

DOMAIN MODELLING

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The Triptych Dogma

In order to *specify* **Software**,
we must understand its requirements.

In order to *prescribe* **Requirements**
we must understand the domain

So we must **study, analyze** and **describe** **Domains**.

$$\mathbb{D}, \mathbb{S} \models \mathbb{R}$$

1. DOMAINS

Definition 1 . *Domain*

- By a *domain* we shall understand
 - a *rationally describable* segment of
 - a *discrete dynamics* fragment of
 - a *human assisted* reality, i.e., of the world.
- It includes
 - its **endurants**,
i.e., *solid and fluid entities* of
 - * **parts** and
 - * **living species**,
 - and **perdurants** ■

- By **endurants** we shall understand
 - those quantities of domains
 - that we can observe (see and touch), in space,
 - as “complete” entities at no matter which point in time
 - “material” entities that persists, endures.
- By **perdurants** we shall understand an entity
 - for which only a fragment exists
 - if we look at or touch them at any given snapshot in time.
 - Were we to freeze time
 - we would only see or touch a fragment of the perdurant.

- *Endurants* are
 - either *natural* [“God-given”]
 - or *artefactual* [“man-made”].and may be considered
 - *atomic* or *compound* parts,
 - or, as in this talk, further unanalysed *living species*:
 - * **plants** and
 - * **animals** – including *humans*.
- *Perdurants* are here considered to be
 - *actions*,
 - *events* and
 - *behaviours*.
- *Perdurants* are **transcendentally deduced** from *parts*.

Example 1 . Domains: A few, more-or-less self-explanatory examples:

- **Rivers** – with their natural sources, deltas, tributaries, waterfalls, etc., and their man-made dams, harbours, locks, etc. – and their conveyage of materials (ships etc.) [19];
- **Road nets** – with street segments and intersections, traffic lights and automobiles – and the flow of these;
- **Pipelines** – with their wells, pipes, valves, pumps, forks, joins and wells and the flow of fluids [8]; and
- **Container terminals** – with their container vessels, containers, cranes, trucks, etc. – and the movement of all of these [15] ■

2. TWO LANGUAGE CLASSES

2.1 The Languages of Domains

- In naming specific or general instances of endurants
 - whether as a whole [i.e., their external qualities],
 - or their general or specific properties [i.e., their internal qualities],
 - we are normally using **nouns** of the “language” of the domain in question.
- In naming specific or general instances of perdurants
 - whether as a whole,
 - or their general or specific pretties,
 - we are normally using **verbs** of the “language” of the domain in question.
- That is, domain descriptions unveil **languages of domains**,
- i.e., are, in a sense **describing their semantics**.

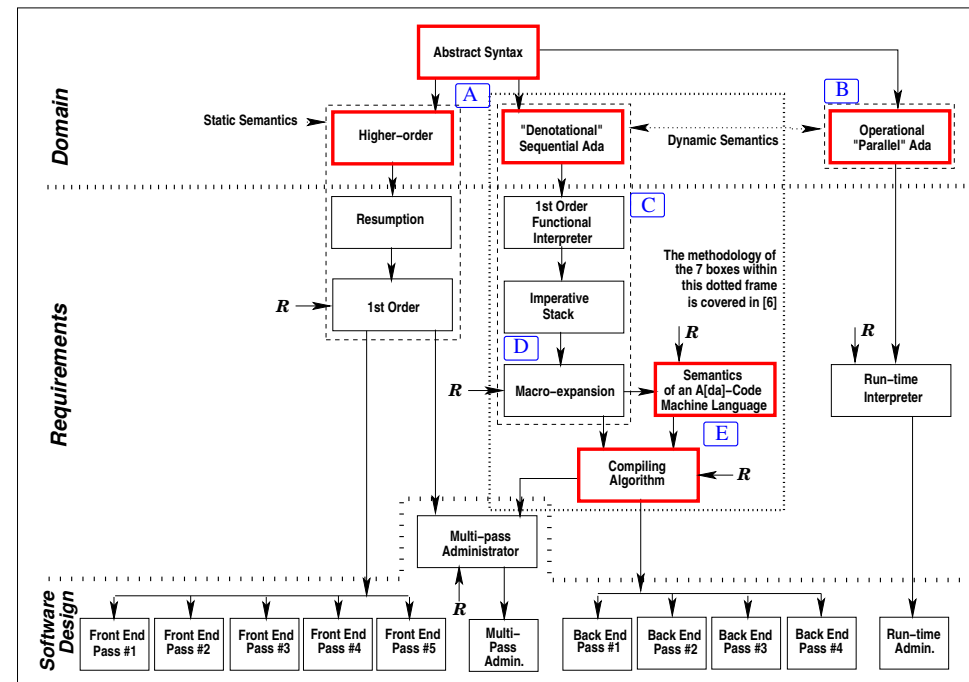


Fig. 2.1: The DDC CHILL and Ada compiler developments graph

2.2 The Languages of Programming

- We contrast that to the languages of programming.
 - For these we have the task of implementing interpreters and compilers.
 - Here is a *software development graph* for the development of compilers for languages such as CHILL and Ada.

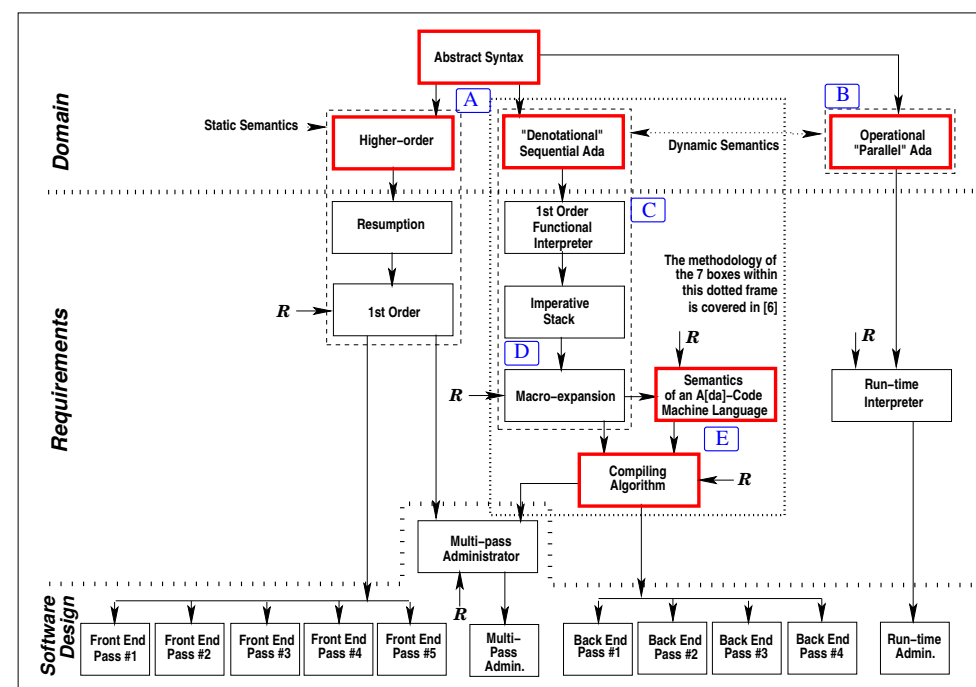


Fig. 2.2: The DDC CHILL and Ada compiler developments graph

- [A] Denotational Semantics, *McCarthy, Scott and Strachey* [40, 41, 45].
- [B] CSP, *Hoare* [34].
- [C] First Order Semantics, *Landin and Reynolds* [38, 43].
- [D] Imperative Stack and Macro-expansion Semantics, *Bekič* [1].
- [E] X Code to Compiling Algorithm, *McCarthy & Painter* [42].

2.3 A First Observation

- Of course we develop interpreters and compilers
 - for programming languages
 - by first describing their [static and dynamic] semantics.
- So, of course, we develop software
 - for any application domain,
 - by first describing its “semantics”,
 - that is: a domain model.

• • •

- Engineers are intimately familiar with their natural science bases:
 - Telecommunications engineers with *Maxwell's Equations*.
 - Aircraft engineers with *Aero Dynamics*.
 - &c.

3. DOMAIN ANALYSIS & DESCRIPTION

3.1 The Natural and Man-made World Around Us!

- We shall focus on the artefactual world, made by us!
 - Some phenomena of that world we can explain, the **entities**,
 - some we cannot.
 - We shall focus on the *entities*.
 - We shall, in particular, focus on
 - * manifest **parts**, i.e., endurants, and their
 - * **behaviours**, i.e., perdurants.

- So how do we analyze & describe a[n application] domain ?
- Is there a **method**, principles, techniques, tools,
- for analysing & describing domains
 - Yes, basically as indicated by the ontology diagram:

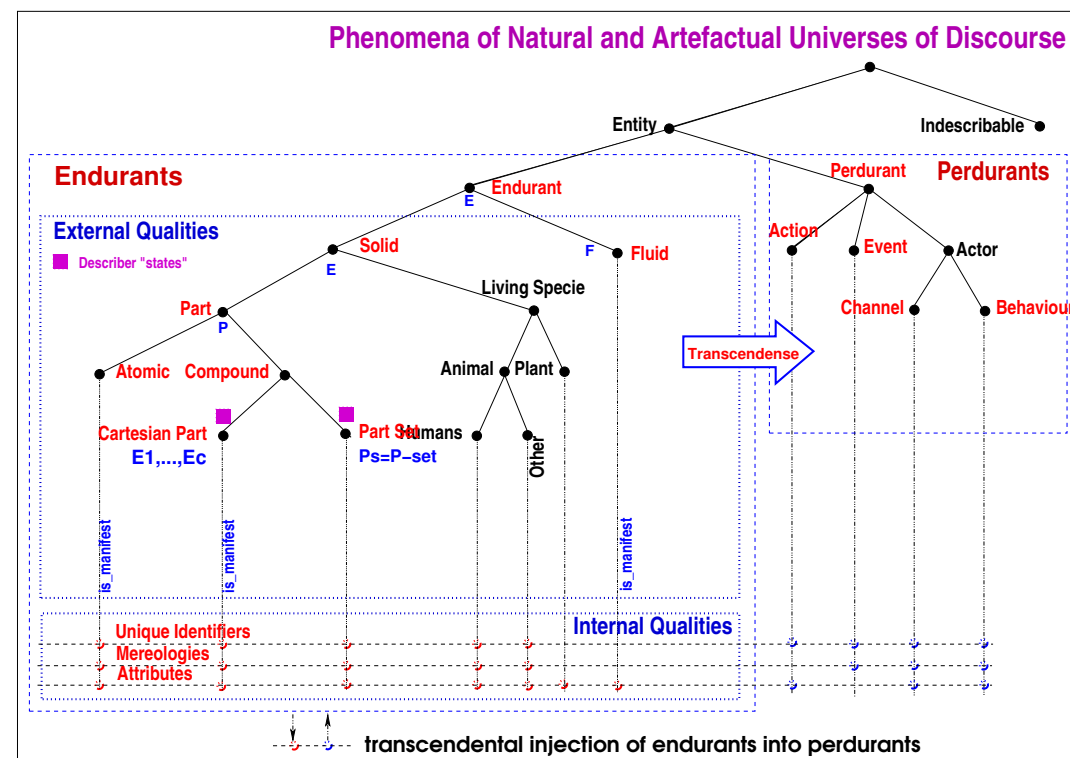


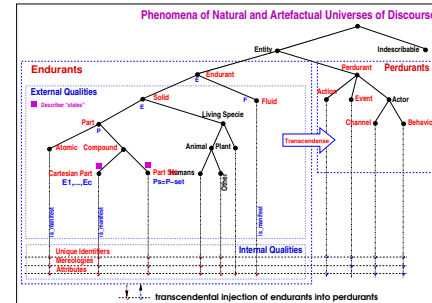
Fig. 3.1: A Domain Analysis & Description Ontology

3.2 Endurants and Perdurants

- There are two sides to the unfolding **analysis & description**:
 - The analysis & description of **endurants**, and
 - the analysis & description of **perdurants**.
- Within endurants there are further two sides:
 - The analysis & description of **external qualities**, those we can see and touch, and
 - the analysis & description of **internal qualities** those properties we can measure and/or speak about.
- And within the analysis & description of internal qualities there are the analysis & description of parts:
 - **uniqueness**, – **mereology** and – **attributes**.
- There is, finally, the concept of **intentional pull**.

3.3 Endurants: External Qualities

3.3.1 Analysis



3.2: Our Domain Analysis & Description Ontology

We analyze what we see and ascertain:

- Endurant Ontology:

- is_entity
- is_endurant
- is_perdurant
- is_solid
- is_fluid
- is_part

- is_living_species
- is_atomic
- is_compound
- is_Cartesian
- is_part_set
- is_plant
- is_animal
- is_human

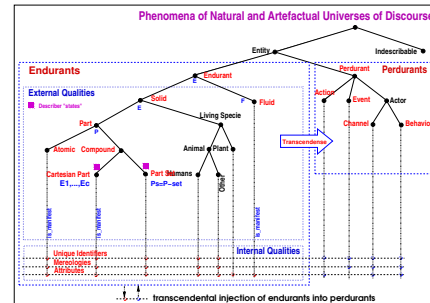
- Location:

- is_stationary
- is_mobile

- Treatment:

- is_manifest
- is_structure

3.3.2 Description



3.3: Our Domain Analysis & Description Ontology

- Auxiliary Description Functions:
 - `determine_Cartesian_part_sorts`
 - `determine_part_set_sort`
- Main description Functions::
 - `calculate_Cartesian_part_sorts`
 - `calculate_part_set_sort`

3.3.3 The Calculate Description Functions

`A calc_Cartesian_parts(e:E) Schema`

let ($_, (\eta E_1, \dots, \eta E_m)$) = `determine_Cartesian_parts_sorts(e)` **in**

“**Narration:**

[s] ... narrative text on sorts ...

[o] ... narrative text on sort observers ...

[p] ... narrative text on proof obligations ...

Formalisation:

type

[s] $E_1, \text{“} \dots \text{”}, E_m$

value

[o] $\text{obs_}E_1: E \rightarrow E_1, \text{“} \dots \text{”}, \text{obs_}E_m: E \rightarrow E_m$

proof obligation

[p] [Disjointedness of endurant sorts] “

end

A `calc_part_set_sort(e:E)` Schema

let ($_, \eta P$) = `determine_part_set_sort(e)` **in**

“**Narration:**

[s] ... narrative text on sort ...

[o] ... narrative text on sort observer ...

[p] ... narrative text on proof obligation ...

Formalisation:

type

[s] P

[s] $P_s = P\text{-set}$

value

[o] $\text{obs_Ps}: E \rightarrow P_s$ “

proof obligation

[p] [“Single sortness” of P_s] “

end

Example

- 1. The domain is that of a generic *road transport system*, RTS.
- 2. Of an RTS we can observe a Cartesian, manifest *road net aggregate*, RNA, and an *automobile aggregate*, AA, structure.
- 3. Of a *road net aggregate* we can observe a structure *set of hub aggregate*, HA, and a structure *set of link aggregate*, LA.
- 4. Of an HA we can observe a part set of atomic *hubs*, H, (i.e., street intersections).
- 5. Of an LA we can observe a part set of atomic *links*, L, (i.e., street segments).

Formalisation

type	value
1. RTS	2. obs_RNA: RTS→RNA
2. RNA, AA	2. obs_AA: RTS→AA
3. HA, LA	3. obs_HA: RNA→HA
4. Hs = H-set	3. obs_LA: RNA→LA
4. H	4. obs_Hs: HA→Hs
5. Ls = L-set	5. obs_Ls: LA→Ls
5. L	

3.4 Endurants: Internal Qualities

3.4.1 Unique Identifiers

- All entities are unique.
- Hence we associate unique identifiers with manifest parts.

A `calc_unique_identifier` Schema

“Narration:

[s] ... narrative text on unique identifier sort EI ...

[u] ... narrative text on unique identifier observer uid_E ...

[a] ... axiom on uniqueness of unique identifiers ...

Formalisation:

type

[s] UI

value

[u] uid_E: $E \rightarrow EI$ ”

Example

6. Road net aggregates have unique identifiers.
7. Hubs have unique identifiers.
8. Links have unique identifiers.
9. They are all distinct.

Formalisation

```

type
6. RNI
7. HI
8. LI
value
6. uid_RN: RN  $\rightarrow$  RNI
7. uid_H: H  $\rightarrow$  HI
8. uid_L: L  $\rightarrow$  LI
axiom
9.  $\forall rn:RN \cdot$ 
9.   let his = {uid_H(h) | h  $\in$  obs_Hs(obs_HA(rn))},
9.     lis = {uid_L(l) | l  $\in$  obs_Ls(obs_LA(rn))} in
9.   uid_RN(rn)  $\notin$  his  $\cup$  lis  $\wedge$  his  $\cap$  lis = {} end

```

3.5 An Aside: States

- For practical reasons, in the next formalizations, we need some state notions.
 - The manifest parts form a state, σ , and
 - The unique identifiers of manifest parts form a state, σ_{uid} .
- Given
 - a **unique identifier**, uid , in σ_{uid}
 - we can **retrieve** the corresponding, **unique part** in σ .

value

rts:RTS

rna = obs_RNA(*rts*)

as = obs_As(obs_AA(*rts*))

hs = obs_Hs(obs_HA(obs_RNA(*rts*)))

ls = obs_Ls(obs_LA(obs_RNA(*rts*)))

$\sigma = \{rna\} \cup as \cup hs \cup ls$

rnai = uid_RNA(*rna*)

ais = $\{ uid_A(a) \mid a:A \cdot a \in as \}$

his = $\{ uid_H(h) \mid h:H \cdot h \in hs \}$

lis = $\{ uid_A(a) \mid l:L \cdot l \in ls \}$

$\sigma_{uid} = \{rna\} \cup ais \cup his \cup lis$

retr_part: (RNA|A|H|L) $\rightarrow \Sigma \rightarrow P$

retr_part(uid)(σ) \equiv **let** *p*:(RNA|A|H|L) $\cdot p \in \sigma \wedge uid_P(p)=uid$ **in** *p* **end**

3.5.1 Mereology

Definition 2 . *Mereology*:

Mereology is the study and knowledge of parts and part relations ■

- Mereology was introduced in the form we shall use it by the Polish mathematician Stanisław Leśniewski (1886–1939) [27, 9].
- Which are the relations that can be relevant for “endurant-hood” ?
- There are basically two relations:
 - (i) **spatial** and
 - (ii) **conceptual**.

- (i) **Spatially** two or more endurants may be *topologically*
 - either adjacent to one another, like rails of a line,
 - or within an endurant, like links and hubs of a road net,
 - or an atomic part is conjoined to one or more fluids,
 - or a fluid is conjoined to one or more parts.
- (ii) **Conceptually** some parts, like automobiles,
 - “belong” to an embedding endurant,
 - * like to an automobile club, or
 - * are registered in the local department of vehicles,
 - or are ‘intended’ to drive on roads.

A calculate_mereology(p:P) Schema

“Narration:

[t] ... narrative text on mereology type ...

[m] ... narrative text on mereology observer ...

[a] ... narrative text on mereology type constraints ...

Formalisation:

type

[t] $MT = \mathcal{M}(UI_i, UI_j, \dots, UI_k)$

value

[m] mereo_P: $P \rightarrow MT$

axiom [Well-formedness of Domain Mereologies]

[a] $\mathcal{A}: \mathcal{A}(MT)$ ” ■

An Example

We shall only consider the mereologies of hubs, links and automobiles.

10. The mereology of a hub is a triple of the possibly empty set of the unique identifiers of the links that emanate from / are incident upon the hub, the automobiles that may enter the hub – and the road net unique identifier.
11. The mereology of a link is a triple of the exactly two element set of the unique identifiers of the hubs that are linked by the link, the automobiles that may enter the link– and the road net unique identifier.
12. The mereology of an automobile is the set of unique identifiers of the hubs and links that the automobile may enter.
13. All identifiers must be identifiers of the road transport system of the road net, hubs, links and automobiles; the link identifiers of a hub must be of links whose mereology prescribe those hubs; and, vice versa, the hub identifiers of a link must be of hubs whose mereology prescribe those links.

Formalisation

type

10. $HM = LI\text{-set} \times AI\text{-set} \times RNI$

11. $LM = HI\text{-set} \times AI\text{-set} \times RNI$

12. $AM = (HI|LI)\text{-set}$

value

10. $\text{mereo_H}: H \rightarrow HM$

11. $\text{mereo_L}: L \rightarrow LM$

12. $\text{mereo_A}: A \rightarrow AM$

axiom

11. $\forall l:L \cdot \text{let } (his, _, _) = \text{mereo_L}(l) \text{ in card } his = 2 \text{ end}$

13. $\forall a:A, h:H, l:L \cdot a \in as \wedge h \in hs \wedge l \in ls \Rightarrow$

13. $\text{let } hilis = \text{mereo_A}(a),$

13. $(lis', ais', rni') = \text{mereo_H}(h),$

13. $(his'', ais'', rni'') = \text{mereo_L}(l) \text{ in}$

13. $hilis \subseteq his \cup lis$

13. $\wedge ais' \subseteq ais \wedge ais'' \subseteq ais$

13. $\wedge lis' \subseteq lis \wedge lis'' \subseteq lis$

13. $\wedge his' \subseteq his \wedge his'' \subseteq his$

13. $\wedge rni' = rni \wedge rni'' \subseteq rni^1 \text{ end}$

¹For $as, hs, ls, rn, ais, his, lis$ and rni see Sect. 3.5 on Slide 22.

3.5.2 Attributes

- To recall: there are three sets of **internal qualities**:
 - **unique identifiers**,
 - **mereologies** and
 - **attributes**.
- **Unique identifiers** and **mereologies** are rather definite kinds of internal endurant qualities.
- **Attributes** form “wider-ranging” sets of **internal qualities**.

- We can roughly distinguish between two kinds of attributes:
 - those which can be motivated by **physical** (incl. chemical) **concerns**, and
 - those which, although they embody some form of ‘physics measures’, appear to reflect on **event histories**, i.e., **audit trails**:
 - * “if ‘something’, ϕ , has ‘happened’ to an endurant, e_a ,
 - * then some ‘commensurate thing’, ψ , has ‘happened’ to another (one or more) endurants, e_b .”
 - where the ‘something’ and ‘commensurate thing’
 - usually involve some ‘interaction’ between the two (or more) endurants.
- It can take some reflection and analysis to properly identify
 - endurants e_a and e_b and
 - commensurate events ϕ and ψ .

A calculate_attributes Schema

let $\{\eta A_1, \dots, \eta A_m\} = \text{analyse_attribute_type_names}(e)$ **in**

“**Narration:**

[t] ... narrative text on attribute sorts ...

some A_i s may be concretely defined: $[A_i = \dots]$

[o] ... narrative text on attribute sort observers ...

[p] ... narrative text on attribute sort proof obligations ...

Formalisation:

type

[t] $A_1 [= \dots], \dots, A_m [= \dots]$

value

[o] $\text{attr_}A_1: E \rightarrow A_1, \dots, \text{attr_}A_m: E \rightarrow A_m$

Proof obligation [Disjointness of Attribute Types]

[p] $\mathcal{PO}: \text{is_}A_i(a) \neq \text{is_}A_j(a) \ [i \neq j, i, j: [1..m]]$ ”

end

Example

14. Hubs have signal states: possibly empty sets of pairs of incident link identifiers.
15. Hubs have state spaces: set of hub states.
16. And hubs have traffic histories: time-stamped automobile identifiers
17. Links have traffic histories: time-stamped automobile identifiers
18. Automobiles have location: either at a hub, from a link to a link, or a fraction down a link.
19. Automobile have traffic histories: time-stamped hub or link identifiers

Formalisation

type

14. $H\Sigma = (LI \times LI)\text{-set}$

15. $H\Omega = H\Sigma\text{-set}$

16. $H_Hist = (TIME \times AI)^*$

17. $L_Hist = (TIME \times AI)^*$

18. $A_Loc == \text{at_a_Hub}(fli:LI,hi:HI,tli:LI)$

18. $\quad \quad \quad | \text{on_a_Link}(fhi:HI,li:LI,f:F,thi:HI)$

18. $F = \mathbf{Real}$, **axiom** $0 < f < 1$

19. $A_Hist = (TIME \times (HI | LI))^*$

value

14. $\text{attr_}H\Sigma: H \rightarrow H\Sigma$

15. $\text{attr_}H\Omega: H \rightarrow H\Omega$

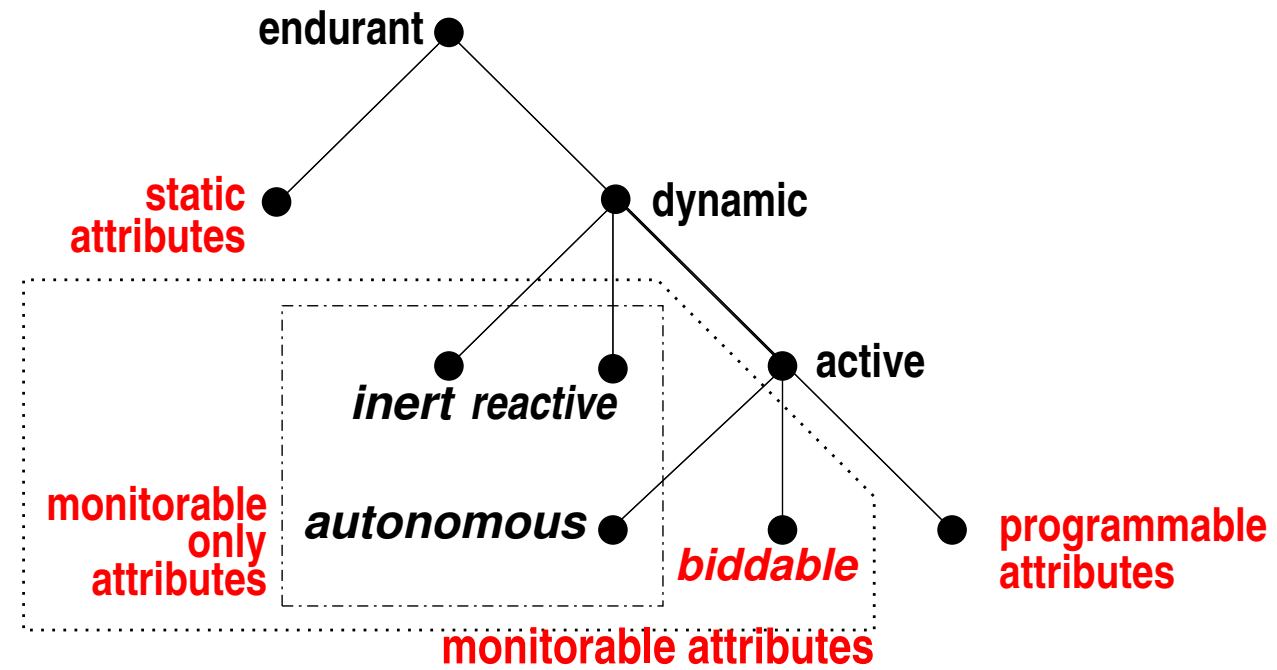
16. $\text{attr_}H_Hist: H \rightarrow H_Hist$

17. $\text{attr_}L_Hist: L \rightarrow L_Hist$

18. $\text{attr_}A_Loc: A \rightarrow A_Loc$

19. $\text{attr_}A_Hist: A \rightarrow A_Hist$

Michael A. Jackson's Attribute Categories



3.4: Attribute Value Ontology [36]

3.5.3 Intentional Pull

- *Intentionality*
 - “expresses” conceptual, abstract relations
 - between otherwise, or seemingly unrelated entities.
- The Oxford English Dictionary [39] characterizes intentionality as follows:
 - *the quality of mental states*
(e.g. *thoughts, beliefs, desires, hopes*)
 - *which consists in their being directed*
 - *towards some object or state of affairs.*

Informal Examples

We present three examples.

- **Automobile Traffic:**

- If an automobile history “records” being on a road or link
- at time τ ,
- then that road or link must “record”
the presence of that automobile
- at that time;

AND:

- If a hub or link history “records” an automobile at time τ ,
- then that automobile must “record”
its presence on that hub, respectively link
- at that time ■

- **Double-entry Bookkeeping:**
 - The outlay/expense sum total
 - must balance
 - the active/passive sum total ■
- **The Henry George Theorem:**
 - The Henry George theorem states that
 - under certain conditions, aggregate spending
 - by government on public goods
 - will increase aggregate rent based on land value (land rent)
 - more than that amount,
 - with the benefit of the last marginal investment equaling its cost ■

3.6 Perdurants

3.6.1 Transcendental Deduction

Some Definitions

Definition 3 . **Transcendental:**

By **transcendental**

we shall understand the philosophical notion:
*the a priori or intuitive basis of knowledge,
independent of experience* ■

Definition 4 . **Transcendental Deduction:**

By a **transcendental deduction**

we shall understand the philosophical notion:
*a transcendental “conversion”
of one kind of knowledge
into a seemingly different kind of knowledge* ■

Examples

- Trains, in a service center, being maintained, can be considered *endurants*.
- Those same trains, now in operation, “speeding” down the rail tracks, can, by transcendental deduction, be considered *perdurants*.
- And: “trains” referred to in time-tables can be considered *time-table attributes*.

3.7 Morphing Endurants into Perdurants: Parts into Behaviours

- Thus, to every **endurant part** we shall associate, by *transcendental deduction*, a **perdurant behaviour**.

3.8 Analysis of Perdurants

- *Part behaviours* are characterized by
 - **actions** pertaining to the individual part behaviours and
 - **events** pertaining to the (**channel**) *interaction* between part behaviours – with
 - part behaviours “alternating”, non-deterministically, externally (\sqcap) or internally (\sqcup), between two or more actions and/or two or more events.
- We shall describe these behavioral issues using CSP [34].

3.9 Description Details

3.9.1 Channel Description

- So behaviours interact via channels.
- In general any two [part] behaviours may communicate.
- So we consider the channels to be a double-indexed array of simple channels:

channel { $ch[\{u_i, u_j\}] \mid u_i, u_j:PI \cdot \text{of any domain parts}$ }

Example

- The channel array of the road transport system.

Formalisation

channel { $ch[\{u_i, u_j\}] \mid u_i, u_j: (RNI|AI|HI|LI) \cdot u_i \neq u_j \wedge \{u_i, u_j\} \subseteq \sigma_{uid}$ }

3.9.2 Actions and Events I

- Actions pertain to one or more behaviours.
- Actions **are planned** and may change the state of its related behaviours.
- Events pertain to one behaviour.
- Events **are not planned**, but **occur surreptitiously** and may change the state of its behaviour.

Examples

- **Actions:**

- An automobile remains in a hub.
- An automobile remains on a link.
- An automobile leaves a hub and enters a link.
- An automobile leaves a link and enters a hub.
- An automobile exits the road net.

- **Events:**

- An automobile ceases to be an automobile².
- A link, for example a tunnel or a bridge, breaks down³.

²motor breaks down, or crashes

³fire, mud slide, or other

3.9.3 Behaviour Signatures

- schematic form of part (p) behaviour signatures is:

$b: bi:Bl \rightarrow me:Mer \rightarrow svl:StaV^* \rightarrow mvl:MonV^* \rightarrow prl:PrgV^* \text{ channels } \mathbf{Unit}$

- b : name of part p behaviour
- bi : p unique identifier
- mer : p mereology
- svl : p static attributes
- mvl : p monitorable attributes
- prl : p programmable attrs.
- channels: subset of channels
- **Unit**: the () state value

Example: Behaviour Signatures

20. automobile:

- (a) unique identifier, mereology, static (...) and monitorable (...) attributes;
- (b) programmable attributes: automobile location and automobile history;
- (c) and channel references
allowing communication between the automobile and the hub and link behaviours.

21. Similar for link and hub behaviours.

Formalisation

value

20a automobile: $ai:AI \rightarrow (_,uis):AM \rightarrow \dots \rightarrow \dots$

20b $\rightarrow (A_Loc \times A_Hist)$

20c **out** $\{ch[\{ai,ui\}] | ui:(HI|LI) \cdot ui \in hisUlis\}$ **Unit**

21a hub: $hi:HI \rightarrow (lis,ais,rni):HM \rightarrow (H\Omega \times \dots) \rightarrow \dots$

21b $\rightarrow (H\Sigma \times H_Hist)$

21c **in** $\{ch[\{hi,ui\}] | ui:(|HI|RNI)\text{-set} \cdot ui \in lisUlisUrni\}$ **Unit**

21a link: $li:LI \rightarrow (his,ais,rni):LM \rightarrow (LEN \times L\Omega \times \dots)$

21b $\rightarrow (L\Sigma \times L_Hist)$

21c **in** $\{ch[\{li,ui\}] | ui:(|HI|RNI)\text{-set} \cdot li \in lisUhisUrni\}$ **Unit**

3.9.4 Behaviour Invocation

$b(bi)(me)(svl)(mvl)(prl)$

3.9.5 Behaviour and Action Definition Schemes

Behaviours

behaviour $(bi)(me)(svl)(mvl)(prl) \equiv$
 $\quad nd_action_1(bi)(me)(svl)(mvl)(prl)$
 $\quad \sqcap nd_action_2(bi)(me)(svl)(mvl)(prl)$
 $\quad \dots$
 $\quad \sqcap nd_action_n(bi)(me)(svl)(mvl)(prl)$
 $\quad \sqcap d_action_1(bi)(me)(svl)(mvl)(prl)$
 $\quad \sqcap d_action_2(bi)(me)(svl)(mvl)(prl)$
 $\quad \dots$
 $\quad \sqcap d_action_d(bi)(me)(svl)(mvl)(prl)$

Actions

$action(bi)(me)(svl)(mvl)(prl) \equiv$
 $\quad \text{let } prl' = act(bi)(me)(svl)(mvl)(prl) \text{ in } \text{behaviour}(bi)(me)(svl)(mvl)(prl') \text{ end}$

Example: Behaviour Definitions

- 22. We abstract automobile behaviour at a Hub (hi).
- 23. Internally non-deterministically, an automobile either
- 24. either progresses around the hub
- 25. or leaves the hub to enter a link.

Formalisation

```

22 automobile(ai)(aai,uis)(...)(apos:at_a_Hub(fli,hi,tli),ahist) ≡
24   automobile_progress_around_hub(ai)(aai,uis)(...)(a_loc:at_a_Hub(fli,hi,tli),ahist)
23   ⊔
25   automobile_leave_hub_enter_link(ai)(aai,uis)(...)(a_loc:at_a_Hub(fli,hi,tli),ahist)

```

26. The automobile progresses around the hub:

- (a) the automobile at that hub,
- (b) informing (“first”) the hub behaviour.

Formalisation

```

26 automobile_progress_around_hub(ai)(aai,uis)(...)(at_a_Hub(fli,hi,tli),ahist) ≡
26   let  $\tau$  = record_TIME() in
26b   ch[ ai,hi ] !  $\tau$  ;
26a   automobile(ai)(aai,uis)(...)(at_a_Hub(fli,hi,tli),upd_hist( $\tau$ ,hi)(ahist))
26   end

26a upd_hist: (TIME×UI) → (A_Hist→A_Hist)|(H_Hist→H_Hist)|(L_Hist→L_Hist)
26a upd_hist( $\tau$ ,ui)(hist) ≡ hist † [ ui ↦  $\langle \tau \rangle$  hist(ui) ]

```

27. The automobile leaves the hub entering a link:

- (a) tli, whose “next” hub, identified by thi, is obtained from the mereology of the link identified by tli;
- (b) informs the hub it is leaving and the link it is entering,
- (c) “whereupon” the vehicle resumes (i.e., “while at the same time” resuming) the vehicle behaviour positioned at the very beginning (0) of that link.

Formalisation

```

27 automobile_leave_hub_enter_link(ai)(aai,uis)(...)(a_loc:at_a_Hub(fli,hi,tli),ahist) ≡
27a  (let ({fhi,thi},ais) = mereo_L(retr_L(tli)(σ)) in assert: fhi=hi
27b  ( ch[ ai,hi ] ! τ || ch[ ai,tli ] ! τ ) ;
27c  automobile(ai)(aai,uis)(...)(on_a_Link(tli,(hi,thi),0),upd_hist(τ,tli)(upd_hist(τ,hi)(ahist))) end)

```

28. Or the automobile “disappears — off the radar” !

Formalisation

```

28 automobile_stop(ai)(aai,uis),(...)(apos:atH(fli,hi,tli),ahist) ≡ stop

```

3.9.6 Domain Instantiation

- For every manifest part sort
 - there is a single description: signature and definition (i.e., its syntax).
- For every manifest part
 - there is a behaviour (i.e., its semantics “realization”).
- For the total of all manifest domain parts there is their initialization:
 - the parallel “execution”
 - of the behaviour of each manifest part,
 - properly initialized.

Example: Domain Initialization

- 29. Let us refer to the system initialization as a behaviour.
- 30. All links are initialized,
- 31. all hubs are initialized,
- 32. all automobiles are initialized,
- 33. etc.

Formalisation

value

29. `rts_initialisation: Unit → Unit`

29. `rts_initialisation() ≡`

30. `|| { link(uid_L(l))(mereo_L(l))(attr_LEN(l),attr_LΩ(l))(attr_L_Traffic(l),attr_LΣ(l)) | l:L · l ∈ ls }`

31. `|| || { hub(uid_H(l))(mereo_H(l))(attr_HΩ(l))(attr_H_Traffic(l),attr_HΣ(l)) | h:H · h ∈ hs }`

32. `|| || { automobile(uid_A(a))(mereo_A(a))(attr_RegNo(a))(attr_APos(a)) | a:A · a ∈ as }`

33. `|| ...`

4. SUMMING UP!

4.1 What has [Not] been Achieved?

4.1.1 Achieved

- We have outlined a method —
 - hinting at its principles, procedures, techniques and tools —
 - for analyzing and describing a certain class of domains.
- This method “heralds” an extension within software development:
 - before there was *requirements engineering* and *software design*,
 - now *domain engineering*, and its *science*, is prefixed that approach.

4.1.2 Not Achieved

- The next-but-following section of the talk hints at some open issues.

4.2 Experimental Domain Models

- The domain analysis & description method, its
 - principles,
 - techniques and
 - procedures,
 - tools
- have been honed over many years
- through domain modelling experiments – some are:
 - *railways* [2, 23, 4],
 - *“The Market”* [3],
 - *container shipping* [5],
 - *Web systems* [6],
 - *stock exchange* [7],
 - *oil pipelines* [8],
 - *credit card systems* [11],
 - *weather information* [12],
 - *swarms of drones* [13],
 - *document systems* [14],
 - *container terminals* [15],
 - *retail systems* [17],
 - *assembly plants* [16],
 - *waterway systems* [19],
 - *shipping* [20], and
 - *urban planning* [26].

4.3 Open Issues

4.3.1 The Rôle of Algorithms

- Which rôle¹ do algorithms play in all this?
 - Basically no role!
 - We describe properties.
 - Not how to compute properties.

4.3.2 RSL, The RAISE Specification Language

- We use a slight extension of the RAISE [32] Specification Language, RSL [31].
- We could as well have used similar extensions to either of
 - VDM [24, 25, 29],
 - Alloy [35],
 - cafeOBJ [30],
 - or other!

¹– or just role!

4.3.3 Continuity

- To model the behaviour of discrete dynamic domains,
 - such as are the main focus of this talk,
 - we use the CSP process concept [34].
- To model the behaviour of continuous dynamic domains,
 - which we really have not,
 - we suggest that You use methods of classical analysis,
 - to wit: *[Partial] Differential Equations, PDEs*.
 - Perhaps also some *Fuzzy Logic* [48, 37].
- That is: We see this as the “dividing line” between
 - discrete and
 - continuousdynamic systems modelling: *CSP versus PDEs*.
- But: Current formal, logic-based specification languages do not mesh easily with classical calculi!
- See, however, [46, 47, *rTiMo*, *BigrTiMo*].

4.3.4 Domain Facets – covered in [18] Chapter 8

.

- By a domain facet we shall understand
one amongst a finite set of generic ways of analyzing a domain:
 - a view of the domain,
 - such that the different facets
 - cover conceptually different views,
 - and such that these views together
 - cover the domain.
- As examples of domain facets we list

<ul style="list-style-type: none"> – intrinsics, – support technologies, – rules & regulations, – scripts, 	<ul style="list-style-type: none"> – license languages, – management & organization, and – human behaviour.
--	--

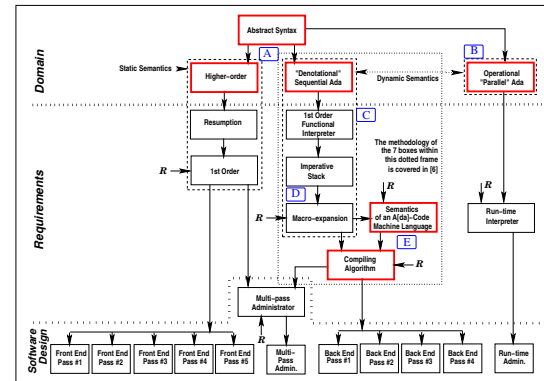
as such facets.

4.3.5 Requirements Engineering– covered in [18] Chapter 9

- We shall view requirements from three “sides”:
 - (α) domain requirements,
 - (β) interface requirements, and
 - (γ) machine requirements.
- But first a definition of the term ‘machine’.
 - By machine we shall understand
 - * a, or the, combination of hardware and software
 - * that is the target for, or result of
 - * the required computing systems development.
- By a *requirements* we shall understand (cf., IEEE Standard 610.12):
 - “A condition or capability
 - needed by a user
 - to solve a problem
 - or achieve an objective.” ■

- By a *domain requirements* we shall understand
 - those requirements
 - which can be expressed
 - solely using terms of the domain ■
- By an *interface requirements* we shall understand
 - those requirements
 - which can be expressed
 - only using technical terms of both the domain and the machine ■
- By a *machine requirements* we shall understand
 - those requirements which, in principle,
 - can be expressed
 - solely using terms of the machine ■

- The *domain requirements* stage of requirements development
 - starts with a basis in the domain engineering's domain description.
 - It is, so-to-speak, a first step in the development of a requirements prescription.
 - From there follows, according to [18, Chapter 9] a number of (five) steps:
 - * (1.) *projection*
 - * (2.) *instantiation*
 - * (3.) *determination*
 - * (4.) *extension*
- The *interface and machine requirements* stages of requirements development can be decomposed “similarly” !



4.1: The DDC CHILL and Ada compiler developments graph

4.3.6 From Programming to Domains

Theories of Compiler Development

- Trustworthy compiler development is based on many theories; to wit:
 - [A] Denotational Semantics, *McCarthy, Scott and Strachey* [40, 41, 45].
 - [B] CSP, *Hoare* [34].
 - [C] First Order Semantics, *Landin and Reynolds* [38, 43].
 - [D] Imperative Stack and Macro-expansion Semantics, *Bekič* [1].
 - [E] X Code to Compiling Algorithm, *McCarthy & Painter* [42].
- A trustworthy progress, from “top” to “bottom” of the above diagram reflects **Unifying Theories of Programming** [33].

Domain Specific Languages

- A domain specific language, generically referred to as a DSL,
 - is a language whose basic syntactic elements directly reflect endurants and perdurants of a specific domain.
 - *Actulus*, a language in which to express calculations of actuarial character [28], is a DSL.
- The semantics of a DSL, obviously, must relate to a model for the domain in question.
- In fact, we advice, that DSLs be developed from the basis of relevant domain models.
- A guiding rule for the development of DSLs is their adherence to *The Dogma of Unifying Theories of Programming*

Philosophy of Computing

- The Danish philosopher Kai Sørlander
 - has shown that there is a foundation in philosophy
 - for domain analysis and description.
- We refer to [18, *Chapter 2*] for a summary of his findings.

4.3.7 Possible PhD Topics

- Domain science & engineering offers scientific challenges:
 - Domain Specific Languages & Unifying Theories of Programming
 - Role of $\mathbb{D}, S \models \mathbb{R}$ in Program Verification
 - Intentional Pull
 - Continuous Behaviours [46, 47, *rTiMo*, *BigrTiMo*]
 - Towards a Calculus of Perdurants
 - Modelling Human Interaction
 - Further Study of Domain Facets [18, Chapter 8]
 - Further Study of Domain Requirements [18, Sect. 9.4]
 - Further Study of Interface Requirements [18, Sect. 9.5]
 - Formalizing Domain Calculi [10]
 - Transcendental Deduction
 - Kai Sørlander's Philosophy

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¹This book is currently being translated into Chinese by Dr. Yang ShaoFa, IoS/CAS, Beijing and into Russian by Dr. Mikhail Chupilko, ISP/RAS, Moscow

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5.1: **Left:** The gypsum model, Thorvaldsens Museum, Copenhagen
Right: My grandfather, 1911, Modlin Castle, Homel, Count Paszkiewicz.