a state •

EXAMPLE 29. *Road Net Events*: These are some road net events: The blocking of a link due to a mud slide; the failing of a hub traffic signal due to power outage; an automobile failing to drive; and the blocking of a link due to an automobile accident •

We shall not formalize events.

8.3.3 Behaviour.

CHARACTERIZATION 28. *Behaviours*: Behaviours are sets of sequences of actions, events and behaviours •

Concurrency is modeled by the *sets* of sequences. Synchronization and communication of behaviours are effected by CSP *output/inputs*: $ch[\{i,j\}]!value/ch[\{i,j\}]?$.

EXAMPLE 30. *Road Net Traffic*: Road net traffic can be seen as a behaviour of all the behaviours of automobiles, where each automobile behaviour is seen as sequence of start, stop, turn right, turn left, etc., actions; of all the behaviours of links where each link behaviour is seen as a set of sequences (i.e., behaviours) of “following” the link entering, link leaving, and movement of automobiles on the link; of all the behaviours of hubs (etc.); of the behaviour of the aggregate of roads, viz. *The Department of Roads*, and of the behaviour of the aggregate of automobiles, viz. *The Department of Vehicles*.

8.4 Channel

CHARACTERIZATION 29. *Channel*: A channel is anything that allows synchronization and communication of values between behaviours •

SCHEMA 8. *Channel*

We suggest the following schema for describing channels:

“channel { $ch[\{ui,uj\}] \mid ui,ij:UI \bullet \dots$ } M

where ch is the describer-chosen name for an array of channels, ui,uj are channel array indices of the unique identifiers, UI , of the chosen domain •

EXAMPLE 31. *Road Transport Interaction Channel*:

channel { $ch[\{ui,uj\}] \mid \{ui,ij\}:(HI|LI|AI)\text{-set} \bullet ui \neq uj \wedge \{ui,uj\} \subseteq \sigma_{uids}$ } M

Channel array ch is indexed by a “pair” of distinct unique part identifiers of the domain. We shall later outline M , the type of the “messages” communicated between behaviours •

8.5 Behaviours

We single out the perdurants of behaviours – as they relate directly to the parts of Sect. 7. The treatment is “divided” into three sections.

8.5.1 Behaviour Signature.

SCHEMA 9. *Behaviour Signature*

By the *behaviour signature*, for a part p , we shall understand a pair: the name of the behaviour, B_p , and a function type expression as indicated:

value

$B_p: Uid_p \rightarrow^{33} Mereop_p \rightarrow Sta_Vals_p \rightarrow Inert_Vals_p \rightarrow Mon_Refs_p \rightarrow Prgr_Vals_p \rightarrow \{ ch[\{i,j\}] \mid \dots \} \mathbf{Unit}$

³³We have Schönfinckel’ed https://en.wikipedia.org/wiki/Moses_Schönfinckel#Further_reading (Curried <https://en.wikipedia.org/wiki/Currying>) the function type

We explain:

- Uid_p is the type of unique identifiers of part p , $\mathbf{uid}_P(p) = \text{Uid}_p$;
- Mereo_p is the type of the mereology of part p , $\mathbf{mereo}_P(p) = \text{Mereo}_p$;
- Sta_Vals_p is a Cartesian of the type of inert attributes of part p . Given $\text{record_attribute_type_names}(p)$ $\text{static_attributes}(\text{record_attribute_type_names}(p))$ yields Sta_Vals_p ;
- Inert_Vals_p is a Cartesian of the type of static attributes of part p . Given $\text{record_attribute_type_names}(p)$ $\text{inert_attributes}(\text{record_attribute_type_names}(p))$ yields Inert_Vals_p ;
- Mon_Refs_p is a Cartesian of the $\mathbf{attr_ibute}$ observer functions of the types of monitorable attributes of part p . Given $\text{record_attribute_type_names}(p)$ analysis function $\text{monitorable_attributes}(\text{record_attribute_type_names}(p))$ yields Mon_Vals_p ;
- Prgr_Vals_p is a Cartesian of the type of programmable attributes of part p . Given $\text{record_attribute_type_names}(p)$ analysis function $\text{programmable_attributes}(\text{record_attribute_type_names}(p))$ yields Prgr_Vals_p ;
- $\{ \text{ch}[\{i,j\}] \mid \dots \}$ specifies the channels over which part p behaviours, B_p , may communicate;

and:

- **Unit** is the type name for the () value³⁴ •

The Cartesian arguments may “degenerate” to the non-Cartesian of no, or just one type identifier, In none, i.e., (), then () may be skipped. If one, e.g., (a), then (a) is listed.

EXAMPLE 32. *Road Transport Behaviour Signatures:*

value

```

hub: H1 → MereoH → (HΩ × ...) → (...) → (HHist × ...)
      → {ch[ {uid_H(p), ai} ] | ai: A1 • ai ∈ asuid } Unit
link: L1 → MereoL → (LEN × ...) → (...) → (LHist × ...)
      → {ch[ {uid_L(p), ai} ] | ai: A1 • ai ∈ asuid } Unit
automobile: A1 → MereoA → (...) → (attr_AVel × attr_HAcc × ...) → (APos × AHist × ...)
      → {ch[ {uid_H(p), ri} ] | ri: (H1 | L1) • ri ∈ hsuid Ulsuid } Unit

```

Here we have suggested additional part attributes: monitorable automobile velocity and acceleration, AVel, AAcc, and omitted other attributes •

8.5.2 *Inert Arguments: Some Examples.* Let us give some examples of inert attributes of automobiles. (i) Driving uphill, one a level road, or downhill, exert some inert “drag” or “pull”. (ii) Velocity can be treated as a reactive attribute – but it can be [approximately] calculated on the basis of, for example, these inert attributes: drag/pull and accelerator pedal pressure, and the static engine power attribute.

8.5.3 *Behaviour Invocation.*

SCHEMA 10. *Behaviour Invocation*

Behaviours are invoked as follows:

$$\begin{aligned}
 & \text{“}B_p(\mathbf{uid}_p(p))\text{”}^{35} \\
 & \quad (\mathbf{mereo}_P(p)) \\
 & \quad \quad (\mathbf{attr_sta}A_1(p), \dots, \mathbf{attr_sta}A_s(p)) \\
 & \quad \quad \quad (\mathbf{attr_inert}A_1(p), \dots, \mathbf{attr_inert}A_i(p)) \\
 & \quad \quad \quad \quad (\mathbf{attr_mon}A_1, \dots, \mathbf{attr_mon}A_m)
 \end{aligned}$$

³⁴– You may “read” () as the value yielded by a statement, including a never-terminating function

$$(\mathbf{attr_prgA}_1(p), \dots, \mathbf{attr_prgA}_p(p))$$

- All arguments are passed *by value*.
- The *uid* value is never changed.
- The *mereology* value is usually not changed.
- The *static attribute* values are fixed, never changed.
- The *inert attribute* values are fixed, but can be updated by receiving explicit input communications.
- The *monitorable attribute* values are functions, i.e., it is as if the “actual” monitorable values are passed *by name*!
- The *programmable attribute* values are usually changed, “updated”, by actions described in the behaviour definition •

8.5.4 Argument References. Within behaviour descriptions, see next section, references are made to the behaviour arguments. References, *a*, to *unique identifier*, *mereology*, *static* and *programmable attribute* arguments yield their value. References, *a*, to *monitorable attribute* arguments also yield their value. This value is an **attr_A** observer function. To yield, i.e., read, the monitorable attribute value this function is applied to that behaviour’s uniquely identified part, p_{uid} , in the global part state, σ . To update, i.e., write, say, to a value v , for the case of a *biddable*, *monitorable* attribute, that behaviour’s uniquely identified part, p_{uid} , in the global part state, σ , shall have part p_{uid} ’s A attribute changed to v – with all other attribute values of p_{uid} unchanged. Common to both the read and write functions is the *retrieve part* function:

- (1) Given a unique part identifier, pi , assumed to be that of an existing domain part,
- (2) *retr_part* reads the global [all parts] variable σ to retrieve that part p whose unique part identifier is pi .

value

2. *retr_part*: $PI \rightarrow P$ **read**
2. *retr_part*(pi) \equiv **let** $p:P \bullet p \in \mathbf{c} \sigma \wedge uid_P(p)=pi$ **in** p **end**
1. **pre**: $\exists p:P \bullet p \in \mathbf{c} \sigma \wedge uid_P(p)=pi$

You may think of the functions being illustrated in this section, Sect. 8.5.4, *retr_part*, *read_A_from_P* and *update_P_with_A*, as “belonging” to the description language, but here suitably expressed for any domain, that is, with suitable substitutions for A and P.

8.5.4.1 Evaluation of Monitorable Attributes.

- (3) Let $pi:PI$ be the unique identifier of any part, p , with monitorable attributes, let A be a monitorable attribute of p , and let ηA be the name of attribute A.
- (4) Evaluation of the [current] attribute A value of p is defined by function *read_A_from_P*.

value

3. $pi:PI, a:A, \eta A:\eta T$
4. *read_A_from_P*: $PI \times T \rightarrow$ **read** σA
4. *read_A*($pi, \eta A$) \equiv *attr_A*(*retr_part*(pi))

8.5.4.2 Update of Biddable Attributes.

- (5) The update of a monitorable attribute A, with attribute name ηA of part p , identified by pi , to a new value **writes** to the global part state σ .
- (6) Part p is retrieved from the global state.
- (7) A new part, p' is formed such that p' is like part p :

³⁵We show the arguments of the invocation on separate lines only for readability. That is: normally we show the invocation arguments as $B(\dots)(\dots)(\dots)(\dots)(\dots)$.

- (a) same unique identifier,
 - (b) same mereology,
 - (c) same attributes values,
 - (d) except for A.
- (8) That new p' replaces p in σ .

value

5. $\sigma, a:A, pi:PI, \eta A:\eta T$
5. $update_P_with_A: PI \times A \times \eta T \rightarrow \mathbf{write} \sigma$
5. $update_P_with_A(pi,a,\eta A) \equiv$
6. **let** $p = retr_part(pi)$ **in**
7. **let** $p':P \bullet$
- 7a. $uid_P(p')=pi$
- 7b. $\wedge mereo_P(p)=mereo_P(p')$
- 7c. $\wedge \forall \eta A' \in record_attribute_type_names(p) \setminus \{\eta A\} \Rightarrow attr_A'(p)=attr_A'(p')$
- 7d. $\wedge attr_A(p')=a$ **in**
8. $\sigma := c \sigma \setminus \{p\} \cup \{p'\}$
5. **end end**
6. **pre:** $\exists p:P \bullet p \in c \sigma \wedge uid_P(p)=pi$

8.5.5 *Behaviour Description – Examples.* Behaviour descriptions rely strongly on CSPs' [30] expressivity. Leaving out some details ($_$, $' \dots'$), and *without “further ado”*, we exemplify.

EXAMPLE 33. *Automobile Behaviour at Hub:*

- (9) We abstract automobile behaviour at a Hub (hi).
- (a) Either the automobile remains in the hub,
 - (b) or, internally non-deterministically,
 - (c) leaves the hub entering a link,
 - (d) or, internally non-deterministically,
 - (e) stops.
- 9 $automobile(ai)(ris)(...)(atH(hi),ahis,_) \equiv$
- 9a $automobile_remains_in_hub(ai)(ris)(...)(atH(hi),ahis,_)$
- 9b \sqcap
- 9c $automobile_leaving_hub(ai)(ris)(...)(atH(hi),ahis,_)$
- 9d \sqcap
- 9e $automobile_stop(ai)(ris)(...)(atH(hi),ahis,_)$
- (10) [9a] The automobile remains in the hub:
- (a) time is recorded,
 - (b) the automobile remains at that hub, “idling”,
 - (c) informing (“first”) the hub behaviour.
- 10 $automobile_remains_in_hub(ai)(ris)(...)(atH(hi),ahis,_) \equiv$
- 10a **let** $\tau = record_TIME$ **in**
- 10c $ch[\{ai,hi\}] ! \tau ;$
- 10b $automobile(ai)(ris)(...)(atH(hi),\langle \tau,hi \rangle \hat{~} ahis,_) \mathbf{end}$

- (11) [9c] The automobile leaves the hub entering link li :
- (a) time is recorded;
 - (b) hub is informed of automobile leaving and link that it is entering;
 - (c) “whereupon” the vehicle resumes (i.e., “while at the same time” resuming) the vehicle behaviour positioned at the very beginning (0) of that link.

```

11 automobile_leaving_hub(ai)({li}∪ris)(...)(atH(hi),ahis,_) ≡
11a  let  $\tau = \text{record\_TIME}$  in
11b  (ch[ {ai,hi} ] !  $\tau$  || ch[ {ai,li} ] !  $\tau$  );
11c  automobile(ai)(ris)(...)(onL(li,(hi,0,_)),(( $\tau$ ,li))^ahis,_) end
11  pre: [hub is not isolated]

```

The choice of link entered is here expressed (11) as a non-deterministic choice³⁶. One can model the leave hub/enter link otherwise.

- (12) [9e] Or the automobile “disappears — off the radar” !

```

12 automobile_stop(ai)(ris),(...)(atH(hi),ahis,_) ≡ stop •

```

8.6 Behaviour Initialization.

For every manifest part it must be described how its behaviour is initialized.

EXAMPLE 34. *Road Transport Initialization*: We “wrap up” the main example of this paper: We omit treatment of monitorable attributes.

- (13) Let us refer to the system initialization as an action.
- (14) All hubs are initialized,
- (15) all links are initialized, and
- (16) all automobiles are initialized.

value

```

13. rts_initialisation: Unit → Unit
13. rts_initialisation() ≡
14.  || { hub(uid_H(l))(mereo_H(l))(attr_HΩ(l),...)(attr_HΣ(l),...) | h:H • h ∈ hs }
15.  || || { link(uid_L(l))(mereo_L(l))(attr_LEN(l),...)(attr_LΣ(l),...) | l:L • l ∈ ls }
16.  || || { automobile(uid_A(a))(mereo_A(a))(attr_APos(a)attr_AHis(a),...) | a:A • a ∈ as }

```

We have here omitted possible monitorable attributes. For hs, ls, as we refer to Sect. 7.1.4 •

9 CONCLUSION

We have summarized a method to be used by [human] domain analyzers cum describers in studying and modelling domains. Our previous publications [12–14] have, with this paper, found its most recent, we risk to say, for us, final form.

Of course, domain models can be developed without the calculi presented in this paper. And was for many years. From the early 1990s a number of formal models of railways were worked out [4–6; 19; 27]. The problem, though, was still, between 1992 and 2016, “*where to begin, how to proceed and when to end*”. The domain analysis & description ontology and, hence calculus, of this paper shows how. The systematic approach to domain modelling of this ontology and calculus has stood its test of time. The Internet ‘publication’ [18] presents 19 domain models from the 2007–2024 period.

³⁶ – as indicated by the **pre**- condition: the hub mereology must specify that it is not isolated. Automobiles can never leave isolated hubs.

9.1 Previous Literature

To the best of my knowledge there is no prior, comparable publications in the field of domain science and engineering. Closest would be Michael A. Jackson's [35]. Well, most computer scientists working in the field of correctness of programs, from somewhat "early on", stressed the importance of making proper assumptions about the domain, They would then express these "in-line", as appropriate predicates, with their proofs. Michael A. Jackson, lifted this, to a systematic treatment of the domain in his triplet 'Problem Frame Approach': *program, machine, domain* [34]. But Jackson did not lift his problem frame concern into a proper study of domains.

9.2 Domain Facets

There is more to *domain modelling* than covered in this paper. In [9] and in [14, *Chapter 8*] we cover the concept of *domain facets*. General examples of domain facets are *support technologies, rules & regulations, scripts, license languages, management & organization, and human behaviour*.

9.3 Perspectives

Domain models can be developed for either of a number of reasons:

- (i) in order to *understand* a human-artifact domain;
- (ii) in order to *re-engineer the business processes* of a human-artifact domain; or
- (iii) in order to develop *requirements prescriptions* and, subsequently *software application* "within" that domain.

[(ii)] We refer to [28; 29] and [7, *Vol. 3, Chapter 19, pages 404–412*] for the concept of *business process engineering*.

[(iii)] We refer to [14, *Chapter 9*] for the concept of *requirements engineering*.

9.4 The Semantics of Domain Models

The meaning of domain models, such as we describe them in this paper, is, "of course", the actual, real domain "out there"! One could, and, perhaps one should, formulate a mathematical semantics of the models, that is, of the **is_...**, **obs_...**, **uid_...**, **mereo_...** and **attr_...** analysis and description functions and what they entail (e.g., the type name labels: ηT 's; etc.). An early such semantics description is given in [11].

9.5 Further on Domain Modelling

Additional facets of domain modelling are covered in [8] and [14, *Chapter 8: Domain Facets*.]

9.6 Software Development

[8] and [14, *Chapter 9 Requirements*] show how to develop *Requirements prescriptions* from *Domain descriptions*. [7] shows how to develop *Software designs* from *Requirements prescriptions*.

9.7 Modeling

Domain descriptions, such as outlined in this paper, are models of domains, that is, of some reality. They need not necessarily lead to or be motivated by possible development of software for such domains. They can be experimentally researched and developed just for the sake of understanding domains in which man has had an significantly influence. They are models. We refer to [23] for complementary modeling based on Petri nets. The current author is fascinated by the interplay between graphical and textual descriptions of HERAKLIT, well, in general Petri Nets.

9.8 Philosophy of Computing

The Danish philosopher Kai Sørlander [39–42] has shown that there is a foundation in philosophy for domain analysis and description. We refer to [15, *Chapter 2*] for a summary of his findings.

REFERENCES

- [1] H. Bekič, D. Bjørner, W. Henhapl, C.B. Jones, and P. Lucas. A Formal Definition of a PL/I Subset. Technical Report 25.139, Vienna, Austria, December 1974.
- [2] Hans Bekič, Peter Lucas, Kurt Walk, and Many Others. Formal Definition of PL/I, ULD Version III. IBM Laboratory, Vienna, 1969.
- [3] D. Bjørner and O. Oest. *Towards a Formal Description of Ada*, volume 98 of *LNCS*. Springer–Verlag, 1980.
- [4] Dines Bjørner. Formal Software Techniques in Railway Systems. In Eckehard Schnieder, editor, *9th IFAC Symposium on Control in Transportation Systems*, pages 1–12, Technical University, Braunschweig, Germany, 13–15 June 2000. VDI/VDE-Gesellschaft Mess- und Automatisierungstechnik, VDI-Gesellschaft für Fahrzeug- und Verkehrstechnik. Invited talk.
- [5] Dines Bjørner. Dynamics of Railway Nets: On an Interface between Automatic Control and Software Engineering. In *CTS2003: 10th IFAC Symposium on Control in Transportation Systems*, Oxford, UK, August 4-6 2003. Elsevier Science Ltd. Symposium held at Tokyo, Japan. Editors: S. Tsugawa and M. Aoki. www2.imm.dtu.dk/~dibj/ifac-dynamics.pdf.
- [6] Dines Bjørner. New Results and Trends in Formal Techniques for the Development of Software for Transportation Systems. In *FORMS2003: Symposium on Formal Methods for Railway Operation and Control Systems*. Institut für Verkehrssicherheit und Automatisierungstechnik, Techn.Univ. of Braunschweig, Germany, 15–16 May 2003. Conf. held at Techn.Univ. of Budapest, Hungary. Editors: G. Tarnai and E. Schnieder, Germany. www2.imm.dtu.dk/~dibj/dines-amore.pdf.
- [7] Dines Bjørner. *Software Engineering, Vol. 1: Abstraction and Modelling; Vol. 2: Specification of Systems and Languages; Vol. 3: Domains, Requirements and Software Design*. Texts in Theoretical Computer Science, the EATCS Series. Springer, Heidelberg, Germany, 2006.
- [8] Dines Bjørner. From Domains to Requirements www.imm.dtu.dk/~dibj/2008/ugo/ugo65.pdf. In *Montanari Festschrift*, volume 5065 of *Lecture Notes in Computer Science* (eds. Pierpaolo Degano, Rocco De Nicola and José Meseguer), pages 1–30, Heidelberg, May 2008. Springer.
- [9] Dines Bjørner. Domain Engineering. In Paul Boca, Jonathan Bowen, and Jawed Siddiqi, editors, *Formal Methods: State of the Art and New Directions*, Eds. Paul Boca and Jonathan Bowen, pages 1–42, London, UK, December 2009. Springer.
- [10] Dines Bjørner. A Rôle for Mereology in Domain Science and Engineering. In *Mereology and the Sciences*, Synthese Library (eds. Claudio Calosi and Pierluigi Graziani), pages 323–357, Amsterdam, The Netherlands, October 2014. Springer. <https://www.imm.dtu.dk/~dibj/2011/urbino/urbino-colour.pdf>.
- [11] Dines Bjørner. Domain Analysis: Endurants – An Analysis & Description Process Model www.imm.dtu.dk/~dibj/2014/kanazawa/kanazawa-p.pdf. In Shusaku Iida and José Meseguer and Kazuhiro Ogata, editor, *Specification, Algebra, and Software: A Festschrift Symposium in Honor of Kokichi Futatsugi*. Springer, Heidelberg, Germany, May 2014.
- [12] Dines Bjørner. Manifest Domains: Analysis & Description www.imm.dtu.dk/~dibj/2015/faoc/faoc-bjorner.pdf. *Formal Aspects of Computing*, 29(2):175–225, March 2017. Online: 26 July 2016.
- [13] Dines Bjørner. Domain Analysis & Description – Principles, Techniques and Modeling Languages. www.imm.dtu.dk/~dibj/2018/tosem/Bjorner-TOSEM.pdf. *ACM Trans. on Software Engineering and Methodology*, 28(2):66 pages, March 2019.
- [14] Dines Bjørner. *Domain Science & Engineering – A Foundation for Software Development*. EATCS Monographs in Theoretical Computer Science. Springer, Heidelberg, Germany, 2021. A revised version of this book is [16].
- [15] Dines Bjørner. Domain Modelling – A Primer. A short version of [16]. xii+202 pages³⁷, May 2023.
- [16] Dines Bjørner. Domain Science & Engineering – A Foundation for Software Development. Revised edition of [14]. xii+346 pages³⁸, January 2023.
- [17] Dines Bjørner. Domain Models – A Compendium. Internet: <http://www.imm.dtu.dk/~dibj/2024/models/domain-models.pdf>, March 2024. This is a very early draft. 19 domain models are presented.
- [18] Dines Bjørner. Domain Models – A Compendium. Internet: <http://www.imm.dtu.dk/~dibj/2024/models/domain-models.pdf>, March 2024. This is a very early draft. 19 domain models are presented.
- [19] Dines Bjørner, Chris W. George, and Søren Prehn. Computing Systems for Railways – A Rôle for Domain Engineering. Relations to Requirements Engineering and Software for Control Applications. In *Integrated Design and Process Technology*. Editors: Bernd Kraemer and John C. Petterson, P.O.Box 1299, Grand View, Texas 76050-1299, USA, 24–28 June 2002. Society for Design and Process Science. www2.imm.dtu.dk/~dibj/pasadena-25.pdf.

³⁷This book is currently being translated into Chinese by Dr. Yang ShaoFa, IoS/CAS, Beijing and into Russian by Dr. Mikhail Chupilko, ISP/RAS, Moscow

³⁸Due to copyright reasons no URL is given to this document’s possible Internet location. A primer version, omitting certain chapters, is [15]

- [20] Dines Bjørner and Cliff B. Jones, editors. *The Vienna Development Method: The Meta-Language*, volume 61 of *LNCS*. Springer, Heidelberg, Germany, 1978.
- [21] Dines Bjørner and Cliff B. Jones, editors. *Formal Specification and Software Development*. Prentice-Hall, London, England, 1982.
- [22] Patrick Cousot. *Principles of Abstract Interpretation*. The MIT Press, 2021.
- [23] Peter Fettke and Wolfgang Reisig. *Understanding the Digital World – Modeling with HERAKLIT*. Springer, 2024. To be published.
- [24] K. Futatsugi, A.T. Nakagawa, and T. Tamai, editors. *CAFE: An Industrial-Strength Algebraic Formal Method*, Sara Burgerhartstraat 25, P.O. Box 211, NL-1000 AE Amsterdam, The Netherlands, 2000. Elsevier. Proceedings from an April 1998 Symposium, Numazu, Japan.
- [25] Chris W. George, Peter Haff, Klaus Havelund, Anne Elisabeth Haxthausen, Robert Milne, Claus Bendix Nielsen, Søren Prehn, and Kim Ritter Wagner. *The RAISE Specification Language*. The BCS Practitioner Series. Prentice-Hall, Hemel Hempstead, England, 1992.
- [26] Chris W. George, Anne Elisabeth Haxthausen, Steven Hughes, Robert Milne, Søren Prehn, and Jan Storbank Pedersen. *The RAISE Development Method*. The BCS Practitioner Series. Prentice-Hall, Hemel Hempstead, England, 1995.
- [27] Chris W. George, Hung Dang Van, Tomasz Janowski, and Richard Moore. *Case Studies using The RAISE Method*. FACTS (Formal Aspects of Computing: Theory and Software) and FME (Formal Methods Europe). Springer-Verlag, London, 2002. This book reports on a number of case studies using RAISE (Rigorous Approach to Software Engineering). The case studies were done in the period 1994–2001 at UNU/IIST, the UN University’s International Institute for Software Technology, Macau (till 20 Dec., 1997, Chinese Territory under Portuguese administration, now a Special Administrative Region (SAR) of (the so-called People’s Republic of) China).
- [28] Michael Hammer and James A. Champy. *Reengineering the Corporation: A Manifesto for Business Revolution*. HarperCollinsPublishers, 77–85 Fulham Palace Road, Hammersmith, London W6 8JB, UK, May 1993. 5 June 2001, Paperback.
- [29] Michael Hammer and Stephen A. Stanton. *The Reengineering Revolution: The Handbook*. HarperCollinsPublishers, 77–85 Fulham Palace Road, Hammersmith, London W6 8JB, UK, 1996. Paperback.
- [30] Charles Anthony Richard Hoare. *Communicating Sequential Processes*. C.A.R. Hoare Series in Computer Science. Prentice-Hall International, London, England, 1985. Published electronically: usingcsp.com/cspbook.pdf (2004).
- [31] Gerard J. Holzmann. *The SPIN Model Checker, Primer and Reference Manual*. Addison-Wesley, Reading, Massachusetts, 2003.
- [32] Daniel Jackson. *Software Abstractions: Logic, Language, and Analysis*. The MIT Press, Cambridge, Mass., USA, April 2006. ISBN 0-262-10114-9.
- [33] Michael A. Jackson. *Software Requirements & Specifications: a lexicon of practice, principles and prejudices*. ACM Press. Addison-Wesley, Reading, England, 1995.
- [34] Michael A. Jackson. *Problem Frames — Analyzing and Structuring Software Development Problems*. ACM Press, Pearson Education. Addison-Wesley, England, 2001.
- [35] Michael A. Jackson. Program Verification and System Dependability. In Paul Boca, Jonathan Bowen, and Jawed Siddiqi, editors, *Formal Methods: State of the Art and New Directions*, pages 43–78, London, UK, December 2009. Springer.
- [36] W. Little, H.W. Fowler, J. Coulson, and C.T. Onions. *The Shorter Oxford English Dictionary on Historical Principles*. Clarendon Press, Oxford, England, 1973, 1987. Two vols.
- [37] R. Milne and C. Strachey. *A Theory of Programming Language Semantics*. Chapman and Hall, London, Halsted Press/John Wiley, New York, 1976.
- [38] Karl R. Popper. *Conjectures and Refutations. The Growth of Scientific Knowledge*. Routledge and Kegan Paul Ltd. (Basic Books, Inc.), 39 Store Street, WC1E 7DD, London, England (New York, NY, USA), 1963, ..., 1981.
- [39] Kai Sørlander. *Det Uomgængelige – Filosofiske Deduktioner [The Inevitable – Philosophical Deductions, with a foreword by Georg Henrik von Wright]*. Munksgaard · Rosinante, Copenhagen, Denmark, 1994. 168 pages.
- [40] Kai Sørlander. *Indføring i Filosofien [Introduction to The Philosophy]*. Informations Forlag, Copenhagen, Denmark, 2016. 233 pages.
- [41] Kai Sørlander. *Den rene fornufts struktur [The Structure of Pure Reason]*. Ellekær, Slagelse, Denmark, 2022. See [42].
- [42] Kai Sørlander. *The Structure of Pure Reason*. Publisher to be decided, 2023. This is an English translation of [41] – done by Dines Bjørner in collaboration with the author.
- [43] Achille C. Varzi. *On the Boundary between Mereology and Topology*, pages 419–438. Hölder-Pichler-Tempsky, Vienna, 1994.
- [44] James Charles Paul Woodcock and James Davies. *Using Z: Specification, Proof and Refinement*. Prentice Hall International Series in Computer Science, London, England, 1996.