Domain Science & Engineering

A Primer¹

¹ – first inteded for a M.Sc./Ph.D. course at the Technical University of Vienna, 24.10–4.11, 2022
Preface

The Triptych Dogma

In order to specify software, we must understand its requirements.
In order to prescribe requirements we must understand the domain.
So we must study, analyse and describe domains.

Domains – What Are They?

By a domain we shall understand a rationally describable segment of a discrete dynamics segment of a human assisted reality, i.e., of the world, its solid or fluid entities: natural [“God-given”] and artefactual [“man-made”], and its living species entities: plants and animals including, notably, humans. Examples of domains are: rail, road, sea and air transport; water, oil and gas pipelines; industrial manufacturing; consumer, retail and wholesale markets; health care; et cetera.

Aim and Objectives

• The aim of this monograph is to contribute to a methodology for analysing and describing domains.
• The objectives – in the sense of ‘how is the aim achieved’ – is reflected in the structure and contents and the didactic approach of this monograph.
• The main elements of my approach – along one concept-axis – can be itemized:
  ◦ There is the founding of our analysis & description approach in providing a base philosophy, cf. Chapter 2.
  ◦ There is the application of ideas of taxonomy to understand the possibly hierarchical structuring of domain phenomena respectively the understanding of properties of phenomena and relations between them.
  ◦ There are the notions endurants and perdurants – with endurants being the phenomena that can be observed, or conceived and described, as a “complete thing” at no matter which given snapshot of time [90, Vol. I, pg. 656], and perdurants being the phenomena for which only a fragment exists if we look at or touch them at any given snapshot in time [90, Vol. II, pg. 1552].
  ◦ There is the introduction of base elements of calculi for analysing and describing domains.
  ◦ There is the application of ideas of ontology to understand the possibly hierarchical structuring of these calculi.
  ◦ And finally there is the notion of transcendental deduction, cf. Sect. 2.1.2, for “morphing” certain kinds of endurants into certain kinds of perdurants, Chapter 6.
• Along another conceptual-axis the below are further elements of our approach:
  ◦ We consider domain descriptions, requirements prescriptions and software design specifications to be mathematical quantities.
  ◦ And we consider them basically in the sense of recursive function theory [102, Hartley Rogers, 1952] and type theory [97, Benjamin Pierce, 1997].
Methodology

By a method we shall understand a set of principles and procedures for selecting and applying a set of techniques and tools to a problem in order to achieve an orderly construction of a solution, i.e., an artefact.

By methodology we shall understand the study & application of one or more methods.

By a formal method we shall understand a method whose decisive principles include that of considering its artefacts as mathematical quantities; whose decisive procedures include those of whose decisive techniques include those of whose decisive tools include those of one or more formal languages.

By a language we shall here understand a set of strings of characters, i.e., sentences, sentences which are structured accordin to some syntax, i.e., grammar, are given meaning by some semantics, and are used according to some pragmatics.

By a formal language we shall here understand a languages whose syntax and semantics can both be expressed mathematically and for whose sentences one can rationally reason (argue, prove) properties.

We refer to Chapter 1 of [41] for an 8 page, approximately 50 entries set of concept definitions such as the above.

We refer to the ‘Method’ index, Sect. C.3 on page 166.

•••

In this primer we shall use the formal specification language, RSL, the RAISE Specification Language, RSL [66] – and we shall notably rely on RSL’s adaptation of CSP, Tony Hoare’s Communicating Sequential Processes [78]; and we shall propagate a definitive method for the study and description of domains.

An Emphasis

When we say domain analysis & description we mean that the result of such a domain analysis & description is to be a model that describes a usually infinite set of domain instances. Domains exhibit endurants and perdurants. A domain model is therefore something that defines the nouns (roughly speaking the endurants) and verbs (roughly speaking the – and their combination – of a language spoken in and used in writing by the practitioners of the domain. Not an instantiation of nouns, verbs and their combination, but all possible and sensible instantiations.

September 8, 2022: 10:01 am

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2 By a principle we mean: a principle is a proposition or value that is a guide for behavior or evaluation [Wikipedia], i.e., code of conduct
3 By a procedure we mean: instructions or recipes, a set of commands that show how to achieve some result, such as to prepare or make something [Wikipedia], i.e., an established way of doing something
4 By a technique we mean: a technique, or skill, is the learned ability to perform an action with determined results with good execution often within a given amount of time, energy, or both [Wikipedia], i.e., a way of carrying out a particular task
5 By a tool we mean: a tool is an object that can extend an individual’s ability to modify features of the surrounding environment [Wikipedia]
6 RAISE: Rigorous Approach(es) in Software Engineering, [67]
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Chapter 1
Introduction

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The Triptych Dogma

In order to specify software, we must understand its requirements.

In order to prescribe requirements we must understand the domain.

So we must study, analyse and describe domains.

This primer is both a significantly reduced version of the scientific monograph [41] and a revision of some of its findings.

1.1 Why This Primer?

This primer is intended as a textbook. The courses that I have in mind are, in the lectures, to focus on Chapters 3–6, i.e., Pages 23–108. The serious students, whether just readers or actual, physical course lecture attendants, are expected to study Chapters 1–2 as well as Chapter 7 and the Bibliography (Chapter 8) and the appendices on their own!

The primer is about how to analyse & describe man-made domains (including their possible interaction with nature). We emphasize the ampersand: ‘&’.7 We justify competency

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7 By not writing ‘and’, but ‘&’, we shall emphasize that in A&B we are dealing with one concept which consists of both A and B “tightly interacting”.

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in Domain Science & Engineering for two reasons. (i) For reasons of proper engineering software development – as indicated by the above Triptych Dogma. In possible proofs of software properties references are made, not only to the software code itself and the requirements, but also to the domain, the latter in the form of assumptions about the domain. In our mind no software development project ought be undertaken unless it more-or-less starts with a proper domain engineering phase. And (ii) for reasons of scientifically understanding our own everyday practical world: financial institutions, the transport industry (road, rail and air traffic, shipping), feeder systems (such as oil, gas, water and other such pipeline systems), etc.

1.2 Structure

The primer, beyond the present chapter, has, syntactically speaking, three elements:

1. **Chapter 2** covers the philosophy of Kai Sørlander [108, 109, 110, 111, 112]. Yes, a major contribution of [41] and this primer is to justify important domain concepts by their sheer inevitability in any world description.

2. **Chapters 3–6** presents the methodology of domain engineering. It is split into four chapters for practical and pragmatic reasons. Chapter 3 gives a “capsule introduction” into Chapters 4–6.

3. **Chapters 7–8** and Appendices A–C cover such things as ‘closing remarks’, a ‘bibliography’, a ‘Road Transport’ example, an ‘RSL formal specification language’ primer, and ‘Indexes’ to definitions, concepts, etc.

1.3 Prerequisite Skills

The reader is expected to possess the following skills:

- To be reasonably versed in discrete mathematics: mathematical logic and set theory.
- To have had, even if only a fleeting, acquaintance with abstract specifications in the style of VDM [48, 49, 64], Z [119], CafeObj [65], Maude [92, 55], or the like – and thus to enjoy abstractions.
- To have reasonable experience with functional programming a la Standard ML or F [94, 72, 69] respectively [70] – or similar such language.
- To have reasonable experience with CSP [77, 79, 78, 103, 107].

The reader is further expected to possess the following mindset:

- To basically consider software as mathematical objects. That is: as quantities about which one can (and must) reason logically.
- To think and “act” abstractly. An essence of abstraction is expressed in the next section.
- To act responsibly, that is to make sure that You have indeed understood Your domain, that You have indeed reasoned about adequacy of your requirements, and You have indeed model-checked, proved and formally tested your specifications.

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8 Some say: “Mathematics is the Science of Abstractions”! Others say that both “Mathematics and Physics are Abstractions of Reality”.

9 In is, today, 31 July 2022, very fashionable to propagate messages of ‘ethics’ to programmers – without even touching upon issues such as “have You understood your application domain thoroughly?”, or “have You reasoned about adequacy of your requirements?”, or “have You model-checked, proved and formally tested your specifications (descriptions and prescriptions) and Your code?”, etc.
1.4 Abstraction

Conception, my boy, fundamental brain-work,
is what makes the difference in all art
D.G. Rossetti\textsuperscript{10}, letter to H. Caine\textsuperscript{11}

Abstraction is a tool, used by the human mind, and to be applied in the process of
describing (understanding) complex phenomena.
Abstraction is the most powerful such tool available to the human intellect.
Science proceeds by simplifying reality. The first step in simplification is abstraction.
Abstraction (in the context of science) means leaving out of account all those empirical data
which do not fit the particular, conceptual framework within which science at the moment
happens to be working.
Abstraction (in the process of specification) arises from a conscious decision to advocate
certain desired objects, situations and processes as being fundamental; by exposing, in a
first, or higher, level of description, their similarities and — at that level — ignoring possible
differences.

[From the opening paragraphs of [76, C.A.R. Hoare Notes on Data Structuring]]

1.5 Software Engineering

1.5.1 Domain Science & Engineering

This primer covers only the Domain of software development. There is two things to say
about that. One is that facets of requirements, essential ones, is covered in [41, Chapter
8], general ones in [14, Software Engineering, III, Part V]; the other is that the pursuit of
developing domain models is not just for the sake of software development, but also for the
sake of just understanding the man-made world around us. Domain science and engineering
can thus be pursued in-and-by itself. Such as [the study of] most basic and theoretical physics.

1.5.2 Software Engineering

In 2006 I published these books: [12, 13, 14]:

\textsuperscript{10} Dante Gabrielli Rosetti, 1828–1882, English poet, illustrator, painter and translator.
\textsuperscript{11} T. Hall Caine, 1853–1931, British novelist, dramatist, short story writer, poet and critic.
1.5.2.1 Domain Engineering: 2016–2022

The first inklings of the domain science and engineering of [41] appeared in [23, 27, 2010]. More-or-less “final” ideas were published, first in [34, 2017], then in [37, March 2019]. The book [41] with updates in this primer then constitutes the most recent status of our work in domain science & engineering.

[14, Software Engineering, III, Part V] does not cover the Domain Engineering material covered in [41, Chapter 8], that latter was researched and developed between the appearance of [14] and, obviously, [41]. Part V of [14], except for Chapters 17–18 is still relevant. Chapters 17–18 of [14] are now to be replaced in any study by Chapters 4–7 of [41] and this primer!

1.5.2.2 Requirements Engineering

This primer does not show You how to proceed into software development according to the Triptych Dogma. This is strongly hinted at in [41, Chapter 9]. (That chapter is an adaptation of [16, May 2008].) Our approach to requirements engineering is rather different from that of both [88, A. van Laamswerde] and [85, M. A. Jackson] – to cite two relevant works. It is, I strongly think, commensurate with these works. I wish that someone could take up this line of research: making more precise, perhaps more formal, the ideas of projection, initialisation, determination, extension and fitting; and comparing, perhaps unifying our approach with that of Lamsweerde and Jackson.

1.5.2.3 Software Design

For the software design phase, after requirements engineering, we, of course, recommend [12, 13, Software Engineering vols. 1–2]

1.6 The Structuring of The Text

The reader will find that this text consists of “diverse” kinds of usually small paragraphs of texts: definitions – properly numbered and labeled; examples – properly numbered and
1.8 Lectures

labeled; analysis predicate, function, and description prompt “formalisations”; method principle, procedure, technique and tool paragraphs; – all of these delineated by closing – with short, usually one or two small paragraphs of introductory or otherwise explaining texts. All of this is “brought to You in living colours”! So be prepared: Study such paragraphs: paragraph-by-paragraph. Each form a separate “whole”.

1.7 Self-Study

This primer is primarily intended to support actual, physical lectures. For self-study by B.Sc. and M.Sc. students and practicing novice software engineers we recommend to use this primer in connection with its “origin” [41]. For self-study by Ph.D. students and graduated computer scientist we recommend going directly to the source: [41].

1.8 Lectures

This primer was developed during the summer of 2022. The author was to give a set of seven double lectures at the Technical University of Vienna October 24 – November 4, 2022. These lectures were:

- Week 1:
  - Lecture Day 1: Chapter 3:
    - Domains – lecture “doubles” as a Faculty Seminar!
    - and Appendix A:
    - An Example, I
  - Lecture Day 2: Chapter 4:
    - Endurants: External Qualities
  - Lecture Day 3: Sections 5.1–5.3:
    - Unique Identifiers
    - Merology
  - Lecture Day 4: Sections 5.4–5.6:
    - Attributes,
    - Intentional Pull,
    - A Domain Discovery Process

- Week 2:
  - Lecture Day 5: Appendix A:
    - An Example, II Further on Road Transport, and
    - Chapter 6, Sect. 6.1:
    - A General Analysis of Part Behaviours
  - Lecture Day 6: Chapter 6, Sects. 6.2–6.3.3:
    - Channels and
    - Behaviours Definitions: Signatures and Bodies

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12 as the NBC Television Network programmes would “proudly” announce in the 1960s!
Lecture Day 7: Chapter 6, Sects. 6.4–6.8.2:
- Domain Behaviour Initialisation,
- A Domain Discovery Process,

1.9 Relation to [41]

This primer is based on [41, Nov. 2021]. Chapter 2 is a complete rewrite of [41, Chapter 2]. Chapters 4–6 is a “condensation” of [41, Chapters 4–7]: [41, Chapter 6] has been shortened and appears in this primer as Sect. 2.1.2. From [41, Chapter 4] we have, in Chapter 4, omitted all material on – what is there referred to as Conjoins. And we have further sharpened the notion of type names. We have sharpened the focus on methods: principle, procedures, techniques and tools. You will find, in the Indexes section, Sect. C.3 on page 166, a summary of references to these. Work is still in progress on highlighting more of the method steps. Section 6.5 is new.

1.10 Closing

The purpose of this introduction is to place the present primer in the context of my other books on software development and possible lectures and self study.
Chapter 2
Kai Sørlander’s Philosophy

Definition 1. Philosophy is the study of general and fundamental questions, such as those about existence, reason, knowledge, values, mind, and language.

2.1 Introduction

In philosophising questions are asked. One does not necessarily get answers to these questions. Questions are examined. Light is thrown on the questions and their derivative questions.

Philosophy is man’s endeavour, our quest, for uncovering the necessary characteristics of our world and our situation as humans is that world.

We shall focus on the issues of existence, i.e., metaphysics.

The treatment in this chapter is based very much on the works of the Danish philosopher Kai Sørlander (1944) [108, 109, 110, 111, 112, 1994–2022] both in contrast to and inspired by the German philosopher Immanuel Kant (1724–1804) [68].

The reason why I, as a computer scientist, is interested in philosophy, is that philosophers over more than 2500 years have thought about existence: why is the world as it is – and computer scientists, like other scientists (notably physicists and economists), repeatedly model fragments of the world; and the reason why I focus on Kai Sørlander, is that his philosophy addresses issues that are crucial to our understanding how we must proceed when modelling domains – and, I think, in a way that helps us model domains with a high assurance that our models are reasonable, can withstand close scrutiny. Kai Sørlander thinks and writes logically, rationally. The area of his philosophy that I am focusing on here is metaphysics.

14 From Greek: φιλοσοφία, philosophia, ‘love of wisdom’
15 Many of the ‘definitions’ in this primer are in the style used in philosophy. They are not in the ‘precise’ style commonly used in mathematics and computer science. You may wish to call them characterisations.
In mathematics and computer science the definer usually has a formal base on which to build. In domain science & engineering we do not have a formal base, we have the “material” world of natural and man-made phenomena.
16 – starting, one could claim, with:
2.1.1 Metaphysics

The branch of philosophy that we are focusing on is referred to as metaphysics. To explain that concept I quote from [Wikipedia]:

"Metaphysics is the branch of philosophy that studies the fundamental nature of reality, the first principles of being, identity and change, space and time, causality, necessity, and possibility. It includes questions about the nature of consciousness and the relationship between mind and matter, between substance and attribute, and between potentiality and actuality. The word “metaphysics” comes from two Greek words that, together, literally mean “after or behind or among [the study of] the natural”. It has been suggested that the term might have been coined by a first century CE editor who assembled various small selections of Aristotle’s works into the treatise we now know by the name Metaphysics (μετα τα φυσικα, meta ta physika, lit. ‘after the Physics’, another of Aristotle’s works) [56].

Metaphysics studies questions related to what it is for something to exist and what types of existence there are. Metaphysics seeks to answer, in an abstract and fully general manner, the questions: • What is there? • What is it like?"
2.1 Introduction

2.1.2 Transcendental Deductions

A crucial element in Kant’s and Sørlander’s philosophies is that of *transcendental deduction*. It should be clear to the reader that in domain analysis & description we are reflecting on a number of philosophical issues; first and foremost on those of ontology. For this chapter we reflect on a sub-field of epistemology, we reflect on issues of transcendental nature. Should you wish to follow-up on the concept of transcendental, we refer to [68, Immanuel Kant], [81, Oxford Companion to Philosophy, pp 878–880], [3, The Cambridge Dictionary of Philosophy, pp 807–810], [53, The Blackwell Dictionary of Philosophy, pp 54–55 (1998)], and [111, Sørlander].

2.1.2.1 Some Definitions

**Definition 2.** Transcendental: By *transcendental* we shall understand the philosophical notion: the a priori or intuitive basis of knowledge, independent of experience.

A priori knowledge or intuition is central: By *a priori* we mean that it not only precedes, but also determines rational thought.

**Definition 3.** Transcendental Deduction: By a *transcendental deduction* we shall understand the philosophical notion: a transcendental “conversion” of one kind of knowledge into a seemingly different kind of knowledge.

2.1.2.2 Some Informal Examples

**Example 1.** Transcendental Deductions – Informal Examples: We give some intuitive examples of transcendental deductions. They are from the “domain” of programming languages. There is the syntax of a programming language, and there are the programs that supposedly adhere to this syntax. Given that, the following are now transcendental deductions.

The software tool, a *syntax checker*, that takes a program and checks whether it satisfies the syntax, including the statically decidable context conditions, i.e., the *statics semantics* – such a tool is one of several forms of transcendental deductions.

The software tools, an automatic theorem prover and a model checker, for example SPIN [80], that takes a program and some *theorems*, respectively a *Promela* statement, and proves, respectively checks, the program correct with respect the theorem, or the statement.

A *compiler* and an *interpreter* for any programming language.

Yes, indeed, any abstract interpretation [58, 52] reflects a transcendental deduction: firstly, these examples show that there are many transcendental deductions; secondly, they show that there is no single-most preferred transcendental deduction.

A transcendental deduction, crudely speaking, is just any abstraction that can be “linked” to another, not by logical necessity, but by logical (and philosophical) possibility!

**Definition 4.** Transcendentality: By *transcendentality* we shall here mean the philosophical notion: “the state or condition of being transcendental”.

**Example 2.** Transcendentality: We can speak of a bus in at least three senses:

(i) The bus as it is being “maintained, serviced, refueled”;

(ii) the bus as it "speeds" down its route; and
(iii) the bus as it "appears" (listed) in a bus time table.

The three senses are:
(i) as an endurant (here a part),
(ii) as a perdurant (as we shall see, a behaviour), and
(iii) as an attribute.21

The above example, we claim, reflects transcendentality as follows:
(i) We have knowledge of an endurant (i.e., a part) being an endurant.
(ii) We are then to assume that the perdurant referred to in (ii) is an aspect of the endurant mentioned in (i) – where perdurants are to be assumed to represent a different kind of knowledge.
(iii) And, finally, we are to further assume that the attribute mentioned in (iii) is somehow related to both (i) and (ii) – where at least this attribute is to be assumed to represent yet a different kind of knowledge.

In other words: two (i–ii) kinds of different knowledge; that they relate must indeed be based on a priori knowledge. Someone claims that they relate! The two statements (i–ii) are claimed to relate transcendentally.22

2.1.2.3 Bibliographical Note

The philosophical concept of transcendent deduction is a subtle one. Arguments of transcendental nature, across the literature of philosophy, does not follow set principles and techniques. We refer to [3, The Cambridge Dictionary of Philosophy, pages 807–810] and [53, The Blackwell Companion to Philosophy, Chapter 22: Kant (David Bell), pages 589–606, Bunnin and Tsui-James, eds.] for more on ‘transcendence’.

2.2 The Philosophical Question

Sørlander focuses on the philosophical question of “what is thus necessary that it could not, under any circumstances, be otherwise?”. To study and try answer that question Sørlander thinks rationally, that is, reasons, rather than express emotions. The German philosopher Immanuel Kant (1724–1804) suggests that our philosophising as to the philosophical question above must build on “something which no person can consistently can deny, and thus, something that every person can rationally justify, as a consequence of be able to think at all”. Kant then goes on to build his philosophy [87] on the possibility of self-awareness – something of which we all are aware. Sørlander then, in for example [111], shows that this leads to solipsism23, i.e., to nothing.

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20 I first came across this example when it was presented to me by Paul Lindgreen, an early Danish computer scientist (1936–2021) – and then as a problem of data modelling [89, 1983].
21 - in this case rather: as a fragment of a bus time table attribute.
22 - the attribute statement was “thrown” in “for good measure”, i.e., to highlight the issue!
23 Solipsism: the view or theory that the self is all that can be known to exist.
2.3 Three Principles

2.3.1 The Possibility of Truth

Instead Sørlander suggests that the possibility of truth be the basis for the thinking of an answer to the highlighted question above. The possibility of truth is shared by all of us.

2.3.2 The Principle of Contradiction

Once we accept that the possibility of truth cannot be denied, we have also accepted the principle of contradiction, that is, that an assertion and its negation cannot both be true.

2.3.3 The Implicit Meaning Theory

We must thus also accept the implicit meaning theory.

Definition 5. The Implicit Meaning Theory implies that there is a mutual relationship between the (α) meaning of designations and (β) consistency relations between assertions.

As an example of what “goes into” the implicit meaning theory, we bring, albeit from the world of computer science, that of the description of the stack data type (its endurant data types and perdurant operations).

Example 3. The Implicit Meaning Theory. Narrative:

α The Designations:

1. Stacks, s:S, have elements, e:E;
2. the empty_S operation takes no arguments and yields a result stack;
3. the is_empty_S operation takes an argument stack and yields a Boolean value result.
4. the stack operation takes two arguments: an element and a stack and yields a result stack.
5. the unstack operation takes a non-empty argument stack and yields a stack result.
6. the top operation takes a non-empty argument stack and yields an element result.

β The Consistency Relations:

7. an empty stack is empty, and a stack with at least one element is not;
8. unstacking an argument stack, stack(e,s), results in the stack s; and
9. inquiring the top of a non-empty argument stack, stack(e,s), yields e.

Formalisation.

The designations:

<table>
<thead>
<tr>
<th>Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>E, S</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Axiom</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>is_empty(empty_S()) = true</td>
<td></td>
</tr>
<tr>
<td>is_empty(stack(e,s)) = false</td>
<td></td>
</tr>
<tr>
<td>unstack(stack(e,s)) = s</td>
<td></td>
</tr>
<tr>
<td>top(stack(e,s)) = e</td>
<td></td>
</tr>
</tbody>
</table>
2.3.4 A Domain Analysis & Description Core

The three concepts: (i) the possibility of truth, (ii) the principle of contradiction and (iii) the implicit meaning theory thus form the core – and imply that (a) the indispensably necessary characteristics of any possible world, i.e., domain, are equivalent with (b) the similarly indispensably necessary conditions for any possible domain description.

2.4 The Deductions

2.4.1 Assertions

Definition 6. **Assertion:** An assertion is a declaration, an utterance, that something is the case.

Assertions may typically be either propositions or predicates.

2.4.2 The Logical Connectives

Any domain description must necessarily contain assertions. Assertions are expressed in terms of negation, \( \sim \), conjunction, \( \land \), disjunction, \( \lor \), and implication, \( \Rightarrow \).

2.4.2.1 \( \sim \): Negation

Negation is defined by the principle of contradiction. If an assertion, \( a \), holds, then its negation, \( \sim a \), does not hold.

2.4.2.2 Simple Assertions

Simple assertions, i.e., propositions, are formed from assertions, f.x. \( a,b \), by means of the logical connectives.

2.4.2.3 \( \land \): Conjunction

The simple assertion \( a \land b \) holds if both \( a \) and \( b \) holds.

2.4.2.4 \( \lor \): Disjunction

The simple assertion \( a \lor b \) holds if either or both \( a \) and \( b \) holds.

2.4.2.5 \( \Rightarrow \): Implication

The simple assertion \( a \Rightarrow b \) holds if \( a \) is inconsistent with the negation of \( b \).
2.4 The Deductions

2.4.2.6 Model Theory Explication of The Logical Connectives

A model theory explication of the binary logical connectives is given on Page 143.

2.4.3 Modalities

2.4.3.1 Necessity

Definition 7. **Necessity:** An assertion is necessarily true if its truth ("true") follows from the definition of the designations by means of which it is expressed. Such an assertion holds under all circumstances.

Example 4. **Necessity:** “It may rain someday” is necessarily true.

2.4.3.2 Possibility

Definition 8. **Possibility:** An assertion is possibly true if its negation is not necessarily true.

Example 5. **Possibility:** “it will rain tomorrow” is possibly true.

2.4.4 Empirical Assertions

Definition 9. **Empirical Knowledge:** In philosophy, knowledge gained from experience rather than from innate ideas or deductive reasoning is empirical knowledge. In the sciences, knowledge gained from experiment and observation rather than from theory is empirical knowledge.

Example 6. **Expressing Empirical Knowledge:** There are innumerable ways of expressing empirical knowledge.

a. There are two automobiles in that garage.

b. The two automobiles in that garage are distinct.

c. The two automobiles in that garage are parked next to one another.

d. That automobile, the one to the left, in that garage is [painted] red.

e. The automobile to the right in that garage has just returned from a drive.

---

24 The automobiles are solid endurants, and so is the garage, that is, they are both parts.
25 Their distinctness gives rise to their respective, distinct, i.e., unique identifiers.
26 The topological ordering of the two automobiles is an example of their mereology.
27 The red colour of the automobile is an attribute of that automobile.
28 The fact that that automobile, to the right in the garage, has just returned from a drive, is a possibly time-stamped attribute of that automobile.
f. The automobile, with Danish registration number AB12345, is currently driving on the Copenhagen area city Holte road Fredsvej at position ‘top of the hill’.29

g. The automobile on the roof of that garage is pink. The pronoun ‘that’ shall be taken to mean that someone gestures at, points out, the garage in question. If there is no such garage then the assertion denotes the chaos value! Statements (a.–g.) are assertions. The assertions contain references to quantities “outside the assertions” — ‘outside’ in the sense that they are not defined in the assertions. Assertion (g.) does not make sense, i.e., yields chaos. The term ‘roof’ has not been defined.

The Object Language. The language used in the above assertions is quite ‘free-wheeling’. The language to be used in “our” domain descriptions is, i.e., will be, more rigid.

**Definition 10. Empirical Assertion:** The domain description language of assertions, contain references, i.e., designators, and operators. All of these shall be properly defined in terms of names of endurants and their unique identifers, mereologies and attributes; and in terms of their perdurant “counterparts”.

***

From Possible Predicates to Conceptual Logic Description Framework. The ability to deduce which type of predicates that a phenomena of any domain can be ascribed is thus equivalent to deducing the conceptual logical conditions for every possibly possible domain description.

***

By a so-called transcendental deduction we have shown that simple empirical assertions consist of a subject which refers to an independently existing entity and a predicate which ascribes a property to the referred entity [111, π 146 ℓ 1–5].

The world, or as we shall put it, the domains, that we shall be concerned with, are what can be described in simple assertions, then any possible such world, i.e., domain must primarily consist of such entities [111, π 146 ℓ 5–7].

We shall therefore, in the following, explicate a system of concepts by means of which the entities, that may be referred to in simple assertions, can be described [111, π 146 ℓ 8–11].

These concepts are those of entities, endurants, perdurants, unique identity, mereology and attributes.

### 2.4.5 Identity and Difference

We can now assume that the world consists of an indefinite number of entities: Different empirical assertions may refer to distinct entities. Most immediately we can define two interconnected concepts: identity and diversity.

#### 2.4.5.1 Identity

**Definition 11. Identical:** “An entity referred to by the name A is identical to an entity referred to by the name B if A cannot be ascribed a property which is incommensurable with a property ascribed to B” [111, π 146 ℓ 14-23].

29 The automobile in question is now a perdurant having a so-called time-stamped programmable event attribute of the Copenhagen area city of Holte, “top of the hill”.
2.4 The Deductions

2.4.5.2 Difference

Definition 12. Different: “A and B are distinct, differs from one another, if the can be ascribed incommensurable properties.” [111, π 146 ℓ 23-26]

“These formal definitions, by transcendental deduction, introduces the concepts of of identity and difference. “They can thus be assumed in any transcendental deduction of a domain description which, in principle, must be expressed in any possible language”. [111, π 147 ℓ 1-5]

Definition 13. Unique Identification: By a transcendental deduction we introduce the concept of manifest, physical entities each being uniquely identified.

We make no assumptions about any representation of unique identifiers.

2.4.6 Relations

2.4.6.1 Identity and Difference

Definition 14. Relation: “Implicitly”, from the two concepts of identity and difference, follows the concept of relations. “A identical to B is a relational assertion. So is A different from B” [111, π 147 ℓ 6-10]

2.4.6.2 Symmetry

Definition 15. Symmetry: If A is identical to B then B must be identical to A. This expresses that the identical to relation is symmetric. And, If A is different from B then B must be different from A. This expresses that the different from relation is also symmetric.

2.4.6.3 Asymmetry

Definition 16. Asymmetry: A relation which holds between A and B but does not hold between B and A is asymmetric [111, π 147 ℓ 25-27]

2.4.6.4 Transitivity

Definition 17. Transitivity: “If A is identical to B and if B is identical to C then A must be identical to C. So the relation identical to is transitive” [111, π 147-148 ℓ 28-30,1-4]

The relation different from is not transitive.

2.4.6.5 Intransitivity

Definition 18. In-transitivity: If, on the other hand, we can logically deduce that a relation, \( R \) holds’ from A to B and the same relation, \( R \), holds from B to C but \( R \) does not hold from A to C then relation \( R \) is intransitive [111, π 148 ℓ 9–12].
2.4.7 Sets, Quantifiers and Numbers

2.4.7.1 Sets

The possibility now exists that two or more entities may be prescribed the same property.

**Definition 19. Sets:** The “same properties” could, for example, be that two or more uniquely distinguished entities, \(x, y, \ldots, z\), have [at least] one attribute kind (type) and value, \((t, v)\), in common. This means that \((t, v)\) distinguishes a set \(s_{(t,v)}\) – by a *transcendental deduction*. A fact, just \(t\) likewise distinguishes a possibly other, most likely “larger”, set \(s\).

From the transcendentally deduced notion of set follows the relations: equality, \(=\), inequality, \(\neq\), proper subset, \(\subset\), subset, \(\subseteq\), set membership, \(\in\), set intersection, \(\cap\), set union, \(\cup\), set subtraction, \(\setminus\), set cardinality, \(\text{card}\), etc.!

2.4.7.2 Quantifiers

By a further *transcendental deduction* we can place the *quantifiers* among the concepts that are necessary in order to describe domains.

**Definition 20. The Universal Quantifier:** The universal quantifier expresses that all members, \(x\), of a set, \(s\), possess a certain \(P\)roperty: \(\forall x : S \bullet P(x)\)

**Definition 21. The Existential Quantifier:** The existential quantifier expresses that at least one member, \(x\), of a set, \(s\), possess a certain \(P\)roperty: \(\exists x : S \bullet P(x)\)

2.4.7.3 Numbers

Numbers can, again by *transcendental deduction*, be introduced, not as observable phenomena, but as a rational, logic consequence of sets.

**Definition 22. Numbers:** Numbers can be motivated, for example, as follows:

- Start with an empty set, say \(\{}\). It can be said to represent the number zero.\(^{30}\)
- Then add the empty set \(\{}\) to \(\{}\) and You get \(\{\{}\}\) said to represent 1.
- Continue with adding \(\{}\) to \(\{\{}\}\) and You get \(\{\{}\}, \{\{}\}\}\), said to represent 2.
- And so forth – ad infinitum.

In this way one\(^{31}\) can define the natural numbers. We could also do it by just postulating distinct entities which are then added, one by one to a an initially empty set \([111, \pi 150 \ell 8-13]\).

We can then, still in the realm of philosophy, proceed with the introduction of the arithmetic operations designated by: addition, \(+\), subtraction, \(-\), multiplication, \(\times\), division, \(\div\), equality, \(=\), inequality, \(\neq\), larger than, \(>\), larger than or equal, \(\geq\), smaller than, \(<\), smaller than or equal, \(\leq\), etcetera!

From explaining numbers on a purely philosophical basis one can now proceed mathematically into the realm of *number theory* \([71]\).

---

\(^{30}\) Which, in the decimal notation is written as 0.

2.4 The Deductions

2.4.8 Primary Entities

We now examine the concept of *primary objects*.

The next two definitions, in a sense, “fall outside” the line of the present philosophical inquiry. They will be “corrected” to then “fall inside” our inquiry.

**Definition 23. **Object: By an *object* we, in our context, mean something material that may be perceived by the senses.

**Definition 24. **Primary Object: By a *primary object* we mean an object that exists as its own entity independent of other objects.

In the last definition we have used the term *entity*. That term, ‘entity’, will be used henceforth instead of the term ‘object’.

We have deduced the relations *identity, difference, symmetry, asymmetry, transitivity* and *intransitivity* in Sects. 2.4.5–2.4.6. You may ask: for what purpose? And our answer is: to justify the next set of deductions. First we reason that there is the possibility of there being many entities. We argue that that is possible due to there being the relation of asymmetry. If it holds between two entities then they must necessarily be ascribed different predicates, hence be distinct.

Similarly we can argue that two entities, B and C which both are asymmetric wrt. to an entity A may stand in a symmetric relation to one another. This opens for the possibility that every pair of distinct entities may stand in a pair of mutual relations. First the asymmetry relation that expresses their distinctness. Secondly, the possibility of a symmetry relation which expresses the two entities individually with respect to one-another. *The above forms a transcendental basis for how two or more [primary] entities must necessarily be characterised by predicates.*

2.4.9 Space and Time

The asymmetry and symmetry relations between entities cannot be necessary characteristics of every possibly reality if they cannot also posses an *unavoidable rôle* in our own concrete reality. Next we examine two such *unavoidable rôles*.

2.4.9.1 Space

One pair of such rôles are *distance* and *direction*. *Distance* is a relation that holds between any pair of distinct entities. It is a symmetric relation. *Direction* is an asymmetric relation that also holds between pair of distinct entities. Hence we conclude that *space* is an unavoidable characteristics of every possibly reality. Hence we conclude that entities exist in space. They must “fill” some space, have *extension*, they must fill some space, have *surface* and *form*. From this we can define the notions of spatial point, spatial straight line, spatial surface, etcetera. Thus we can philosophically motivate geometry.

---

34. Yes, we know: we have not defined what is meant by ‘as its own’ and ‘independent’!
2.4.9.2 Time

Primary empirical entities may be accrue predicates that it is not logically necessary that they accrue. That is, it is logically possible that primary entities accrue predicates that they actually accrue. How is it possible that one and the same primary entity may accrue incommensurable predicates?

That is only possible if one and the same primary entity can exist in different states. It may exist in one state in which it accrue a certain predicate. And it may exist in another state in which it accrue a therefrom incommensurable predicate.

What can we say about these states? First that these states accrue different, incommensurable predicates. How can we assure that? Only if the states stand in an asymmetric relation to one another. From this we can conclude that primary entities necessarily may exist in a number of states each of which stand in an asymmetric relation to their predecessor state. So these states also stand in a transitive relation.

This is a necessary characteristics of any possible world. So it is also a characteristics of our world. That relation is time. It possesses the before, after, in-between, and other [temporal] relations. We have thus deduced that every possible world must “occur in time” and that primary entities may exist in before or after states.

From the above we can derive a whole algebra of temporal types and operations, for example:

- TIME and TIME INTERVAL types;
- addition of TIME and TIME INTERVAL to obtain TIME;
- addition of TIME INTERVALs to obtain TIME INTERVALs;
- subtraction of two TIMEs to become TIME INTERVALs; and
- subtraction of of TIME INTERVALs to obtain TIME INTERVALs.

2.4.10 The Causality Principle

But what is it that cause primary entities to undergo state changes? Assertions about how a primary entity is at different times, such assertions must necessarily be logically independent. That follows from primary entities necessarily must accrue incommensurable predicates at different times. It is therefore logically impossible to conclude from how a primary entity is at one time to how it is at another time. How, therefore, can assertions about a primary entity at different times be about the same entity?

We can therefore transcendentally deduce that there must be a special implication-relationship between assertions about how a primary entity is at different times. Such a special implication-relationship must depend on the empirical circumstances under which the primary entity exists. That is, we must deduce the conditions under which it is, at all, possible to consistently make statements about primary entities going from one state in which it accrues a specific predicate to another state in which it accrues a therefrom incommensurable predicate. There must be something in the empirical circumstances which implicates the state transition. If the the empirical circumstances are stable then Thebes is nothing in these circumstances that imply entity changes. If the primary entity changes, then that assumes that there must have been a prior change in the circumstances – with those changes having that consequence. . . .

We name such a change of the circumstances a cause. And we conclude that every change of a primary entity must have a cause. We also conclude that equivalent cause imply equivalent effects.

35 We skip some of Sørlander’s reasoning, [111, Page 162, lines 1–12]
This form of implication is called the causality principle. It assumes logical implication. But it cannot be reduced to logical implication. It is logically necessary that every primary entity – and therefore every possible world – is subject to the causality principle. In this way Kai Sørlander transcendentally deduce the principle of causality. Every change has a cause. The same cause under the same circumstances lead to same effects.

2.4.11 Newton’s Laws

Sørlander then shows how Newton’s laws can be deduced. These laws, in summary, are:

- **Newton’s First Law**: An entity is at rest or moving at a constant speed in a straight line, it will remain at rest or keep moving in a straight line at constant speed unless it is acted upon by a force.
- **Newton’s Second Law**: When an entity is acted upon by a force, the time rate of change of its momentum equals the force.
- **Newton’s Third Law**: To every action there is always opposed an equal reaction; or, the mutual actions of two bodies upon each other are always equal, and directed to contrary entities.

2.4.11.1 Kinematics

Above we have deduced that primary entities are in both space and time. They have extent in both space and time. That means that they must change with respect to their spatial properties: place and form. The change in place is the fundamental. A primary entity which changes place is said to be in movement. A primary entity in movement must follow a certain geometric route. It must move a certain length of route in a certain interval of time, i.e., have a velocity: speed and direction. A primary entity which changes velocity has an acceleration. That is, we have deduced he basics of kinematics.

2.4.11.2 Dynamics

When we to the above add that primary entities are in time, then they are subject to causality. That means that we are entering the doctrine of the influence of forces on primary entities. That is, dynamics. Kinematics imply that an entity changes if it goes from being at rest to move, or if it goes from moving to being at rest. An entity also changes if it goes from moving at one velocity to moving at a different velocity. We introduce the notion of momentum. An entity has same momentum if at two times it has the same velocity and acceleration.

2.4.11.3 Newton’s First Law

When we combine kinematics with causality then we can deduce that if an entity changes momentum then there must be a cause in the circumstances which causally implies the change. We call that cause a force. The force must be proportional to the change in momentum. This implies that an entity which is not subject to an external force remains in the same momentum. This is The Law of Inertia, Newtons First Law.
2.4.11.4 **Newton’s Second Law**

That a certain force is necessary in order to change an entity’s momentum must imply that such an entity must provide a certain resistance against change of momentum. It must have a mass. From this it follows that the change of an entity’s momentum not only must be proportional to the applied force but also inversely proportional to that entity’s mass. This is Newton’s Second Law.

2.4.11.5 **Newton’s Third Law**

Where do the forces that influence the momentum of entities come from? It must, it can only, be from primary entities. Primary entities must be the source of the forces that influence other entities. Here we shall argue one such reason. The next section, on universal gravitation, presents a second reason.

Primary entities may be in one another’s way. Hence they may eventually collide. If a primary entity has a certain velocity it may collide with another primary entity crossing its way. In the mutual collision the two entities influence one another such that they change momentum. They influence each other with forces. Since none of the two entities have any special position, i.e., rank, the forces by means of which they affect one another must be equal and oppositely directed. This is Newton’s Third Law.

2.4.12 **Universal Gravitation**

But really, how can primary entities be the source of forces that affects one another? We must dig deeper! How can primary entities have mass such that it requires force to change their momentum? Our answer is that the reason they have mass must be due to mutual influence between the primary objects themselves. It must be an influence which is oppositely directed to that which they expose on one another when they collide. Because this, in principle, applies to all primary entities, these must be characterised by a mutual universal attraction. And that is what we call universal gravitation. That concept has profound implications.

We shall not go into details here but just, casually, as it were, mention that such concepts as speed limit, elementary particles and Einstein’s theories are “more-or-less” transcendentally deduced!

2.4.13 **Purpose, Life and Evolution**

We shall briefly summarise Sørlander’s analysis and deductions with respect to the concepts of living species: plants and animals, the latter including humans.

Up till now Sørlander’s analyses and deductions have focused on the physical world, “culminating” in Newton’s Laws and Einstein’s theories.

---

36 This section is from [111, Pages 168–173]
If there is to be language and meaning then, as a first condition, there must be the possibility that there are primary entities which are not locked-in “only” in that physical world deduced till now. This is only possible if such primary entities are additionally subject to a purpose-causality, one that is so constructed as to strive to maintain its own existence. We shall refer to this kind of primary entities as living species.

2.4.13.1 Living Species

As living species they must be subject to all the physical conditions for existence and mutual influence. Additionally they must have a form which they are causally determined to reach and maintain. This development and maintenance must take place in a substance exchange with its surroundings. Living species need these substances in order to develop and maintain their form.

It must furthermore be possible to distinguish between two forms of living species: (i) one form which is characterised only by development, form and substance exchange; and (ii) another form which, additional to (i), is characterised by being able to move. The first form we call plants. The second form we call animals.

2.4.13.2 Animals

For animals to move they must (i) possess sense organs, (ii) organs of movement and (iii) instincts, incentives, or feelings. All this still subject to the physical laws and to satisfy motion.

This is only possible if animals are not built (like the elementary particles of physics) but by special physical units. These cells must satisfy the purpose-causality of animals. And we know, now, from the biological sciences that something like that is indeed the case. Indeed animals are built from cells all of which possess genomes for the whole animal and, for each such cell, a proper fraction of its genome controls whether it is part of a sensory organ, or a nerve, or a motion organ, or a more specific function. Thus it has transcendentally been deduced that such must be the case and biology has confirmed this.

2.4.13.2.1 Humans

We briefly summarise, in six steps, (i–vi), Sørlander’s reasoning that leads from animals, in general, see above, to humans, in particular.

(i) First the concept of level of consciousness is introduced. On the basis of animals being able to learn from experience the concept of consciousness level is introduced. It is argued that neurons provide part of the basis for learning and the consciousness level.

(ii) Secondly the concept of social instincts is introduced. For animals to interact social instincts are required.

(iii) Thirdly the concept of sign language is introduced. In order for animals to interact some such animals, notably the humans, develop a sign language.

(iv) Fourthly the concept of language is introduced. The animals that we call humans finally develop their sign language into a language that can be spoken, heard and understood. Such a language, regardless of where it is developed, that is, regardless of which language it is, must assume, i.e., build on the same set of basic concepts as had been uncovered so far in our deductions of what must necessarily be in any description of any world.

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37 We now treat the material of [111, Chapter 10, Pages 174–179].
38 [111, Chapter 11, Pages 180–183]
We continue summarising Sørlander’s reasoning that leads from generalities about humans to humans with knowledge and responsibility.

(v) Fifthly the concept of knowledge is introduced. An animal which is conscious must sense and must react to what it senses. To do so it must have incentives as causal conditions for its specific such actions. If the animal has, possess, language, then it must be able to express that and what it senses and that it acts accordingly, and why it does so. It must be able to express that it can express this. That is, that what it expresses, is true. To express such assertions, with sufficient reasons for why they are true, is equivalent to knowing that they are true. Such animals, as possess the above “skills”, become persons, humans.

(vi) Sixthly the concept of responsibility is introduced. Humans conscious of their concrete situation, must also know that these situations change. They are conscious of earlier situations. Hence they have memory. So that can formulate experience with respect to the consequences of their actions. Thus humans are (also) characterised by being able to understand the consequences of future actions. A person who considers how he ought act, can also be ascribed responsibility – and can be judged morally.

... This ends our esopée of Sørlander’s metaphysics wrt. living species. That is, we shall not cover neither non-human animals, nor plants.

2.5 Philosophy, Science and the Arts

We quote extensively from [109, Kai Sørlander, 1997].

[109, pp 178] “Philosophy, science and the arts are products of the human mind.”
[109, pp 179] “Philosophy, science and the arts each have their own goals.”

- Philosophers seek to find the inescapable characteristics of any world.
- Scientists seek to determine how the world actually and our situation in that world.
- Artists seek to create objects for experience.

We shall elaborate. [109, pp 180] “Simplifying, but not without an element of truth, we can relate the three concepts by the modalities:”

- philosophy is the necessary,
- science is the real, and
- art is the possible.

... Here we have, then, a distinction between philosophy and science. ... From [108] we can conclude the following about the results of philosophy and science. These results must be consistent [with one another]. This is a necessary condition for their being correct. ... The real must be a concrete realisation of the necessary.

2.6 A Word of Caution

The present chapter represents an attempt to give an English interpretation of Kai Sørlander’s Philosophy. I will “mull” over this interpretation for a while. Then I will present it to Kai Sørlander for his comments. We shall see.
Chapter 3
Domains

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This chapter is informal. Here we introduce you to important main concepts of domains. Subsequent chapters will be more technical. They will define most of the domain concepts of this chapter properly.
3.1 Domain Definition

Definition 25. Domain By a domain we shall understand a rationally describable segment of a discrete dynamics segment of a human assisted reality, i.e., of the world; its solid or fluid entities: natural ["God-given"] and artefactual ["man-made"], and its living species entities: plants and animals – including, notably, humans.

Example 7. Domains: A few, more-or-less self-explanatory examples:

- **Rivers** – with their natural sources, deltas, tributaries, waterfalls, etc., and their man-made dams, harbours, locks, etc. [42]
- **Road nets** – with street segments and intersections, traffic lights, and automobiles.
- **Pipelines** – with their wells, pipes, valves, pumps, forks, joins and wells [28].
- **Container terminals** – with their container vessels, containers, cranes, trucks, etc. [36].

The definition relies on the understanding of the terms ‘rationally describable’, ‘discrete dynamics’, ‘human assisted’, ‘solid’ and ‘fluid’. The last two will be explained later. By rationally describable we mean that what is described can be understood, including reasoned about, in a rational, that is, logical manner. By discrete dynamics we imply that we shall basically, in this primer, rule out such domain phenomena which have properties which are continuous with respect to their time-wise, i.e., dynamic, behaviour. By human-assisted we mean that the domains – that we are interested in modelling – have, as an important property, that they possess man-made entities.

This primer presents a method, its principles, procedures, techniques and tools, for analysing & describing domains.

3.2 Phenomena and Entities

Definition 26. Phenomena By a phenomenon we shall understand a fact that is observed to exist or happen.

Some phenomena are rationally describable – to a large or full degree – others are not.

Definition 27. Entities By an entity we shall understand a more-or-less rationally describable phenomenon.

Example 8. Phenomena and Entities: Some, but not necessarily all aspects of a river can be rationally described, hence can be still be considered entities. Similarly, many aspects of a road net can be rationally described, hence will be considered entities.

3.3 Endurants and Perdurants

3.3.1 Endurants

Definition 28. Endurants those quantities of domains that we can observe (see and touch), in space, as “complete” entities at no matter which point in time – “material” entities that persists, endures.

---

40 We use here the ampersand, ‘&’, as in A&B, to emphasize that we are treating A and B as one concept.
Example 9. **Endurants**: a street segment [link], a street intersection [hub], an automobile.

Domain endurants, when eventually modelled in software, typically become data. Hence the careful analysis of domain endurants is a prerequisite for subsequent careful conception and analyses of data structures for software, including databases.

### 3.3.2 Perdurants

Definition 29. **Perdurants**: those quantities of domains for which only a fragment exists, in space, if we look at or touch them at any given snapshot in time.

Example 10. **Perdurant**: a moving automobile.

Domain perdurants, when eventually modelled in software, typically become processes. Hence the careful analysis of domain perdurants is a prerequisite for subsequent careful conception and analyses of functions (procedures).

### 3.4 External and Internal Endurant Qualities

#### 3.4.1 External Qualities

Definition 30. **External qualities**: of endurants of a manifest domain are, in a simplifying sense, those we can see, touch and have spatial extent. They, so to speak, take form.

Example 11. **External Qualities**: The Cartesian of sets of solid atomic street intersections, and of sets of solid atomic street segments, and of sets of solid automobiles of a road transport system where the Cartesian, sets, atomic, and solid reflect external qualities.

#### 3.4.1.1 Discrete or Solid Endurants

Definition 31. **Discrete or Solid Endurants**: By a solid [or discrete] endurant we shall understand an endurant which is separate, individual or distinct in form or concept, or, rephrasing: have ‘body’ [or magnitude] of three-dimensions: length, breadth and depth.

Example 12. **Solid Endurants**: The wells, pipes, valves, pumps, forks, joins and sinks of pipelines are solids. [These units may, however, and usually will, contain fluids, e.g., oil, gas or water.]

We shall mostly be analysing and describing solid endurants.

As we shall see, in the next chapter, we analyse and describe solid endurants as either parts or living species: animals and humans. We shall mostly be concerned with parts. That is, we shall just, as: “in passing”, for sake of completeness, mention living species!

---

41 Cartesian after the French philosopher, mathematician, scientist René de Descartes (1596–1650)
3.4.1.2 Fluids

**Definition 32.** Fluid Endurants: By a *fluid endurant* we shall understand an endurant which is prolonged, without interruption, in an unbroken series or pattern; or, rephrasing: a substance (liquid, gas or plasma) having the property of flowing, consisting of particles that move among themselves [90, Vol. I, pg. 774].

**Example 13.** Fluid Endurants: water, oil, gas, compressed air, smoke.

Fluids are otherwise liquid, or gaseous, or plasmatic, or granular, or plant products, i.e., chopped sugar cane, threshed, or otherwise, etcetera. Fluid endurants will be analysed and described in relation to solid endurants, viz. their “containers”.

3.4.1.3 Parts

**Definition 33.** Parts: The non-living species solids are what we shall call parts.

Parts are the “work-horses” of man-made domains. That is, we shall mostly be concerned with the analysis and description of endurants into parts.

**Example 14.** Parts: The previous example of solids was also an example of parts.

We distinguish between atomic and compound parts.

3.4.1.3.1 Atomic Parts

**Definition 34.** Atomic Part, I: By an *atomic part* we shall understand a part which the domain analyser considers to be indivisible in the sense of not meaningfully, for the purposes of the domain under consideration, that is, to not meaningfully consist of sub-parts.

3.4.1.3.2 Compound Parts

We, pragmatically, distinguish between Cartesian-product-, and set-oriented parts. If Cartesian-oriented, to consist of two or more distinctly sort-named endurants (solids or fluids), If set-oriented, to consist of an indefinite number of zero, one or more parts.

**Definition 35.** Compound Part, I: *Compound parts* are those which are either Cartesian-product- or are set-oriented parts.

**Example 15.** Compound Parts: A road net consisting of a set of hubs, i.e., street intersections or “end-of-streets”, and a set of links, i.e., street segments (with no contained hubs), is a Cartesian compound; and the sets of hubs and the sets of links are part set compounds.

---

42 This is a purely pragmatic decision. “Of course” sand, gravel, soil, etc., are not fluids, but for our modelling purposes it is convenient to “compartmentalise” them as fluids!

43 See footnote 42.
3.4 External and Internal Endurant Qualities

3.4.2 An Aside: An Upper Ontology

We have been reasonably careful to just introduce and state informal definitions of phenomena and some classes thereof. In the next chapter we shall, in a sense, “repeat” coverage of these phenomena. But now in a more analytic manner. Figure 3.1 is intended to indicate this.

![Phenomena of Natural and Artefactual Universes of Discourse](image)

**Fig. 3.1 Upper Ontology**

So far we have only touched upon the ‘External Qualities’ labeled, dotted-dashed box of the ‘Endurants’ label-led dashed box of Fig. 3.1. In Chapter 4 we shall treat external qualities in more depth — more systematically: analytically and descriptively.

3.4.3 Internal quality

**Definition 36.** **Internal qualities:** those properties [of endurants] that do not occupy space but can be measured or spoken about.

**Example 16.** **Internal qualities:** the unique identity of a part, the relation of part to other parts, and the endurant attributes such as temperature, length, colour.
3.4.3.1 **Unique identity**

**Definition 37.** **Unique identity:** an immaterial property that distinguishes two *spatially* distinct solids.

**Example 17.** **Unique identities:** Each hub in a road net is unique identified, so is each link and automobile.

3.4.3.2 **Mereology**

**Definition 38.** **Mereology:** a theory of *endurant* part-hood relations: of the relations of an *endurant* parts to a whole and the relations of *endurant* parts to *endurant* parts within that whole.

**Example 18.** **Mereology:** that a link is topologically *connected* to exactly two specific hubs, that hubs are *connected* to zero, one or more specific links, and that links and hubs are *open* to specific subsets of automobiles.

3.4.3.3 **Attribute**

**Definition 39.** **Attributes:** Properties of endurants that are not *spatially* observable, but can be either physically (electronically, chemically, or otherwise) measured or can be objectively spoken about.

**Example 19.** **Attribute:** Links have lengths, and, at any one time, zero, one or more automobiles are occupying the links.

3.5 **Prompts**

3.5.1 **Analysis Prompts**

**Definition 40.** **Analysis prompt:** a predicate or a function that may be posed by humans to facets of a domain. Observing the domain the analyser may then act upon the combination of the particular prompt (whether a predicate or a function, and then what particular one of these it is) thus “applying” it to a domain phenomena, and yielding, in the minds of the humans, either a truth value or some other form of value.

3.5.1.1 **Analysis Predicate**

**Definition 41.** **Analysis predicates:** an analysis prompt which yields a truth value.

**Example 20.** **Analysis Predicates:** General examples are can an observable phenomena be rationally described, i.e., an entity, is an entity a solid or a fluid, is a solid endurant a part or a living species.

3.5.1.2 **Analysis Function**

**Definition 42.** **Analysis function:** an analysis prompt which yields some RSL-Text.
3.6 Perdurant Concepts

**Example 21. Analysis Functions:** Two examples: one yields the endurants of a Cartesian part and their respective sort names, another yields the set of a parts of a part set and their common type.

3.5.2 Description Prompt

**Definition 43. Description prompt:** A function that may be posed by humans who may then act upon it: “applying” it to a domain phenomena, and “yielding” narrative and formal RSL-Texts describing what is being observed.

**Example 22. Description Prompts:** Result in RSL-Texts describing for example a (i) Cartesian endurant, or (ii) its unique identifier, (iii) or its mereology, or (iv) its attributes, (iv) or other.

3.6 Perdurant Concepts

3.6.1 “Morphing” Parts into Behaviours

As already indicated we shall transcendently deduce (perdurant) behaviours from those (endurant) parts which we, as domain analysers cum describers, have endowed with all three kinds of internal qualities: unique identifiers, mereologies and attributes. Chapter 6, will show how.

3.6.2 State

**Definition 44. State:** A state is any set of the parts of a domain.

**Example 23. A Road System State:** The domain analyser cum describer may, in brief, decide that a road system state consists of the road net aggregate (of hubs and links), all the hubs, and all the links, and the automobile aggregate (of all the automobiles), and all the individual automobiles.

3.6.3 Actors

**Definition 45. Actors:** An actor is anything that can initiate an action, an event or a behaviour.

44 The road net aggregate, in its perdurant form, may “model” the Department of Roads of some country, province, or town.

45 The automobile aggregate aggregate, in its perdurant form, may “model” the Department of Vehicles of some country, province, or town.
3.6.3.1 Action

Definition 46. Actions: An action is a function that can purposefully change a state.

Example 24. Road Net Actions: These are some road net actions: The insertion of a new or removal of an existing hub; or the insertion of a new, or removal of an existing link.

3.6.3.2 Event

Definition 47. Events: An event is a function that surreptitiously changes a state.

Example 25. Road Net Events: These are some road net events: The blocking of a link due to a mud slide; the failing of a hub traffic signal due to power outage; the blocking of a link due to an automobile accident.

3.6.3.3 Behaviour

Definition 48. Behaviours: A behaviour is a set of sequences of actions, events and behaviours.

Example 26. Road Net Traffic: Road net traffic can be seen as a behaviour of all the behaviours of automobiles, where each automobile behaviour is seen as sequence of start, stop, turn right, turn left, etc., actions; of all the behaviours of links where each link behaviour is seen as a set of sequences (i.e., behaviours) of “following” the link entering, link leaving, and movement of automobiles on the link; of all the behaviours of hubs (etc.); of the behaviour of the aggregate of roads, viz. The Department of Roads, and of the behaviour of the aggregate of automobiles, viz, The Department of Vehicles.

3.6.4 Channel

Definition 49. Channel: A channel is anything that allows synchronisation and communication of values between two behaviours.

We shall use Tony Hoare’s CSP concept [78] to express synchronisation and communication of values between behaviours. Hence the behaviour $i$ statement $ch[index]!value$ to state that behaviour $i$ offers, “outputs”: $i$, $value$ to behaviours indicated by $index$. And behaviour $j$ expresses $ch[index]?value$ that it is willing to accept “input from & synchronise with” behaviour $i$, $?$, any $value$.

3.7 Domain Analysis & Description

3.7.1 Domain Analysis

Definition 50. Domain Analysis is the act of studying a domain as well as the result of that study in the form of informal statements.
3.7.2 Domain Description

Definition 51. **Domain Description** is the act of describing a domain as well as the result of that act in the form of narratives and formal RSL-Text.

3.8 Closing

This chapter has introduced the main concepts of domains such as we shall treat (analyse and describe) domains. The next three chapters shall now systematically treat the analysis and description of domains. That treatment takes concept by concept and provides proper definitions and introduces appropriate analysis and description prompts; one-by-one, in an almost pedantic, hence perhaps “slow” progression! The reader may be excused if they, now-and-then, lose sight of “their way”. Hence the present chapter. To show “the way”: that, for example, when we treat external endurant qualities, there is still the internal endurant qualities, and that the whole thing leads of to perdurants: actors, actions, events and behaviours.

---

46 We have omitted treatment of living species: plants and animals – the latter including humans. They will be treated in the next chapter!
# Chapter 4

*Endurants: External Domain Qualities*

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This, the present chapter, as well as Chapter 3, is based on Chapter 4 of [41]. You may wish to study that chapter for more detail.

4.1 Universe of Discourse

Definition 52. **Universe of Discourse, UoD** By a *universe of discourse* we shall understand the same as the *domain of interest*, that is, the *domain* to be *analysed & described*.

4.1.1 Identification

The **first task** of a domain analyser cum describer is to settle upon the domain to be analysed and described. That domain has first to be given a name.

4.1.2 Naming

A **first decision** is to give a name to the overall domain sort, that is, the type of the domain seen as an endurant, with that sort, or type, name being freely chosen by the analyser cum describer – with no such sort names having been chosen so far!

4.1.3 Examples

**Examples of UoDs** We refer to a number of Internet accessible experimental reports of descriptions of the following domains:

- *railways* [8, 46, 10],
- *“The Market”* [9],
- *container shipping* [15],
- *Web systems* [24],
- *stock exchange* [25],
- *oil pipelines* [28],
4.1 Universe of Discourse

- credit card systems \[31\],
- weather information \[32\],
- swarms of drones \[33\],
- document systems \[35\],
- container terminals \[36\],
- retail systems \[39\],
- assembly plants \[40\],
- waterway systems \[42\],
- shipping \[43\],
- urban planning \[51\].

4.1.4 Sketching

The second task of a domain analyser cum describer is to develop a rough sketch narrative of the domain. The rough-sketching of what a domain is, is not a trivial matter. It is not done by a committee! It usually requires repeated “trial sketches”. To carry it out, i.e., the sketching, normally requires a combination of physical visits to domain examples, if possible; talking with domain professionals, at all levels; and reading relevant literature. It also includes searching the Internet for information. We shall show an example next.

Example 27. Sketch of a Road Transport System UoD: The road transport system that we have in mind consists of a road net and a set of automobiles (private, trucks, buses, etc.) such that the road net serves to convey automobiles. We consider the road net to consist of hubs, i.e., street intersections, including street segment connection points, and links, i.e., street segments between adjacent hubs\(^{49}\).

4.1.5 Universe of Discourse Description

The general universe of discourse, i.e., domain, description prompt can be expressed as follows:

Domain Description Prompt 1 \(\text{calc.Universe.of.Discourse()}\) describer

- Naming:
  - type UoD
- Rough Sketch:
  - Text

The above \(\text{RSL-Text}\) expresses that the \(\text{calc.Universe.of.Discourse()}\) domain describer generates RSL-Text. Here is another example rough sketch:

Example 28. A Rough Sketch Domain Description: The example is that of the production of rum, say of a Rum Production domain. From

10 the sowing, watering, and tending to of sugar cane plants;
11 via the “burning” of these prior to harvest;

\(^{49}\) This “rough” narrative fails to narrate what hubs, links, vehicles, automobiles are. In presenting it here we rely on your a priori understanding of these terms. But that is dangerous! The danger, if we do not painstakingly narrate and formalise what we mean by all these terms, then readers (software designers, etc.) may make erroneous assumptions.
12 the harvest;
13 the collection of harvest from sugar cane fields to
14 the chopping, crushing, (and sometimes repeated) boiling, cooling and centrifuging of
sugar cane when making sugar and molasses (into A, B, and low grade batches);
15 the fermentation, with water and yeast, producing a ‘wash’;
16 the (pot still or column still) distilling of the wash into rum;
17 the aging of rum in oak barrels;
18 the charcoal filtration of rum;
19 the blending of rum;
20 the bottling of rum;
21 the preparation of cases of rum for sales/export; and
22 the transportation away from the rum distiller of the rum

Some comments on Example 28: Each of the enumerated items above is phrased in terms
of perdurants. Behind each such perdurant lies some endurant. That is, in English, “every
noun can be verbed”, and vice-versa. So we anticipate the transcendence, from endurants to
perdurants.

Method Principle 1. From the “Overall” to The Details: Our first principle, as the first
task in any new domain modelling project, is to “focus” on the “overall”, that is, on the
“entire”, though specific domain

4.2 Entities
A core concept of domain modelling is that of an entity.

Definition 53. Entity By an entity we shall understand a phenomenon, i.e., something
that can be observed, i.e., be seen or touched by humans, or that can be conceived as an
abstraction of an entity; alternatively, a phenomenon is an entity, if it exists, it is “being”;
it is that which makes a “thing” what it is: essence, essential nature [90, Vol. I, pg. 665]. If a
phenomenon cannot be so observed and described then it is not an entity.

Analysis Predicate Prompt 1 is_entity: The domain analyser analyses “things” (θ) into
either entities or non-entities. The method provides the domain analysis prompt:
• is_entity – where is_entity(θ) holds if θ is an entity.50

is_entity is said to be a prerequisite prompt for all other prompts. is_entity is a method
tool.

The is_entity predicate function represents the first of a number of analysis prompts. They
are “applied” by the domain analyser to phenomena of domains. They yield truth values,
true or false, “left” in the mind of the domain analyser.

50 marks the end of an analysis prompt definition.
We have just shown how the is_entity predicate prompt can be applied to a universe of discourse. From now on we shall see prompts being applicable to successively more analysed entities. Figure 4.1 [Page 37] diagrams a domain description ontology of entities. That ontology indicates the sub-classes of endurants for which we shall motivate and for which we shall introduce prompts, predicates and functions.

The present chapter shall focus only on the external qualities, that is, on the “contents” of the leftmost dotted box.


dd

Method Principle 2. Justifying Analysis along Philosophical Lines: The concept of entities as a main focal point is justified in Kai Sørlander’s philosophy[108, 109, 110, 111, 112, 1994–2022]. Entities are there referred to as primary objects. They are the ones about which we express predicates.

4.3 Endurants and Perdurants

Method Principle 3. Separation of Endurants and Perdurants: As we shall see in this primer, the domain analysis & description method calls for the separation of first considering the careful analysis & description of endurants, then considering perdurants. This principle is based on the transcendental deduction of the latter from the former.

---

51 This ontology was first shown, as Fig. 3.1 [Page 27]
4.3.1 Endurants

**Definition** 54. **Endurant** By an **endurant**, to repeat, we shall understand an entity that can be observed, or conceived and described, as a “complete thing” at no matter which given snapshot of time; alternatively an entity is endurant if it is capable of **enduring**, that is persist, “hold out” [90, Vol. I, pg. 656]. Were we to “freeze” time we would still be able to observe the entire endurant.

**Example 29.** **Natural and Artefactual Endurants:**
- **Geography Endurants**: fields, meadows, lakes, rivers, forests, hills, mountains, et cetera.
- **Railway Track Endurants**: a railway track, its net, its individual tracks, switch points, trains, their individual locomotives, signals, et cetera.
- **Road Transport System Endurants**: the transport system, its road net aggregate and the aggregate of automobiles, the set of links (road segments) and hubs (road intersections) of the road net aggregate, these links and hubs, and the automobiles.

**Analysis Predicate Prompt 2** $\text{is\_endurant}$: The domain analyser analyses an entity, $\phi$, into an endurant as prompted by the **domain analysis prompt**:

- $\text{is\_endurant} - \phi$ is an endurant if $\text{is\_endurant}(\phi)$ holds.

**4.3.2 Perdurants**

**Definition** 55. **Perdurant** By a **perdurant** we shall understand an entity for which only a fragment exists if we look at or touch them at any given snapshot in time. Were we to freeze time we would only see or touch a fragment of the perdurant [90, Vol. II, pg. 1552].

**Example 30.** **Perdurants:**
- **Geography Perdurants**: the continuous changing of the weather (meteorology); the erosion of coastlines; the rising of some land area and the “sinking” of other land area; volcanic eruptions; earthquakes; et cetera.
- **Railway System Perdurants**: the ride of a train from one railway station to another; and the stop of a train at a railway station from some arrival time to some departure time.

**Analysis Predicate Prompt 3** $\text{is\_perdurant}$: The domain analyser analyses an entity $e$ into perdurants as prompted by the **domain analysis prompt**:

- $\text{is\_perdurant} - e$ is a perdurant if $\text{is\_perdurant}(e)$ holds.

**is\_entity** is a **prerequisite prompt** for $\text{is\_endurant}$. $\text{is\_endurant}$ is a method tool.
4.4 Solids and Fluids

We repeat method principle 3 on page 37:

**Method Principle 4. **Separation of Endurants and Perdurants: First domain analyse & describe endurants; then domain analyse & describe perdurants.

4.4 Solids and Fluids

For pragmatic reasons we distinguish between solids and fluids.

**Method Principle 5. **Abstraction, I: The principle of abstraction is now brought into “full play”: In analysing & describing entities the domain analyser cum describer is “free” to not consider all facets of entities, that is, to abstract. We refer to our characterisation of abstraction in Sect. 1.4 on page 3.

4.4.1 Solids

**Definition 56. **Solid Endurant: By a solid endurant we shall understand an endurant which is separate, individual or distinct in form or concept, or, rephrasing: a body or magnitude of three-dimensions, having length, breadth and thickness [90, Vol. II, pg. 2046].

**Analysis Predicate Prompt 4. is solid: **The domain analyser analyses endurants, $e$, into solid entities as prompted by the domain analysis prompt:

- $is\_solid \rightarrow e$ is solid if $is\_solid(e)$ holds.

To simplify matters we shall allow separate elements of a solid endurant to be fluid! That is, a solid endurant, i.e., a part, may be conjoined with a fluid endurant, a fluid. $is\_solid$ is a method tool.

**Example 31. **Artefactual Solid Endurants: The individual endurants of the above example of railway system endurants, Example 29 on the facing page, were all solid. Here are examples of solid endurants of pipeline systems. A pipeline and its individual units: wells, pipes, valves, pumps, forks, joins, regulator, and sinks.

4.4.2 Fluids

**Definition 57. **Fluid Endurant By a fluid endurant we shall understand an endurant which is prolonged, without interruption, in an unbroken series or pattern; or, rephrasing: a substance (liquid, gas or plasma) having the property of flowing, consisting of particles that move among themselves [90, Vol. I, pg. 774].
**Analysis Predicate Prompt 5** `is_fluid`: The domain analyser analyses endurants $e$ into fluid entities as prompted by the *domain analysis prompt*:

- **is_fluid** – $e$ is fluid if `is_fluid(e)` holds

`is_fluid` is a method tool. Fluids are otherwise liquid, or gaseous, or plasmatic, or granular\(^52\), or plant products\(^53\), et cetera.

**Example 32** **Fluids**: Specific examples of fluids are: water, oil, gas, compressed air, etc. A container, which we consider a solid endurant, may be *conjoined* with another, a fluid, like a gas pipeline unit may “contain” gas.

### 4.5 Parts and Living Species

We analyse endurants into either of two kinds: *parts* and *living species*. The distinction between *parts* and *living species* is motivated in Kai Sørlander’s Philosophy [108, 109, 110, 111, 112].

#### 4.5.1 Parts

**Definition 58** **Parts** By a *part* we shall understand a solid endurant existing in time and subject to laws of physics, including the *causality principle* and *gravitational pull*\(^54\).

**Analysis Predicate Prompt 6** `is_part`: The domain analyser analyses “things” (e) into part. The method can thus be said to provide the *domain analysis prompt*:

- **is_part** – where `is_part(e)` holds if e is a part

`is_part` is a method tool. *Parts* are either *natural* parts, or are *artefactual* parts, i.e. man-made. Natural and man-made parts are either *atomic* or *compound*.

#### 4.5.1.1 Atomic Parts

The term ‘atomic’ is, perhaps, misleading. It is not used in order to refer to nuclear physics. It is, however, chosen in relation to the notion of *atomism*: a doctrine that the physical or physical and mental universe is composed of simple indivisible minute particles [Merriam Webster].

\(^52\) This is a purely pragmatic decision. “Of course” sand, gravel, soil, etc., are not fluids, but for our modelling purposes it is convenient to “compartmentalise” them as fluids!

\(^53\) i.e., chopped sugar cane, threshed, or otherwise. See footnote 52.

\(^54\) This characterisation is the result of our study of relations between philosophy and computing science, notably influenced by Kai Sørlander’s Philosophy [108, 109, 110, 111, 112]
4.5 Parts and Living Species

Definition 59. Atomic Part, II By an atomic part we shall understand a part which the domain analyser considers to be indivisible in the sense of not meaningfully, for the purposes of the domain under consideration, that is, to not meaningfully consist of sub-parts.

Example 33. Atomic Parts: We refer to Example ?? on page ??: pipeline systems. The wells, pumps, valves, pipes, forks, joins and sinks can be considered atomic.

Analysis Predicate Prompt 7 is_atomic: The domain analyser analyses “things” (e) into atomic part. The method can thus be said to provide the domain analysis prompt:

• is_atomic – where is_atomic(e) holds if e is an atomic part.

is_atomic is a method tool.

4.5.1.2 Compound Parts, II

We, pragmatically, distinguish between Cartesian-product-, and set- oriented parts. That is, if Cartesian-product-oriented, to consist of two or more distinctly sort-named endurants (solids or fluids), or, if set-oriented, to consist of an indefinite number of zero, one or more identically sort-named parts.

Definition 60. Compound Part Compound parts are those which are either Cartesian-product- or are set- oriented parts.

Analysis Predicate Prompt 8 is_compound: The domain analyser analyses “things” (e) into compound part. The method can thus be said to provide the domain analysis prompt:

• is_compound – where is_compound(e) holds if e is a compound part.

is_compound is a method tool.

4.5.1.2.1 Cartesian Parts

Definition 61. Cartesian Part Cartesian parts are those (compound parts) which consists of an “indefinite number” of two or more parts of distinctly named sorts.

Some clarification may be needed. (i) In mathematics, as in RSL [66], a value is a Cartesian value if it can be expressed, for example as (a,b,...,c), where a,b,...,c are mathematical (or, which is the same, RSL) values. Let the sort names of these be A,B,...,C – with these being required to be distinct. We wrote “indefinite number”: the meaning being that the number is fixed, finite, but not specific. (ii) The requirement: ‘distinctly named’ is pragmatic. If the domain analyser cum describer thinks that two or more of the components of a Cartesian part are [really] of the same sort, then that person is most likely confused and must come up with suitably distinct sort names for these “same sort” parts! (iii) Why did we not write “definite number”? Well, at the time of first analysing a Cartesian part, the domain analyser...
cum describer may not have thought of all the consequences, i.e., analysed, the compound part. Additional sub-parts, of the Cartesian compound, may be “discovered”, subsequently and can then, with the approach we are taking wrt. the modelling of these, be “freely” added subsequently!

**Example 34. Cartesian Automobiles:** We refer to Example 29 on page 38, the transport system sub-example. We there viewed (hubs, links and) automobiles as atomic parts. From another point of view we shall here understand automobiles as Cartesian parts: the engine train, the chassis, the car body, four doors (left front, left rear, right front, right rear), and the wheels. These may again be considered Cartesian parts.

**Analysis Predicate Prompt 9 is Cartesian:** The domain analyser analyses “things” (e) into Cartesian part. The method can thus be said to provide the *domain analysis prompt*:

- *is Cartesian* – where *is Cartesian(e)* holds if *e* is a Cartesian part

*is Cartesian* is a method tool.

### 4.5.1.2.2 Calculating Cartesian Part Sorts

The above analysis amounts to the analyser first “applying” the *domain analysis prompt* *is compound(e)* to a solid endurant, *e*, where we now assume that the obtained truth value is *true*. Let us assume that endurants *e:E* consist of sub-endurants of sorts \{E_1,E_2,...,E_m\}. Since we cannot automatically guarantee that our domain descriptions secure that *E* and each *E_i* (1 ≤ i ≤ m) denotes disjoint sets of entities we must prove so!

---

**On Determination Functions**

Determination functions apply to compound parts and yield their sub-parts and the sorts of these. That is, we observe the domain and our observation results in a focus on a subset of that domain and sort information about that subset.

---

**An RSL Extension**

The determine

functions below are expressed as follows:

\[
\text{value determine}_{\ldots}(e) \text{ as (parts.sorts)}
\]

where we focus here on the *sorts* clause. Typically that clause is of the form \(\eta A,\eta B,...,\eta C\). That is, a “pattern” of sort names: A,B,...,C. These sort names are provided by the domain analyser cum describer. They are chosen as “full names”, or as mnemonics, to capture an essence of the (to be) described sort. Repeated invocations, by the domain analyser cum describer, of these \(\ldots\text{sorts}\) analysis functions normally lead to new sort names distinct from previously chosen such names.

\(^{55}\) \(\eta A,\eta B,...,\eta C\) are names of types. \(\eta \theta\) is the type of all type names!
4.5 Parts and Living Species

4.5.1.2.2.1 Cartesian Part Determination

Observer Function Prompt 1 determine\text{\textunderscore Cartesian\textunderscore parts}:
The domain analyser analyses a part into a Cartesian part. The method provides the domain observer prompt:

- \textit{determine\textunderscore Cartesian\textunderscore parts} — it directs the domain analyser to determine the definite number of values and corresponding distinct sorts of the part.

\begin{verbatim}
\text{determine\textunderscore Cartesian\textunderscore parts}: E \rightarrow (E_1 \times E_2 \times \ldots \times E_n) \times (\eta E_1 \times \eta E_2 \times \ldots \times \eta E_n)
\end{verbatim}

where by \(E, E_i\) we mean endurants, i.e., part values, and by \(\eta E_i\) we mean the names of the corresponding types.

\textit{determine\textunderscore Cartesian\textunderscore parts} is a method tool.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
\textbf{Calculation prompts} & \textbf{Description} \\
\hline
\textbf{calc Cartesian parts}: & \textbf{Determine Cartesian parts} \\
\hline
\end{tabular}
\caption{Calculation prompts apply to compound parts: Cartesians and sets, and yield an RSL-Text description.}
\end{table}

Domain Description Prompt 2 \textit{calc\textunderscore Cartesian\textunderscore parts}: If \textit{is\textunderscore Cartesian(e)} holds, then the analyser “applies” the \textit{domain description prompt}

\begin{verbatim}
\text{calc\textunderscore Cartesian\textunderscore parts}:
\end{verbatim}

resulting in the analyser writing down the \textit{endurant sorts and endurant sort observers} domain description text according to the following schema:

1. \textit{calc\textunderscore Cartesian\textunderscore parts(e)} describer

\begin{verbatim}
\let (\_57, (\eta E_1, \ldots, \eta E_m)) = \text{determine\textunderscore Cartesian\textunderscore parts\textunderscore sorts}(e)\_58 in
\end{verbatim}

\begin{verbatim}
\textbf{Narration:}
\textbf{[s]} \ldots narrative text on sorts \ldots
\textbf{[o]} \ldots narrative text on sort observers \ldots
\textbf{[p]} \ldots narrative text on proof obligations \ldots
\end{verbatim}

\begin{verbatim}
\textbf{Formalisation:}
\textbf{type}
\textbf{[s]} \ E_1, \ldots, E_m
\textbf{value}
\textbf{[o]} \ \text{obs}_{E_1}: E \rightarrow E_1, \ldots, \text{obs}_{E_m}: E \rightarrow E_m
\textbf{proof obligation}
\textbf{[p]} \ [\text{Disjointness of endurant sorts}]\_59
\end{verbatim}

\textit{calc\textunderscore Cartesian\textunderscore parts} is a method tool.

---

56 The ordering, \((\theta E_1, \ldots, \theta E_n)\), is pairwise arbitrary.
57 The use of the underscore, \_\_\_, shall inform the reader that there is no need, here, for naming a value.
58 For determine\textunderscore composite\textunderscore parts see Sect. 4.5.1.2.2.1
Elaboration 1  **Type, Values and Type Names:** Note the use of quotes above. Please observe that when we write \( \text{obs}_E \) then \( \text{obs}_E \) is the name of a function. The \( E \), when juxtaposed to \( \text{obs}_ \) is now a name. 

**Observer Function Prompt 2  type\_name, type\_of:**

The definition of type\_name, type\_of implies the informal definition of

\[
\text{obs}_E(e_i) = e_i \equiv \text{type\_name}(e_i) = E_i = E_i \land \text{is}_E(e_i)
\]

**Example 35. A Road Transport System Domain: Cartesians:**

23 There is the *universe of discourse*, RTS. 24 a *road net*, RN, and
It is composed from 25 an *aggregate of automobiles*, AA.

<table>
<thead>
<tr>
<th>type</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RTS</td>
<td>obs_RN: RTS ( \rightarrow ) RN</td>
</tr>
<tr>
<td>RN</td>
<td>obs_AA: RTS ( \rightarrow ) AA</td>
</tr>
<tr>
<td>AA</td>
<td></td>
</tr>
</tbody>
</table>

We continue the analysis & description of “our” road transport system:

26 The road net consists of
a an aggregate, AH, of hubs and
b an aggregate, AL, of links.

<table>
<thead>
<tr>
<th>type</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AH</td>
<td>obs_AH: RN ( \rightarrow ) AH</td>
</tr>
<tr>
<td>AL</td>
<td>obs_AL: RN ( \rightarrow ) AL</td>
</tr>
</tbody>
</table>

**4.5.1.2.3 Part Sets**

**Definition 62. Part Sets** Part sets are those which, in a given context, are deemed to meaningfully consist of separately observable a [“root”] part and an indefinite number of proper [“sibling”] sub-parts.

For pragmatic reasons we distinguish between parts sets all of whose parts are of the same, single, further un-analysed sort, and of two or more distinct atomic sorts.

59 Example 35’ Narration is not representative of what it should be. Here is a more reasonable narration:

- A road net is a set of hubs (road intersections) and links such that links are connected to adjacent hubs, and such that connected links and hubs form roads and where a road is a thoroughfare, route, or way on land between two places that has been paved or otherwise improved to allow travel by foot or some form of conveyance, including a motor vehicle, cart, bicycle, or horse [Wikipedia]

We bring this clarification here, once, and allow ourselves, with the reader’s permission, to narrate only very steno-graphically.
4.5 Parts and Living Species

**Definition 63.** Single Sort Part Sets *Single sort part sets* are those which, in a given context, are deemed to *meaningfully* consist of separately observable a [“root”] part and an indefinite number of proper [“sibling”] sub-parts of the same, i.e., single sort.

**Analysis Predicate Prompt 10** *is_single_sort_set*: The domain analyser analyses a solid endurant, i.e., a part $p$ into a set endurant:

- **is_single_sort_set**: $p$ is a composite endurant if $is_single_sort_set(p)$ holds.

*is_single_sort_set* is a method tool.

The *is_single_sort_set* predicate is informal. So are all the domain analysis predicates (and functions). That is, Their values are “calculated” by a human, the domain analyser. That person observes parts in the “real world”. The determination of the predicate values, hence, are subjective.

**Definition 64.** Alternative Atomic Part Sets *Alternative sorts part set* s are those which, in a given context, are deemed to *meaningfully* consist of separately observable a [“root”] part and an indefinite number of proper [“sibling”] sub-parts of two or more atomic parts of distinct sorts.

**Analysis Predicate Prompt 11** *is_alternative_sorts_set*: The domain analyser analyses a solid endurant, i.e., a part $p$ into a set endurant:

- **is_alternative_sorts_set**: $p$ is a composite endurant if $is_alternative_sorts_set(p)$ holds.

*is_alternative_sorts_set* is a method tool.

### 4.5.1.2.3.1 Determine Same Sort Part Sets

**Observer Function Prompt 3** *determine_same_sort_parts_set*: The domain analyser observes parts into same sorts part sets. The method provides the *domain observer prompt*:

- **determine_alternative_sorts_part_set** directs the domain analyser to determine the values and corresponding sorts of the part.

  value
  
  - determine_same_sort_part_set: $E \rightarrow (P \times \theta P)$
  - determine_same_sort_part_set(e) as (ps, $\eta Pn$)

*determine_same_sort_part_set* is a method tool.

### 4.5.1.2.3.2 Determine Alternative Sorts Part Sets

**Observer Function Prompt 4** *determine_alternative_sorts_part_set*: The domain analyser observes parts into alternative sorts part sets. The method provides the *domain observer prompt*:
determine_alternative_sorts_part_set directs the domain analyser to determine the values and corresponding sorts of the part.

\[ \text{value} \]
\[
\text{determine_alternative_sorts_part_set}: E \rightarrow ((P_1 \times \theta P_1) \times \ldots \times (P_n \times \theta P_n))
\]
\[
\text{determine_alternative_sorts_part_set}(e) \text{ as } ((p_1, \eta P_1), \ldots, (p_n, \eta P_n))
\]

The set of parts, of different sorts, may have more than one element, \( p, p', \ldots, p'' \) being of the same sort \( E_i \).

**determine_alternative_sorts_part_set** is a method tool.

### 4.5.1.2.3.3 Calculating Single Sort Part Sets

**Domain Description Prompt 3** calc_single_sort_parts_sort: If \( \text{is_single_sort_parts}(e) \) holds, then the analyser “applies” the domain description prompt

\[ \text{calc_single_sort_parts}(e) \]

resulting in the analyser writing down the single set sort and sort observers domain description text according to the following schema:

**2. calculate_single_sort_parts_sort(e) Describer**

\[
\text{let } (_\eta P) = \text{determine_single_sort_part}(e)\]

**Narration:**

- \( s \) ... narrative text on sort ...
- \( o \) ... narrative text on sort observer ...
- \( p \) ... narrative text on proof obligation ...

**Formalisation:**

\[
\text{type } [s] P \\\n[s] Ps = P\text{-set} \\\n\text{value } [o] \text{obs}_Ps: E \rightarrow Ps \]

**calculate_single_sort_parts_sort** is a method tool.

**Elaboration 2** Type, Values and Type Names: Note the use of quotes above. Please observe that when we write \( \text{obs}_Ps \) then \( \text{obs}_Ps \) is the name of a function. The \( Ps \), when juxtaposed to \( \text{obs}_ \) is now a name.

**Example 36** Road Transport System: Sets of Hubs, Links and Automobiles: We refer to Example 35 on page 44.

27 The road net aggregate of road net hubs consists of a set of [atomic] hubs,
28 The road net aggregate of road net links consists of a set of [atomic] links,
29 The road net aggregate of automobiles consists of a set of [atomic] automobiles.

\[ \text{For } \text{determine_single_sort_part see Defn. 63 on the preceding page.} \]
4.5 Parts and Living Species

4.5.1.2.3.4 Calculating Alternative Sort Part Sets

We leave it to the reader to decipher the `calculate_alternative_sort_part_sets` prompt.

---

Domain Description Prompt 4 `calculate_alternative_sort_part_sets`: If `is_alternative_sort_part_sets(e)` holds, then the analyser “applies” the domain description prompt

- `calculate_alternative_sort_part_sets(e)`

resulting in the analyser writing down the alternative sort and sort observers domain description text according to the following schema:

3. `calculate_alternative_sort_part_sets(e)` Describer

```plaintext
let ((p1,ηE1),...(pn,ηEn)) = determine_alternative_sort_part_set_sorts(e)61 in
```

**Narration:**
- `[s]` ... narrative text on alternative sorts ...
- `[o]` ... narrative text on sort observers ...
- `[p]` ... narrative text on proof obligations ...

**Formalisation:**

```plaintext
type
[s] Ea = E1 | ... | En
[s] E1 :: End1, ..., En :: Endn
value
[o] obs_Ea: E → Ea
axiom
[p] [ disjointness of alternative sorts ] E1, ..., En "
end
```

The set of parts, of different sorts, may have more than one element, say `p, p', ..., p''` being of the same sort `E_i`. Since parts are not mentioned in the sort description above, cf., only the distinct alternative sort observers appear in that description.

calculate_alternative_sort_part_sets is a method tool.

**Example 37**  Alternative Rail Units:

30 The example is that of a railway system.
31 We focus on railway nets. They can be observed from the railway system.
32 The railway net embodies a set of [railway] net units.

61 For `determine_alternative_sort_part_sets` see Defn. 64 on page 45.
A net unit is either a straight or curved linear unit, or a simple switch, i.e., a turnout, unit or a simple cross-over, i.e., a rigid crossing unit, or a single switched cross-over, i.e., a single slip unit, or a double switched cross-over, i.e., a double slip unit, or a terminal unit.

As a formal specification language technicality disjointness of the respective rail unit types is afforded by RSL’s type definition construct.

We refer to Figure 4.2.

33. A net unit is either a straight or curved linear unit, or a simple switch, i.e., a turnout, unit or a simple cross-over, i.e., a rigid crossing unit, or a single switched cross-over, i.e., a single slip unit, or a double switched cross-over, i.e., a double slip unit, or a terminal unit.

34. As a formal specification language technicality disjointness of the respective rail unit types is afforded by RSL’s type definition construct.

We refer to Figure 4.2.

---

**Method Principle 6**  
*Pedantic Steps of Development*: This section, i.e., Sect. 4.5.1, has illustrated a principle of “small, pedantic” analysis & description steps. You could also call it a principle of separation of concerns.

### 4.5.1.3 Ontology and Taxonomy

We can speak of two kinds of ontologies: the general ontologies of domain analysis & description, cf. Fig. 4.1 on page 37, and a specific domain’s possible endurant ontologies. We shall here focus on a “restricted” concept of taxonomies.

---

62 https://en.wikipedia.org/wiki/Railroad_switch  
63 By taxonomy (or taxonomical classification) we shall here understand a scheme of classification, especially a hierarchical classification, in which things are organized into groups [Wikipedia].
4.5 Parts and Living Species

**Definition 65. Domain Taxonomy** By a domain taxonomy we shall understand a hierarchical structure, usually depicted as an “upside-down” tree, whose “root” designates a compound part and whose “siblings” (proper sub-trees) designate parts or fluids.

The ‘restriction’ amounts to considering only endurants. That is, not considering perdurants. **Taxonomy** is a method technique.

**Example 38. The Road Transport System Taxonomy:** Figure 4.3 shows a schematised, i.e., the . . . , taxonomy for the *Road Transport System* domain of Example 4.1 on page 37.

![Figure 4.3 A Road Transport System Taxonomy](image)

**4.5.1.4 “Root” and “Sibling” Parts**

For compound parts, cf. Definition 60 on page 41, we introduce the specific domain taxonomy concepts of “root” and “sibling” parts. (We also refer to Fig. 4.3.)

When observing, as a human, a compound part one may ask the question “a tree consisting of a specific domain taxonomy node labelled, e.g., *X*, and the sub-trees labelled, e.g., *Y₁, Y₂, . . . , Yₙ* does that tree designate one “indivisible” part or does it designate *n* + 1 parts?” We shall, in general, consider the answer to be the latter: *n + 1*!

We shall, in general, consider compound parts to consist of a “root” parts and *n* “sibling parts and fluids”. What the analyser cum describer observes appears as one part, “the whole”, with *n* “embedded” sub-parts. What the analyser cum describer is asked to model is 1, the root part, and *n*, the sibling, parts and fluids. The fact that the root part is separately modelled from the sibling parts, may seem to disappear in this separate modelling — but, as You shall see, in the next chapter, their relation: the siblings to “the whole”, i.e., the root, will be modelled, specifically through their mereologies, as will be covered in Sect. 5.3, but also through their respective attributes, Sect. 5.4. We shall see this non-embeddedness of root and sibling parts further accentuated in the modelling of their transcendentally deduced respective (perdurant) behaviours as distinct concurrent behaviours in Chapter 6.

**4.5.2 Living Species**
**Living Species** are either *plants* or *animals*. Among animals we have the *humans*.

**Definition 66. Living Species** By a *living species* we shall understand a solid endurant, subject to laws of physics, and additionally subject to *causality of purpose*.

Living species must have some *form* they can be developed to reach; a form they must be *causally determined to maintain*. This development and maintenance must further engage in exchanges of *matter* with an environment. It must be possible that living species occur in two forms: *plants*, respectively *animals*, forms which are characterised by *development*, *form* and *exchange*, which, additionally, can be characterised by the *ability of purposeful movement*.

**Analysis Predicate Prompt 12** *is_living_species*: The domain analyser analyses “things” (*e*) into living species. The method can thus be said to provide the *domain analysis prompt*:

- **is_living_species** – where *is_living_species(e)* holds if *e* is a living species.

*is_living_species* is a method tool.

It is appropriate here to mention Carl Linnaeus (1707–1778). He was a Swedish botanist, zoologist, and physician who formalised, in the form of a binomial nomenclature, the modern system of naming organisms. He is known as the “father of modern taxonomy”. We refer to his ‘Species Plantarum’ gutenberg.org/files/20771/20771-h/20771-h.htm.

4.5.2.1 Plants

**Example 39. Plants:** Although we have not yet come across domains for which the need to model the living species of plants were needed, we give some examples anyway: grass, tulip, rhododendron, oak tree.

**Analysis Predicate Prompt 13** *is_plant*: The domain analyser analyses “things” (*ℓ*) into a plant. The method can thus be said to provide the *domain analysis prompt*:

- **is_plant** – where *is_plant(ℓ)* holds if *ℓ* is a plant.

*is_plant* is a method tool. The predicate *is_living_species(ℓ)* is a prerequisite for *is_plant(ℓ)*.

4.5.2.2 Animals

**Definition 67. Animal** We refer to the initial definition of *living species* above – while emphasizing the following traits: (i) a *form* that *animals* can be developed to reach and (ii) *causally determined to maintain* through (iii) *development and maintenance* in an exchange of *matter* with an environment, and (iv) *ability to purposeful movement*.
4.6 Some Observations

Two observations must be made.

(i) The domain analyser cum describer procedures illustrated by the analysis functions determine Cartesian parts, determine same sort part set and determine alternative sort part set yield names of endurant sorts. Some of these names may have already been encountered, i.e., discovered. That is, the domain analyser cum describer must carefully consider such possibilities.

(ii) Endurants are not recursively definable! This appears to come as a surprise to many computer scientists. Immediately many suggest that “tree-like” endurants like a river, or, indeed, a tree, should be defined recursively. But we posit that that is not the case. A river, for example, has a delta, its “root” so-to-speak, but the sub-trees of a recursively defined river endurant has no such “deltas”! Instead we define such “tree-like” endurants as graphs with appropriate mereologies.

© Dines Bjørner. September 8, 2022: 10:01 am 51 The TUV Lectures, Vienna, Austria, October–November 2022
4.7 States

In our continued modelling we shall make good use of a concept of states.

Definition 69. State By a state we shall understand any collection of one or more parts.

In Chapter 5 Sect. 5.4 we introduce the notion of attributes. Among attributes there are the dynamic attributes. They model that internal part quality values may change dynamically. So we may wish, on occasion, to ‘refine’ our notion of state to be just those parts which have dynamic attributes.

4.7.1 State Calculation

Given any universe of discourse, \(uod:UoD\), we can recursively calculate its “full” state, \(\text{calc}_\text{parts}(uod)\).

35 Let \(e\) be any endurant. Let \(\text{arg}_\text{parts}\) be the parts to be calculated. Let \(\text{res}_\text{parts}\) be the parts calculated. Initialise the calculator with \(\text{arg}_\text{parts}=\{e\}\) and \(\text{res}_\text{parts}=\emptyset\). Calculation stops with \(\text{arg}_\text{parts}\) empty and \(\text{res}_\text{parts}\) the result.

36 If \(\text{is Cartesian}(e)\)

37 then we obtain its immediate parts, determine \(\text{composite}_\text{part}(e)\)

38 add them, as a set, to \(\text{arg}_\text{parts}\), \(e\) removed from \(\text{arg}_\text{parts}\) and added to \(\text{res}_\text{parts}\) calculating the parts from that.

39 If \(\text{is single sort part set}(e)\)

40 then the parts, \(ps\), of the single sort set are determined,

41 added to \(\text{arg}_\text{parts}\) and \(e\) removed from \(\text{arg}_\text{parts}\) and added to \(\text{res}_\text{parts}\) calculating the parts from that.

42 If \(\text{is alternative sorts part set}(e)\) then the parts, \((p_1,\ldots,p_2,\ldots,\ldots,p_n)\), of the alternative sorts set are determined, added to \(\text{arg}_\text{parts}\) and \(e\) removed from \(\text{arg}_\text{parts}\) and added to \(\text{res}_\text{parts}\) calculating the parts from that.

35. \(\text{calc}_\text{parts}: E\text{-set} \rightarrow E\text{-set} \rightarrow E\text{-set}\)

35. \(\text{calc}_\text{parts}(\text{arg}_\text{parts})(\text{res}_\text{parts}) = \emptyset\)

35. if \(\text{arg}_\text{parts} = \emptyset\) then \(\text{res}_\text{parts} = \emptyset\)

35. let \(e\) = \(e\) in \(\text{arg}_\text{parts}\)

36. \(\text{is Cartesian}(e)\) →

37. let \((e_1,e_2,\ldots,\ldots,\ldots,\ldots,\ldots) = \text{observe Cartesian part}(e)\) in

38. \(\text{calc}_\text{parts}(\text{arg}_\text{parts} \setminus \{e\} \cup \{e_1,e_2,\ldots,\ldots,\ldots,\ldots\})(\text{res}_\text{parts} \cup \{e\})\) end

39. \(\text{is single sort part set}(e)\) →

40. let \(ps = \text{observe single sort part set}(e)\) in

41. \(\text{calc}_\text{parts}(\text{arg}_\text{parts} \setminus \{e\} \cup ps)(\text{res}_\text{parts} \cup \{e\})\) end

42. \(\text{is alternative sorts part set}(e)\) →

42. let \((p_1,\ldots,p_2,\ldots,\ldots,p_n) = \text{observe alternative sorts part set}(e)\) in

42. \(\text{calc}_\text{parts}(\text{arg}_\text{parts} \setminus \{e\} \cup p_1,p_2,\ldots,p_n)(\text{res}_\text{parts} \cup \{e\})\) end

end end

\(\text{calc}_\text{parts}\) is a method tool.

Method Principle 7. Domain State: We have found, once all the state components, i.e., the endurant parts, have had their external qualities analysed, that it is then expedient to define the domain state. It can then be the basis for several concepts of internal qualities.
Example 40. Constants and States:

Let there be given a universe of discourse, \( rts \). The set \( \{rts\} \) is an example of a state.

From that state we can calculate other states.

44 The set of all hubs, \( hs \).
45 The set of all links, \( ls \).
46 The set of all hubs and links, \( hls \).
47 The set of all automobiles, \( as \).
48 The set of all parts, \( ps \).

\[
\begin{align*}
\text{value} & \\
rts:\text{UoD} & [43] \\
hs:\text{H-set} & \equiv \text{obs}_SH(\text{obs}_SH(\text{obs}_RN(rts))) \\
ls:\text{L-set} & \equiv \text{obs}_SL(\text{obs}_SL(\text{obs}_RN(rts))) \\
hls:\text{H|L-set} & \equiv hs\cup ls \\
as:\text{A-set} & \equiv \text{obs}_As(\text{obs}_AA(\text{obs}_RN(rts))) \\
ps:\text{(UoB|H|L|A)-set} & \equiv rts\cup hls\cup as
\end{align*}
\]

4.7.2 Update-able States

We shall, in Sect. 5.4, introduce the notion of parts, having dynamic attributes, that is, having internal qualities that may change. To cope with the modelling, in particular of so-called monitor-able attributes, we present the state as a global variable:

\[
\text{variable } \sigma := \text{calc_parts(\{uod\})}
\]

4.8 An External Analysis and Description Procedure

We have covered the individual analysis and description steps of our approach to the external qualities modelling of domain endurants. We now suggest a ‘formal’ description of the process of linking all these analysis and description steps.

4.8.1 An Analysis & Description State

Common to all the discovery processes is an idea of a notice board. A notice board, at any time in the development of a domain description, is a repository of the analysis and description process. We suggest to model the notice board in terms of three global variables. The new variable holds the parts yet to be described, The ans variable holds the sort name of parts that have so far been described, the gen variable holds the parts that have so far been described, and the txt variable holds the RSL-Text so far generated. We model the txt variable as a map from endurant identifier names to RSL-Text.
4.8.2 A Domain Discovery Procedure, I

The `discover_sorts` pseudo program suggests a systematic way of proceeding through analysis, manifested by the `is_···` predicates, to (→) description.

Some comments are in order. The `e-set_u / e-set_v` expression yields a set of endurants that are either in `e-set_u`, or in `e-set_v`, or in both, but such that two endurants, `e_x` and `e_y` which are of the same endurants type, say `E`, and are in respective sets is only represented once in the result; that is, if they are type-wise the same, but value-wise different they will only be included once in the result.

As this is the first time RSL-Text is put on the notice board we express this as:

- `txt := txt \cup [\text{type}_\text{name}(v) \mapsto \langle \text{RSL-Text} \rangle]`

Subsequent insertion of RSL-Text for internal quality descriptions and perdurants is then concatenated to the end of previously uploaded RSL-Text.

```plaintext
variable
  new := \{\text{uod}\},
  asn := \{\text{UoD}\},
  gen := \{\},
  txt:RSL-Text := [\text{uid}_\text{UoD}(_) \mapsto \langle\text{type}_\text{UoD}\rangle]

4.8.2 A Domain Discovery Procedure, I

The `discover_sorts` pseudo program suggests a systematic way of proceeding through analysis, manifested by the `is_···` predicates, to (→) description.

Some comments are in order. The `e-set_u / e-set_v` expression yields a set of endurants that are either in `e-set_u`, or in `e-set_v`, or in both, but such that two endurants, `e_x` and `e_y` which are of the same endurants type, say `E`, and are in respective sets is only represented once in the result; that is, if they are type-wise the same, but value-wise different they will only be included once in the result.

As this is the first time RSL-Text is put on the notice board we express this as:

- `txt := txt \cup [\text{type}_\text{name}(v) \mapsto \langle \text{RSL-Text} \rangle]`

Subsequent insertion of RSL-Text for internal quality descriptions and perdurants is then concatenated to the end of previously uploaded RSL-Text.

```
4.9 Summary

We briefly summarise the main findings of this chapter. These are the main analysis predicates and functions and the main description functions. These, to remind the reader, are the analysis, the is-... predicates, the analysis, the determine-... functions, the state calculation function, the description functions, and the domain discovery procedure. They are summarised in this table:

<table>
<thead>
<tr>
<th>External Qualities Predicates and Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Analysis Predicates:</strong> These are the is-... functions. The domain scientist cum engineer, i.e., the domain analyser cum describer, applies this to entities being observed in the domain. The answer is a truth value. Depending on the truth value that person then goes on to apply, again informally, either a subsequent predicate, or some function.</td>
</tr>
<tr>
<td><strong>Analysis Functions:</strong> These are the determine-... functions. They apply, respectively, to parts satisfying respective predicates.</td>
</tr>
<tr>
<td><strong>State Calculation:</strong> The state calculation function is given generally. The domain analyser cum describer must define this function for each domain studied.</td>
</tr>
<tr>
<td><strong>Description Functions:</strong> These calculation functions, in a sense, are the main “results” of this chapter.</td>
</tr>
<tr>
<td><strong>Domain Discovery:</strong> The procedure here being described, informally, guides the domain analyser cum describer to do the job!</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>#</th>
<th>Name Introduced</th>
</tr>
</thead>
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</tr>
<tr>
<td>2</td>
<td>is_endurant page 38</td>
</tr>
<tr>
<td>3</td>
<td>is_perdurant page 38</td>
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<td>4</td>
<td>is_solid page 39</td>
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<td>5</td>
<td>is_fluid page 40</td>
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<td>6</td>
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<tr>
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</tr>
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<td>8</td>
<td>is_compound page 41</td>
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<td>is_Cartesian page 42</td>
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<td>is_single_sort_set page 45</td>
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Chapter 5
Endurants: Internal Domain Qualities

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In this chapter we introduce the concept of internal qualities of endurants, and cover the analysis and description of unique identifiers, mereologies and attributes of endurants. There is yet another, interrelating internal quality: intentionality, “something” that expresses intention, design idea, purpose of artefacts – well, some would say, also natural endurants.

As it turns out, to analyse and describe mereology we need to first analyse and describe unique identifiers; and to analyse and describe attributes we need to first analyse and describe mereologies. Hence:

**Method Procedure 1. Sequential Analysis & Description of Internal Qualities:** We advise that the domain analysis & description first analyse & describe unique identification of all endurant sorts; then analyse & describe mereologies of all endurant sorts; finally analyse & describe attributes of all endurant sorts.

### 5.1 Internal Qualities

We shall investigate the, as we shall call them, internal qualities of domains. That is the properties of the entities to which we ascribe internal qualities. The outcome of this chapter is that the reader will be able to model the internal qualities of domains. Not just for a particular domain instance, but a possibly infinite set of domain instances.

---

64 You, the first time reader cannot know this, i.e., the “turns out”. Once we have developed and presented the material of this chapter, then you can see it; clearly!

65 By this we mean: You are not just analysing a specific domain, say the one manifested around the corner from where you are, but any instance, anywhere in the world, which satisfies what you have described.
5.1 Internal Qualities

5.1.1 General Characterisation

External qualities of endurants of a manifest domain are, in a simplifying sense, those we can see and touch. They, so to speak, take form. Internal qualities of endurants of a manifest domain are, in a less simplifying sense, those which we may not be able to see or “feel” when touching an endurant, but they can, as we now ‘mandate’ them, be reasoned about, as for unique identifiers and mereologies, or be measured by some physical/chemical means, or be “spoken of” by intentional deduction, and be reasoned about, as we do when we attribute properties to endurants.

5.1.2 Manifest Parts versus Structures

In [41] we covered a notion of ‘structures’. In this primer we shall treat the concept of ‘structures’ differently. We do so by distinguishing between manifest parts and structures.

5.1.2.1 Definitions

Definition 70. Manifest Part By a manifest part we shall understand a part which ‘manifests’ itself either in a physical, visible manner, “occupying” an AREA or a VOLUME and a POSITION in SPACE, or in a conceptual manner forms an organisation in Your mind! As we have already revealed, endurant parts can be transcendentally deduced into perdurant behaviours – with manifest parts indeed being so.

Definition 71. Structure By a structure we shall understand an endurant concept that allows the domain analyser cum describer to rationally decompose a domain analysis and/or its description into manageable, logically relevant sections, but where these abstract endurants are not further reflected upon in the domain analysis and description. Structures are therefore not transcendentally deduced into perdurant behaviours.

5.1.2.2 Analysis Predicates

Analysis Predicate Prompt 16 is_manifest: The method provides the domain analysis prompt:

- is_manifest – where is_manifest(p) holds if p is to be considered manifest.

Analysis Predicate Prompt 17 is_structure: The method provides the domain analysis prompt:

- is_structure – where is_structure(p) holds if p is to be considered a structure.

The obvious holds: is_manifest(p) \equiv \neg is_structure(p).
5.1.2.3 Examples

Example 41. Manifest Parts and Structures:
We refer to Example 35 on page 44: the Road Transport System. We shall consider all atomic parts: hubs, links and automobiles as being manifest. (They are physical, visible and in SPACE.) We shall consider road nets and aggregates of automobiles as being manifest. Road nets are physical, visible and in SPACE. Aggregates of automobiles are here considered conceptual. The road net manifest part, apart from its aggregates of hubs and links, can be thought of as “representing” a Department of Roads. The automobile aggregate apart from its automobiles, can be thought of as “representing” a Department of Vehicles. We shall consider hub and link aggregates and hub and link set as structures.

5.1.2.4 Modelling Consequence
In this chapter we introduce internal endurant qualities. If a part is considered manifest then we shall endow that part with all three kinds of internal qualities. If a part is considered a structure then we shall not endow that part with any of three kinds of internal qualities.

5.2 Unique Identification

This section is based on Sect. 5.2 of [41, Pages 108–112].

The concept of parts having unique identifiability, that is, that two parts, if they are the same, have the same unique identifier, and if they are not the same, then they have distinct identifiers, that concept is fundamental to our being able to analyse and describe internal qualities of endurants. So we are left with the issue of ‘identity’!

5.2.1 On Uniqueness of Endurants

We therefore introduce the notion of unique identification of part endurants. We assume (i) that all part endurants, e, of any domain E, have unique identifiers, (ii) that unique identifiers (of part endurants e:E) are abstract values (of the unique identifier sort UI of part endurants e:E), (iii) that such that distinct part endurant sorts, E_i and E_j, have distinctly named unique identifier sorts, say UI_i and UI_j, and (iv) that all ui_i;UI_i and ui_j;UI_j are distinct.

Representation of Unique Identifiers: Unique identifiers are abstractions. When we endow two endurants (say of the same sort) distinct unique identifiers then we are simply saying that these two endurants are distinct. We are not assuming anything about how these identifiers otherwise come about. Identifiability of Endurants: From a philosophical point of view, and with basis in Kai Serlander’s Philosophy, cf. Paragraph Identity, Difference and Relations (Page 14), one can rationally argue that there are many endurants, and that they are unique, and hence uniquely identifiable. From an empirical point of view, and since one may eventually have a software development in mind, we may wonder how unique identifiability can be accommodated.

---

66 – of some country, state, province, city or other.
67 See above footnote.
68 This restriction is not necessary, but, for the time, we can assume that it is.
Unique identifiability for solid endurants, even though they may be mobile, is straightforward: one can think of many ways of ascribing a unique identifier to any part; solid endurants do not “morph”\(^{69}\). Hence one can think of many such unique identification schemas.

Unique identifiability for fluids may seem a bit more tricky. For this monograph we shall not suggest to endow fluids with unique identification. We have simply not experimented with such part-fluids and fluid-parts domains — not enough — to suggest so.

### 5.2.2 Uniqueness Modelling Tools

The analysis method offers an observer function \(\text{uid}_E\) which when applied to part endurants, \(e\), yields the unique identifier, \(ui: UI\), of \(e\).

---

**Domain Description Prompt 5** *describe_unique_identifier(e)*: We can therefore apply the *domain description prompt*:

- *describe_unique_identifier(e)*

  to endurants \(e:E\) resulting in the analyser writing down the *unique identifier type and observer domain description text* according to the following schema:

  4. *describe_unique_identifier(e)* Observer

  **Narration:**
  
  - \([s]\) ... narrative text on unique identifier sort UI ...
  - \([u]\) ... narrative text on unique identifier observer \(\text{uid}_E\) ...
  - \([a]\) ... axiom on uniqueness of unique identifiers ...

  **Formalisation:**
  
  \[
  \text{type} \\ [s] \ UI \\
  \text{value} \ \ [u] \ \text{uid}_E: E \rightarrow UI
  \]

  \(\text{is_part}(e)\) is a prerequisite for *describe_unique_identifier(e)*.

  The unique identifier type name, \(UI\) above, chosen, of course, by the *domain analyser cum describer*, usually properly embodies the type name, \(E\), of the endurant being analysed and mereology-described. Thus a part of type-name \(E\) might be given the mereology type name \(EI\). Generally we shall refer to these names by \(UI\).

**Observer Function Prompt 5** *type_name, type_of, is_*:

Given *description schema* 5 we have, so-to-speak “in-reverse”, that

\[
\forall e:E \cdot \text{uid}_E(e) = ui \Rightarrow \text{type_of}(ui) = \eta UI \land \text{type_name}(ui) = UI \land \text{is_UI}(ui)
\]

\(\eta UI\) is a variable of type \(\eta T\). \(\eta T\) is the type of all domain endurant, unique identifier, mereology and attribute type names. By the subsequent \(UI\) we refer to the unique identifier type name value of \(\eta UI\).

**Example 42**. **Unique Identifiers:**

\(^{69}\) That is, our domain modelling method is not thought of as being applied to the physics situations of endurants going, for example, from states of being solid, via states of melting, to states of fluid.

\(^{70}\)
49 We assign unique identifiers to all parts.
50 By a road identifier we shall mean a link or a hub identifier.
51 Unique identifiers uniquely identify all parts.
   a All hubs have distinct [unique] identifiers.
   b All links have distinct identifiers.
   c All automobiles have distinct identifiers.
   d All parts have distinct identifiers.

type
49 \( H_{UI} \), \( L_{UI} \), \( A_{UI} \)
50 \( R_{UI} = H_{UI} \cup L_{UI} \)
value
51a \( uid_H : H \rightarrow H_{UI} \)
51b \( uid_L : H \rightarrow L_{UI} \)
51c \( uid_A : H \rightarrow A_{UI} \)

5.2.3 The Unique Identifier State

Given a universe of discourse we can calculate the set of the unique identifiers of all its parts.

value
calculate_all_unique_identifiers: UoD \rightarrow UI-set
calculate_all_unique_identifiers(uod) \equiv
let parts = calc_parts((uod)) in
\{ uid_E(e) | e:E \wedge e \in parts \} end

5.2.4 The Unique Identifier State

We can speak of a unique identifier state:

variable
calculate_all_unique_identifiers: UoD \rightarrow UI-set

discover_uuids: UoD \rightarrow Unit
discover_uuids(uod) \equiv calculate_all_unique_identifiers(uod)

Example 43 . Unique Road Transport System Identifiers:

We can calculate:

52 the set, \( H_{UI} \), of unique hub identifiers;
53 the set, \( L_{UI} \), of unique link identifiers;
54 the set, \( R_{UI} \), of all unique hub and link, i.e., road identifiers;
55 the map, \( li_{ui} \), from unique hub identifiers to the set of unique link identifiers of the links connected to the zero, one or more identified hubs,
56 the map, \( hi_{ui} \), from unique link identifiersto the set of unique hub identifiers of the two hubs connected to the identified link;
5.2 Unique Identification

the set, \(a_{ui}s\), of unique automobile identifiers;

value

\[
\begin{align*}
52 & \quad h_{ui}s:H_{UI}-set \equiv \{uid_H(h)|h:H \in hs\} \\
53 & \quad l_{ui}s:L_{UI}-set \equiv \{uid_L(l)|l:L \in ls\} \\
54 & \quad r_{ui}s:R_{UI}-set \equiv h_{ui}s\cup l_{ui}s \\
55 & \quad h_{ui}m:(H_{UI} \rightarrow mL_{UI}-set) \equiv \\
56 & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad [h_{ui} \mapsto \text{uid}_{ui}\bigcup l_{ui}\bigcup b_{ui}\bigcup b_{ui}\bigcup a_{ui}\bigcup \eta(h_{ui})] \\
57 & \quad l_{ui}m:(L_{UI} \rightarrow mH_{UI}-set) \equiv \\
58 & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad [l_{ui} \mapsto \text{uid}_{ui}\bigcup l_{ui}\bigcup b_{ui}\bigcup b_{ui}\bigcup a_{ui}\bigcup \eta(l_{ui})] \\
59 & \quad a_{ui}s:A_{UI}-set \equiv \{uid_A(a)|a:A \in as\}
\end{align*}
\]

5.2.5 A Domain Law: Uniqueness of Endurant Identifiers

We postulate that the unique identifier observer functions are about the uniqueness of the postulated endurant identifiers, but how is that guaranteed? We know, as “an indisputable law of domains”, that they are distinct, but our formulas do not guarantee that! So we must formalise their uniqueness.

All Domain Parts have Unique Identifiers

A Domain Law: 1 All Domain Parts have Unique Identifiers:

All parts of a described domain have unique identifiers.

Example 44. Uniqueness of Road Net Identifiers: We must express the following axioms:

59 All hub identifiers are distinct.
60 All link identifiers are distinct.
61 All automobile identifiers are distinct.
62 All part identifiers are distinct.

axiom

\[
\begin{align*}
59 & \quad \text{card } hs = \text{card } h_{ui}s \\
60 & \quad \text{card } ls = \text{card } l_{ui}s \\
61 & \quad \text{card } as = \text{card } a_{ui}s \\
62 & \quad \text{card } l_{ui}s\cup b_{ui}s\cup b_{ui}s\cup b_{ui}s\cup a_{ui}s \\
63 & \quad = \text{card } h_{ui}s+\text{card } l_{ui}s+\text{card } b_{ui}s+\text{card } b_{ui}s+\text{card } a_{ui}s
\end{align*}
\]

We ascribe, in principle, unique identifiers to all endurants whether natural or artefactual. We find, from our many experiments, cf. the Universes of Discourse example, Page 34, that we really focus on those domain entities which are artefactual endurants and their behavioural “counterparts”.

Example 45. Rail Net Unique Identifiers:

63 With every rail net unit we associate a unique identifier.
64 That is, no two rail net units have the same unique identifier.
65 Trains have unique identifiers.
We let \( tris \) denote the set of all train identifiers. 
No two distinct trains have the same unique identifier. 
Train identifiers are distinct from rail net unit identifiers.

\[
\text{type} \quad UI \\
\text{value} \quad \text{uid}_{\text{NU}}: \text{NU} \rightarrow \text{UI} \\
\text{axiom} \quad \forall u_i, u_j : \text{UI} \cdot u_i = u_j \equiv \text{uid}_{\text{NU}}(u_i) = \text{uid}_{\text{NU}}(u_j)
\]

### 5.2.5.1 Part Retrieval

Given the unique identifier, \( p_i \), of a part \( p \), but not the part itself, and given the universe-of-discourse (uod) state \( \sigma \), we can retrieve part, \( p \), as follows:

```plaintext
\[
\text{value} \quad p_i : \text{PI}, \text{uod} : \text{UoD}, \sigma \\
\text{retr}_{\text{part}} : \text{UI} \rightarrow \text{P} \\
\text{retr}_{\text{part}}(u_i) = \text{let } p : \text{P} \cdot p \in \sigma \land \text{uid}_{\text{P}}(p) = u_i \text{ in } p \text{ end} \\
\text{pre} : \exists p : \text{P} \cdot p \in \sigma \land \text{uid}_{\text{P}}(p) = u_i
\]
```

### 5.2.5.2 Unique Identification of Compounds

For structures we do not model their unique identification. But their components, whether the structures are “Cartesian” or “sets”, may very well be non-structures, hence be uniquely identifiable.

### 5.3 Mereology

This section is based on Sect. 5.3 of [41, Pages 112–119].

**Definition 72.** Mereology Mereology is the study and knowledge of parts and part relations.

Mereology, as a logical/philosophical discipline, can perhaps best be attributed to the Polish mathematician/logician Stanisław Leśniewski [54, 29].

### 5.3.1 Endurant Relations

Which are the relations that can be relevant for “endurant-hood”? There are basically two relations: (i) physical ones, and (ii) conceptual ones.

(i) Physically two or more endurants may be topologically either adjacent to one another, like rails of a line, or within an endurant, like links and hubs of a road net, or an atomic part is conjoined to one or more fluids, or a fluid is conjoined to one or more parts. The latter two could also be considered conceptual “adjacencies”.

---

54 © Dines Bjørner, September 8, 2022: 10:01 am
5.3 Mereology

(ii) Conceptually some parts, like automobiles, “belong” to an embedding endurant, like to an automobile club, or are registered in the local department of vehicles, or are ‘intended’ to drive on roads.

5.3.2 Mereology Modelling Tools

When the domain analyser decides that some endurants are related in a specifically enunciated mereology, the analyser has to decide on suitable mereology types and mereology observers (i.e., endurant relations).

69 We may, to illustration, define a mereology type of an endurant $e : E$ as a triplet type expression over set of unique [endurant] identifiers.

70 There is the identification of all those endurant sorts $E_{i1}, E_{i2}, ... , E_{im}$ where at least one of whose properties "is of interest" to parts $e : E$.

71 There is the identification of all those sorts $E_{io1}, E_{io2}, ... , E_{iom}$ where at least one of whose properties "is of interest" to endurants $e : E$ and vice-versa.

72 There is the identification of all those endurant sorts $E_{o1}, E_{o2}, ... , E_{onom}$ for whom properties of $e : E$ "is of interest" to endurants of sorts $E_{i1}, E_{i2}, ... , E_{im}$.

73 The mereology triplet sets of unique identifiers are disjoint and are all unique identifiers of the universe of discourse.

The triplet mereology is just a suggestion. As it is formulated here we mean the three ‘sets’ to be disjoint. Other forms of expressing a mereology should be considered for the particular domain and for the particular endurants of that domain. We leave out further characterisation of the seemingly vague notion “is of interest”.

$$\text{type}$$

$$iEi = iEi1 | iEi2 | ... | iEim$$

$$ioEi = ioEi1 | ioEi2 | ... | ioEim$$

$$oEi = oEi1 | oEi2 | ... | oEio$$

$$MT = iEi-set \times ioEi-set \times oEi-set$$

axiom

$$\forall (iset,ioset,oset) : MT \bullet$$

$$\text{card} iset + \text{card} ioset + \text{card} oset = \text{card} \cup \{iset,ioset,oset\}$$

$$\cup \{iset,ioset,oset\} \subseteq \text{calc_all_unique_identifiers(uod)}$$

Domain Description Prompt 6 **describe_mereology(e):** If has_mereology(p) holds for parts $p$ of type $P$, then the analyser can apply the domain description prompt:

- **describe_mereology**

  to parts of that type and write down the mereology types and observer domain description text according to the following schema:

**5. describe_mereology(e) Observer**

- **Narration:**

  [t] ... narrative text on mereology type ...

  [m] ... narrative text on mereology observer ...

  [a] ... narrative text on mereology type constraints ...

- **Formalisation:**

  type
The mereology type name, $MT$, chosen of course, by the domain analyser cum describer, usually properly embodies the type name, $E$, of the endurant being analysed and mereology-described. The mereology type expression $M(U_i, U_j, \ldots, U_k)$ is a type expression over unique identifiers. Thus a part of type-name $P$ might be given the mereology type name $MP$. $A(MT)$ is a predicate over possibly all unique identifier types of the domain description. To write down the concrete type definition for $MT$ requires a bit of analysis and thinking.

Example 46. Mereology of a Road Net:

The mereology of hubs is a pair: (i) the set of all automobile identifiers, and (ii) the set of unique identifiers of the links that it is connected to and the set of all unique identifiers of all automobiles.

The mereology of links is a pair: (i) the set of all bus and automobile identifiers, and (ii) the set of the two distinct hubs they are connected to.

The mereology of an automobile is the set of the unique identifiers of all links and hubs.

We presently omit treatment of road net and automobile aggregate mereologies. For road net mereology we refer to Example 70, Item 139 on page 101.

type
74 $H_{\text{Mer}} = V_{UI-\text{set}} \times L_{UI-\text{set}}$
75 $L_{\text{Mer}} = V_{UI-\text{set}} \times H_{UI-\text{set}}$
76 $A_{\text{Mer}} = R_{UI-\text{set}}$

value
74 $\text{mereo}_H: H \rightarrow H_{\text{Mer}}$
75 $\text{mereo}_L: L \rightarrow L_{\text{Mer}}$
76 $\text{mereo}_A: A \rightarrow A_{\text{Mer}}$

5.3.2.1 Invariance of Mereologies

For mereologies one can usually express some invariants. Such invariants express “law-like properties”, facts which are indisputable. We refer to Sect. 5.3.4 on the next page.

Example 47. Invariance of Road Nets: The observed mereologies must express identifiers of the state of such for road nets:

---

21 We refer to Appendix Sect. B.1.1 on page 139 for more on RSL types.
22 This is just another way of saying that the meaning of hub mereologies involves the unique identifiers of all the vehicles that might pass through the hub is of interest to it.
23 The link identifiers designate the links, zero, one or more, that a hub is connected to is of interest to both the hub and that these links is interested in the hub.
24 — that the automobile might pass through
5.3 Mereology

axiom
74 ∀ (auis,luis):H • luis ⊆ luis' ∧ auis = auis'
75 ∀ (auis,huis):L • auis = auis' ∧ huis ⊆ huis' ∧ card huis' = 2
76 ∀ ruis:A • ruis = ruis' 

77 For all hubs, \( h \), and links, \( l \), in the same road net,
78 if the hub \( h \) connects to link \( l \) then link \( l \) connects to hub \( h \).

axiom
77 ∀ h:H,l:L • h ∈ hs ∧ l ∈ ls ⇒
77 let (_luis)=mereo_H(h), (_huis)=mereo_L(l)
78 in uid_L(l)∈luis ≡ uid_H(h)∈huis end

79 For all links, \( l \), and hubs, \( h_a,h_b \), in the same road net,
80 if the \( l \) connects to hubs \( h_a \) and \( h_b \), then \( h_a \) and \( h_b \) both connects to link \( l \).

axiom
79 ∀ h_a,h_b:H,l:L • {h_a,h_b} ⊆ hs ∧ l ∈ ls ⇒
79 let (_luis)=mereo_H(h), (_huis)=mereo_L(l)
80 in uid_L(l)∈luis ≡ uid_H(h)∈huis end

5.3.2.2 Deductions made from Mereologies

Once we have settled basic properties of the mereologies of a domain we can, like for unique identifiers, cf. Example 42 on page 61, “play around” with that concept: ‘the mereology of a domain’.

Example 48. Consequences of a Road Net Mereology:
81 are there [isolated] units from which one can not “reach” other units ?
82 does the net consist of two or more “disjoint” nets ?
83 et cetera.

We leave it to the reader to narrate and formalise the above properly.

5.3.3 Formulation of Mereologies

The observe_mereology domain descriptor, Page 66, may give the impression that the mereo type MT can be described “at the point of issue” of the observe_mereology prompt. Since the MT type expression may, in general, depend on any part sort the mereo type MT can, for some domains, “first” be described when all part sorts have had their unique identifiers defined.

5.3.4 Fixed and Varying Mereologies

The mereology of parts is not necessarily fixed.
**Definition 73.** **Fixed Mereology** By a *fixed mereology* we shall understand a mereology of a part which remains fixed over time.

**Definition 74.** **Varying Mereology** By a *varying mereology* we shall understand a mereology of a part which may vary over time.

**Example 49.** **Fixed and Varying Mereology:** Let us consider a road net. If hubs and links never change “affiliation”, that is: hubs are in fixed relation to zero one or more links, and links are in a fixed relation to exactly two hubs then the mereology of Example 46 on page 66 is a *fixed mereology*. If, on the other hand hubs may be inserted into or removed from the net, and/or links may be removed from or inserted between any two existing hubs, then the mereology of Example 46 on page 66 is a *varying mereology*.

**5.3.5 No Fluids Mereology**

We comment on our decision, for this monograph, to not endow fluids with mereologies. A first reason is that we “restrict” the concept of mereology to part endurants, that is, to solid endurants – those with “more-or-less” fixed extents. Fluids can be said to normally not have fixed extents, that is, they can “morph” from small, fixed into spatially extended forms. For domains of part-fluid conjoins this is particularly true. The fluids in such domains flow through and between parts. Some parts, at some times, embodying large, at other times small amounts of fluid. Some proper, but partial amount of fluid flowing from one part to a next. Et cetera. It is for the same reason that we do not endow fluids with identity. So, for this monograph we decide to not suggest the modelling of fluid mereologies.

**5.3.6 Some Modelling Observations**

It is, in principle, possible to find examples of mereologies of natural parts: rivers: their confluence, lakes and oceans; and geography: mountain ranges, flat lands, etc. But in our experimental case studies, cf. Example on Page 34, we have found no really interesting such cases. All our experimental case studies appears to focus on the mereology of artefacts. And, finally, in modelling humans, we find that their mereology encompass all other humans and all artefacts! Humans cannot be tamed to refrain from interacting with everyone and everything.

Some domain models may emphasize *physical mereologies* based on spatial relations, others may emphasize *conceptual mereologies* based on logical “connections”. Some domain models may emphasize *physical mereologies* based on spatial relations, others may emphasize *conceptual mereologies* based on logical “connections”.

**Example 50.** **Rail Net Mereology:** We refer to Example 37 on page 47.

84 A linear rail unit is connected to exactly two distinct other rail net units of any given rail net.

85 A point unit is connected to exactly three distinct other rail net units of any given rail net.

---

74 cf. Examples 27 on page 35, 35 on page 44, 36 on page 46, 38 on page 49, 41 on page 60, 42 on page 61, 44 on page 63, 45 on page 63, 46 on page 66 and 47 on page 66.
5.4 Attributes

86 A rigid crossing unit is connected to exactly four distinct other rail net units of any given rail net.
87 A single and a double slip unit is connected to exactly four distinct other rail net units of any given rail net.
88 A terminal unit is connected to exactly one distinct other rail net unit of any given rail net.
89 So we model the mereology of a railway net unit as a pair of sets of rail net unit unique identifiers distinct from that of the rail net unit.

\[ \text{value} \]
89. \( \text{meréo}_\text{NU}: \text{NU} \rightarrow (\text{UI}-\text{set} \times \text{UI}-\text{set}) \)
axiom
89. \( \forall \text{nu}:\text{NU} \cdot \)
89. let \( (\text{uis}_i, \text{uis}_o) = \text{meréo}_\text{NU}(\text{nu}) \) in
89. case (card \( \text{uis}_i \), card \( \text{uis}_o \)) =
84. (\( \text{is}_\text{LU}(\text{nu}) \rightarrow (1,1) \),
85. \( \text{is}_\text{PU}(\text{nu}) \rightarrow (1,2) \lor (2,1) \),
86. \( \text{is}_\text{RU}(\text{nu}) \rightarrow (2,2) \),
87. \( \text{is}_\text{SU}(\text{nu}) \rightarrow (2,2) \), \( \text{is}_\text{DU}(\text{nu}) \rightarrow (2,2) \),
88. \( \text{is}_\text{TU}(\text{nu}) \rightarrow (1,0) \lor (0,1) \),
89. \( \rightarrow \text{chaos} \) end
89. \( \land \text{uis}_i \cap \text{uis}_o = \{\} \)
89. \( \land \text{uid}_\text{NU}(\text{nu}) \notin (\text{uis}_i \cup \text{uis}_o) \)
89. \( \text{end} \)

Figure 5.1 illustrates the mereology of four rail units.

<table>
<thead>
<tr>
<th>Linear Point</th>
<th>Rigid Crossing</th>
<th>Double Slip</th>
</tr>
</thead>
<tbody>
<tr>
<td>ua</td>
<td>ux</td>
<td>ua</td>
</tr>
<tr>
<td>ul</td>
<td>ux</td>
<td>ub</td>
</tr>
<tr>
<td>ui</td>
<td>uy</td>
<td>ui</td>
</tr>
<tr>
<td>ux</td>
<td>uy</td>
<td>ua</td>
</tr>
<tr>
<td>({ua},{ux})</td>
<td>({ua},{ux,uy})</td>
<td>({ua,ub},{ux,uy})</td>
</tr>
<tr>
<td>({ux},{ua})</td>
<td>({ux,uy},{ua})</td>
<td>({ux,uy},{ua,ub})</td>
</tr>
</tbody>
</table>

Fig. 5.1 Four Symmetric Rail Unit Mereologies

5.4 Attributes

This section is based on Sect. 5.4 of [41, Pages 119–139].

To recall: there are three sets of internal qualities: unique identifiers, mereologies and attributes. Unique identifiers and mereologies are rather definite kinds of internal endurant qualities; attributes form more “free-wheeling” sets of internal qualities. Whereas, for this monograph, we suggest to not endow fluids with unique identification and mereologies all endurants, i.e., including fluids, are endowed with attributes.
5.4.1 Inseparability of Attributes from Parts and Fluids

Parts and fluids are typically recognised because of their spatial form and are otherwise characterised by their intangible, but measurable attributes. That is, whereas endurants, whether solid (as are parts) or fluids, are physical, tangible, in the sense of being spatial [or being abstractions, i.e., concepts, of spatial endurants], attributes are intangible: cannot normally be touched\textsuperscript{75}, or seen\textsuperscript{76}, but can be objectively measured\textsuperscript{77}. Thus, in our quest for describing domains where humans play an active rôle, we rule out subjective “attributes”; feelings, sentiments, moods. Thus we shall abstain, in our domain science also from matters of aesthetics.

We equate all endurants — which have the same type of unique identifiers, the same type of mereologies, and the same types of attributes — with one sort. Thus removing an internal quality from an endurant makes no sense: the endurant of that type either becomes an endurant of another type or ceases to exist (i.e., becomes a non-entity)!

We can roughly distinguish between two kinds of attributes: those which can be motivated by physical (incl. chemical) concerns, and those, which, although they embody some form of ‘physics measures’, appear to reflect on event histories: “if ’something’, \(\phi\), has ’happened’ to an endurant, \(e_a\), then some ’commensurate thing’, \(\psi\), has ’happened’ to another (one or more) endurants, \(e_b\)” where the ‘something’ and ‘commensurate thing’ usually involve some ‘interaction’ between the two (or more) endurants. It can take some reflection and analysis to properly identify endurants \(e_a\) and \(e_b\) and commensurate events \(\phi\) and \(\psi\). Example 61 shall illustrate the, as we shall call it, intentional pull of event histories.

5.4.2 Attribute Modelling Tools

5.4.2.1 Attribute Quality and Attribute Value

We distinguish between an attribute (as a logical proposition, of a name, i.e.) type, and an attribute value, as a value in some value space.

5.4.2.2 Concrete Attribute Types

By a concrete type shall understand a sort (i.e., a type) which is defined in terms of some type expression: \(T = T(...)\). This is referred to below as \([=...\].

5.4.2.3 Attribute Types and Functions

Let us recall that attributes cover qualities other than unique identifiers and mereology. Let us then consider that parts and fluids to have one or more attributes. These attributes are qualities which help characterise “what it means” to be a part or a fluid. Note that we expect

\textsuperscript{75} One can see the red colour of a wall, but one touches the wall.

\textsuperscript{76} One cannot see electric current, and one may touch an electric wire, but only if it conducts high voltage can one know that it is indeed an electric wire.

\textsuperscript{77} That is, we restrict our domain analysis with respect to attributes to such quantities which are observable, say by mechanical, electrical or chemical instruments. Once objective measurements can be made of human feelings, beauty, and other, we may wish to include these “attributes” in our domain descriptions.
5.4 Attributes

every part and fluid to have at least one attribute. The question is now, in general, how many
and, particularly, which.

Domain Description Prompt 7 describe attributes: The domain analyser experiments,

  thinks and reflects about endurant, \( e \), attributes. That process is initiated by the domain
description prompt:

- describe attributes(e).

The result of that domain description prompt is that the domain analyser cum describer
writes down the attribute (sorts or) types and observers domain description text according to the
following schema:

\[
\text{let } \{ \eta A_1, ..., \eta A_m \} = \text{analyse_attribute_type_names(e)} \]

* Narration:

  - \( \text{[t]} \) ... narrative text on attribute sorts ...
    - some Ais may be concretely defined: \( [\eta A_i=...] \)
  - \( \text{[o]} \) ... narrative text on attribute sort observers ...
  - \( \text{[p]} \) ... narrative text on attribute sort proof obligations ...

* Formalisation:

  - \( \text{type} \)
    - \( \text{[t]} \) \( A_1[=...] \), ..., \( A_m[=...] \)
  - \( \text{value} \)
    - \( \text{[o]} \) attr\( A_1 : E \rightarrow A_1 \), ..., attr\( A_m : E \rightarrow A_m \)
  - \( \text{proof obligation} \) [Disjointness of Attribute Types]
    - \( \text{[p]} \) PO: let \( P \) be any part sort in [the domain description]
    - \( \text{[p]} \) let \( a : (A_1 | A_2 | ... | A_m) \) in is\( A_i(a) \neq is A_j(a) \) \( [i \neq j, i,j:[1..m]] \) end end

Let \( A_1, ..., A_n \) be the set of all conceivable attributes of endurants \( e:E \). (Usually \( n \) is a rather large
natural number, say in the order of a hundred conceivable such.) In any one domain model
the domain analyser cum describer selects a modest subset, \( A_1, ..., A_m, \) i.e., \( m < n \). Across
many domain models for “more-or-less the same” domain \( m \) varies and the attributes, \( A_1, ..., A_m \), selected for one model may differ from those, \( A'_1, ..., A'_m \), chosen for another model.

The type definitions: \( A_1, ..., A_m \), inform us that the domain analyser has decided to focus on the
distinctly named \( A_1, ..., A_m \) attributes. The value clauses attr\( A_1:P \rightarrow A_1 \), ..., attr\( A_m:P \rightarrow A_m \) are then “automatically” given: if an endurant, \( e:E \), has an attribute \( A_i \) then there is postulated, “by definition” [eureka] an attribute observer function attr\( A_i:E \rightarrow A_i \) et cetera.

We cannot automatically, that is, syntactically, guarantee that our domain descriptions
secure that the various attribute types for a endurant sort denote disjoint sets of values.
Therefore we must prove it.

5.4.2.4 Attribute Categories

Michael A. Jackson [84] has suggested a hierarchy of attribute categories: from static to
dynamic values – and within the dynamic value category: inert values, reactive values,
active values – and within the dynamic active value category: autonomous values, biddable
values and programmable values. We now review these attribute value types. The review is
based on [84, M.A.Jackson]. Endurant attributes are either constant, i.e., static, or varying,
i.e., dynamic attributes.

\[26\] The attribute type names are chosen by the domain analyser to reflect on domain phenomena.
**Attribute Category 1** By a *static attribute*, \( a:A \), we shall understand an attribute whose values are constants, i.e., cannot change.

**Example 51. Static Attributes:** Let us exemplify road net attributes in this and the next examples. And let us assume the following attributes: year of first link construction and link length at that time. We may consider both to be static attributes: The year first established, seems an obvious static attribute and the length is fixed at the time the road was first built.

**Attribute Category 2** By a *dynamic attribute*, \( a:A \), we shall understand an attribute whose values are variable, i.e., can change. Dynamic attributes are either *inert, reactive or active attributes*.

**Attribute Category 3** By an *inert attribute*, \( a:A \), we shall understand a dynamic attribute whose values only change as the result of external stimuli where these stimuli prescribe new values.

**Example 52. Inert Attribute:** And let us now further assume the following link attribute: link name. We may consider it to be an inert attribute: the name is not “assigned” to the link by the link itself, but probably by some road net authority which we are not modelling.

**Attribute Category 4** By a *reactive attribute*, \( a:A \), we shall understand a dynamic attribute whose values, if they vary, change in response to external stimuli, where these stimuli either come from outside the domain of interest or from other endurants.

**Example 53. Reactive Attributes:** Let us further assume the following two link attributes: “wear and tear”, respectively “icy and slippery”. We will consider those attributes to be reactive in that automobiles (another part) traveling the link, an external “force”, typically causes the “wear and tear”, respectively the weather (outside our domain) causes the “icy and slippery” property.

**Attribute Category 5** By an *active attribute*, \( a:A \), we shall understand a dynamic attribute whose values change (also) of its own volition. Active attributes are either autonomous, or biddable or programmable attributes.

**Attribute Category 6** By an *autonomous attribute*, \( a:A \), we shall understand a dynamic active attribute whose values change only “on their own volition”. The values of an autonomous attributes are a “law onto themselves and their surroundings”.
Example 54. **Autonomous Attributes:** We enlarge scope of our examples of attribute categories to now also include automobiles (on the road net). In this example we assume that an automobile is driven by a human [behaviour]. These are some automobile attributes: velocity, acceleration, and moving straight, or turning left, or turning right. We shall consider these three attributes to be autonomous. It is the driver, not the automobile, who decides whether the automobile should drive at constant velocity, including 0, or accelerate or decelerate, including stopping. And it is the driver who decides when to turn left or right, or not turn at all.

**Attribute Category 7** By a *biddable attribute*, a:A, *is_biddable_attribute*(a) we shall understand a dynamic active attribute whose values are prescribed but may fail to be observed as such.

Example 55. **Biddable Attributes:** In the context of automobiles these are some biddable attributes: turning the wheel, to drive right at a hub – with the automobile failing to turn right; pressing the accelerator, to obtain a higher speed – with the automobile failing to really gaining speed; pressing the brake, to stop– with the automobile failing to halt.

**Attribute Category 8** By a *programmable attribute*, a:A, *is_programmable_attribute*(a), we shall understand a dynamic active attribute whose values can be prescribed.

Example 56. **Programmable Attribute:** We continue with the automobile on the road net examples. In this example we assume that an automobile includes, as one inseparable entity, “the driver”. These are some automobile attributes: position on a link, velocity, acceleration (incl. deceleration), and direction: straight, turning left, turning right. We shall now consider these three attributes to be programmable.

Figure 5.2 captures an attribute value ontology.

**Fig. 5.2** Attribute Value Ontology

Figure 5.2 hints at three categories of dynamic attributes: *monitorable only*, *biddable* and *programmable* attributes.
Attribute Category 9 By a monitorable only attribute, \( a:A \), is\_monitorable\_only\( attribute(a) \), we shall understand a dynamic active attribute which is either inert or reactive or autonomous.

That is:

\[
\begin{align*}
\text{value} \quad & \text{is\_monitorable\_only}: E \to \text{Bool} \\
& \text{is\_monitorable\_only}(e) = \text{is\_inert}(e) \lor \text{is\_reactive}(e) \lor \text{is\_autonomous}(e)
\end{align*}
\]

Example 57. Road Net Attributes:

We treat some attributes of the hubs of a road net.

90 There is a hub state. It is a set of pairs, \((l_f, l_t)\), of link identifiers, where these link identifiers are in the mereology of the hub. The meaning of the hub state in which, e.g., \((l_f, l_t)\) is an element, is that the hub is open, “green”, for traffic from link \(l_f\) to link \(l_t\). If a hub state is empty then the hub is closed, i.e., “red” for traffic from any connected links to any other connected links.

91 There is a hub state space. It is a set of hub states. The current hub state must be in its state space. The meaning of the hub state space is that its states are all those the hub can attain.

92 Since we can think rationally about it, it can be described, hence we can model, as an attribute of hubs, a history of its traffic: the recording, per unique bus and automobile identifier, of the time ordered presence in the hub of these vehicles. Hub history is an event history.

\[
\begin{align*}
\text{type} \quad & H\Sigma = (L\text{UI} \times L\text{UI})\text{-set} \\
& H\Omega = H\Sigma\text{-set} \\
& H\text{Traffic} = (A\text{UI} | B\text{UI}) \implies (\text{TIME} \times \text{VPos})^* \\
\text{axiom} \quad & \forall h:H \cdot \text{obs\_H}\Sigma(h) \in \text{obs\_H}\Omega(h) \\
& \forall h:\text{Traffic}, u:(A\text{UI} | B\text{UI}) \cdot u \in \text{dom } h \Rightarrow \text{time\_ordered}(h(u)) \\
\text{value} \quad & \text{attr\_H}\Sigma: H \to H\Sigma \\
& \text{attr\_H}\Omega: H \to H\Omega \\
& \text{attr\_H}\text{Traffic}: H \to H\text{Traffic} \\
& \text{time\_ordered}: (\text{TIME} \times \text{VPos})^* \to \text{Bool} \\
& \text{time\_ordered}(\text{tvpl}) = ...
\end{align*}
\]

In Item 92 we model the time-ordered sequence of traffic as a discrete sampling, i.e., \( \implies \), rather than as a continuous function, \( \rightarrow \).

Example 58. Invariance of Road Net Traffic States: We continue Example 57.

93 The link identifiers of hub states must be in the set, \( L\text{ui}\), of the road net’s link identifiers.

\[
\begin{align*}
\text{axiom} \quad & \forall h:H \cdot h \in Ls \Rightarrow \\
& \text{let } h\sigma = \text{attr\_H}\Sigma(h) \text{ in} \\
& \forall (l_{ui}, l_{ui}'): (L\text{UI} \times L\text{UI}) \cdot (l_{ui}, l_{ui}') \in h\sigma \Rightarrow \{l_{ui}, l_{ui}'\}' \subseteq L\text{ui}\text{ end}
\end{align*}
\]
You may skip Example 59 in a first reading.

**Example 59. Road Transport: Further Attributes:**

### Links:

We show just a few attributes.

94 There is a link state. It is a set of pairs, $(h_f, h_t)$, of distinct hub identifiers, where these hub identifiers are in the mereology of the link. The meaning of a link state in which $(h_f, h_t)$ is an element is that the link is open, “green”, for traffic from hub $h_f$ to hub $h_t$. Link states can have either 0, 1 or 2 elements.

95 There is a link state space. It is a set of link states. The meaning of the link state space is that its states are all those the which the link can attain. The current link state must be in its state space. If a link state space is empty then the link is (permanently) closed. If it has one element then it is a one-way link. If a one-way link, $l$, is imminent on a hub whose mereology designates that link, then the link is a “trap”, i.e., a “blind cul-de-sac”.

96 Since we can think rationally about it, it can be described, hence it can model, as an attribute of links a history of its traffic: the recording, per unique bus and automobile identifier, of the time ordered positions along the link (from one hub to the next) of these vehicles.

97 The hub identifiers of link states must be in the set, $h_{ui}s$, of the road net’s hub identifiers.

$$\textbf{type}$$

94 $\Sigma = H_{UI}$-set  

95 $\Omega = \Sigma$-set 

96 $\text{Traffic} = \text{obs}(\Sigma(l) \in \Omega(l)) \Rightarrow \text{time}_{\text{ordered}}(\text{ht}(ui))$ 

96a $\text{Fraction} = \text{Real}$, axiom $\text{frac} : \text{Fract} \cdot 0 < \text{frac} < 1$

97 $\forall \Sigma(l) \in \Omega(l), \text{let } \sigma = \text{attr}(\Sigma(l))$ in

$$\forall (h_{ui_i}, h_{ui_i}') : (H_{UI} \times K_{UI}) \cdot$$

$$\{h_{ui_i}, h_{ui_i}'\} \subseteq h_{ui}s$$

### Automobiles:

We illustrate but a few attributes:

98 Automobiles have static number plate registration numbers.

99 Automobiles have dynamic positions on the road net:

a or on a link, some fraction, frac:Fract down an identified link, $l_{ui}$, from one of its identified connecting hubs, $h_{ui}$, in the direction of the other identified hub, $h_{ui}$.

c Fraction is a real properly between 0 and 1.

$$\textbf{type}$$

98 $\text{RegNo} = \text{atHub} | \text{onLink}$

99a $\text{atHub} :: h_{ui} : H_{UI}$

99b $\text{onLink} :: h_{ui} : H_{UI} \times l_{ui} : L_{UI} \times \text{frac} : \text{Fract} \times h_{ui} : H_{UI}$

99c $\text{Fract} = \text{Real}$
axiom
99c \frac{Fract}{0<\frac{a}{b}<1}

value
98 \text{attr}_{\text{RegNo}}: A \rightarrow \text{RegNo}
99 \text{attr}_{\text{APos}}: A \rightarrow \text{APos}

Obvious attributes that are not illustrated are those of velocity and acceleration, forward or backward movement, turning right, left or going straight, etc. The acceleration, deceleration, even velocity, or turning right, turning left, moving straight, or forward or backward are seen as command actions. As such they denote actions by the automobile — such as pressing the accelerator, or lifting accelerator pressure or braking, or turning the wheel in one direction or another, etc. As actions they have a kind of counterpart in the velocity, the acceleration, etc. attributes. In Items 92 Pg. 74 and 96 Pg. 75, we illustrated an aspect of domain analysis & description that may seem, and at least some decades ago would have seemed, strange: namely that if we can think, hence speak, about it, then we can model it “as a fact” in the domain. The case in point is that we include among hub and link attributes their histories of the timed whereabouts of buses and automobiles.

5.4.2.5 Calculating Attribute Category Type Names

One can calculate sets of all attribute type names, of static, so-called monitorable and programmable attribute types of parts and fluids with the following domain analysis prompts:

- \text{analyse\_attribute\_type\_names},
- \text{sta\_attr\_types},
- \text{mon\_attr\_types}, and
- \text{pro\_attr\_types}.

\text{analyse\_attribute\_type\_names} applies to parts and yields a set of all attribute names of that part. \text{mon\_attr\_types} applies to parts and yields a set of attribute names of \textit{monitorable} attributes of that part.

Observer Function Prompt 6 \text{analyse\_attribute\_types}:

\begin{align*}
\text{value} \\
\text{analyse\_attribute\_type\_names}: P \rightarrow \eta A\text{-set} \\
\text{analyse\_attribute\_type\_names}(p) \text{ as } \{
\eta A_1, \eta A_2, \ldots, \eta A_m
\}
\end{align*}

Observer Function Prompt 7 \text{sta\_attr\_types}:

\begin{align*}
\text{value} \\
\text{sta\_attr\_types}: P \rightarrow \eta A \times \eta A \times \ldots \times \eta A \\
\text{sta\_attr\_types}(p) \text{ as } (\eta A_1, \eta A_2, \ldots, \eta A_n) \\
\text{where: } (\{\eta A_1, \eta A_2, \ldots, \eta A_n\} \subseteq \text{analyse\_attribute\_type\_names}(p)) \\
\land \text{ let } \text{anms} = \text{analyse\_attribute\_type\_names}(p) \\
\land \forall \text{anm}\in\text{anms} \text{ as } (\eta A_1, \eta A_2, \ldots, \eta A_n)
\end{align*}

\footnote{In this day and age of road cameras and satellite surveillance these traffic recordings may not appear so strange: We now know, at least in principle, of technologies that can record approximations to the hub and link traffic attributes.}

\footnote{\eta A is the type of all attribute types.}
5.4 Attributes

\[ \Rightarrow \sim \text{is_static_attribute}(anm) \]
\[ \land \forall \text{anm}: \eta A \cdot \text{anm} \in \{ \eta A_1, \eta A_2, ..., \eta A_n \} \]
\[ \Rightarrow \text{is_static_attribute}(anm) \text{ end} \]

Observer Function Prompt 8 \textbf{mon_attr_types}: 

\text{value} \quad \text{mon_attr_types: } P \rightarrow \eta A \times \eta A \times ... \times \eta A \\
\text{mon_attr_types(p) as (} \eta A_1, \eta A_2, ..., \eta A_n \text{)}

\text{where: } \{ \eta A_1, \eta A_2, ..., \eta A_n \} \subseteq \text{analyse_attribute_type_names}(p) \\
\land \text{let anms = analyse_attribute_type_names(p)}
\land \forall \text{anm}: \eta A \cdot \text{anm} \in \text{anms} \setminus \{ \eta A_1, \eta A_2, ..., \eta A_n \} \\
\Rightarrow \sim \text{is_monitorable_attribute}(anm) \\
\land \forall \text{anm}: \eta A \cdot \text{anm} \in \{ \eta A_1, \eta A_2, ..., \eta A_n \} \\
\Rightarrow \text{is_monitorable_attribute}(anm) \text{ end} \\

Observer Function Prompt 9 \textbf{pro_attr_types}: 

\text{value} \quad \text{pro_attr_types: } P \rightarrow \eta A \times \eta A \times ... \times \eta A \\
\text{pro_attr_types(p) as (} \eta A_1, \eta A_2, ..., \eta A_n \text{)}

\text{where: } \{ \eta A_1, \eta A_2, ..., \eta A_n \} \subseteq \text{analyse_attribute_type_names}(p) \\
\land \text{let anms = analyse_attribute_type_names(p)}
\land \forall \text{anm}: \eta A \cdot \text{anm} \in \text{anms} \setminus \{ \eta A_1, \eta A_2, ..., \eta A_n \} \\
\Rightarrow \sim \text{is_monitorable_attribute}(anm) \\
\land \forall \text{anm}: \eta A \cdot \text{anm} \in \{ \eta A_1, \eta A_2, ..., \eta A_n \} \\
\Rightarrow \text{is_monitorable_attribute}(anm) \text{ end} \\

Some comments are in order. The \text{analyse_attribute_type_names} function is, as throughout, meta-linguistic, that is, informal, not-computable, but decidable by the domain analyser cum describer. Applying it to a part or fluid yields, at the discretion of the domain analyser cum describer, a set of attribute type names “freely” chosen by the domain analyser cum describer. The \text{sta_attr_type_names}, the \text{mon_attr_type_names}, and the \text{pro_attr_type_names} functions are likewise meta-linguistic; their definition here relies on the likewise meta-linguistic \text{is_static}, \text{is_monitorable} and \text{is_programmable} analysis predicates.

5.4.2.6 Calculating Attribute Values

Let (\eta A_1, \eta A_2, ..., \eta A_n) be a grouping of attribute types for part \( p \) (or fluid \( f \)). Then (\text{attr}_A(p), \text{attr}_A(p), ..., \text{attr}_A(p)) (respectively \( f \)) yields (\text{a1}, \text{a2}, ..., \text{an}), the grouping of values for these attribute types.

We can “formalise” this conversion:

\text{value} \quad \text{types_to_values: } \eta A_1 \times \eta A_2 \times ... \times \eta A_n \rightarrow A_1 \times A_2 \times ... \times A_n
5.4.2.7 Calculating Attribute Names

The meta-linguistic, i.e., “outside” RSL proper, name for attribute type names is introduced here as $\eta_A$.

100 Given endurant $e$ we can *meta-linguistically* calculate names for its *static* attributes.
101 Given endurant $e$ we can *meta-linguistically* calculate name for its *monitorable* attributes.
102 Given endurant $e$ we can *meta-linguistically* calculate names for its *programmable* attributes.
103 These four sets make up all the attributes of endurant $e$.

The type names $ST$, $MA$, $PT$ designate mutually disjoint sets, $ST$, of names of static attributes, sets, $MA$, of names of monitorable, i.e., monitorable-only and biddable, attributes, sets, $PT$, of names of programmable, i.e., fully controllable attributes.

type
100 $ST = \eta_A$-set
101 $MA = \eta_A$-set
102 $PT = \eta_A$-set

value
100 $\text{stat\_attr\_types}: E \rightarrow ST$
101 $\text{moni\_attr\_types}: E \rightarrow MA$
102 $\text{prgr\_attr\_types}: E \rightarrow PT$

axiom
103 $\forall e : E \cdot$
104 $\text{let stat\_nms = stat\_attr\_types(e),}$
105 $\text{moni\_nms = moni\_attr\_types(e),}$
106 $\text{prgr\_nms = prgr\_attr\_types(e) in}$
107 $\text{card stat\_nms} + \text{card moni\_nms} + \text{card prgr\_nms}$
108 $= \text{card(stat\_nms} \cup \text{moni\_nms} \cup \text{prgr\_nms}) \text{ end}$

The above formulas are indicative, like mathematical formulas, they are not computable.

104 Given endurant $e$ we can *meta-linguistically* calculate its static attribute values, $\text{stat\_attr\_vals}$; 105 given endurant $e$ we can *meta-linguistically* calculate its monitorable-only attribute values, $\text{moni\_attr\_vals}$; and 106 given endurant $e$ we can *meta-linguistically* calculate its programmable attribute values, $\text{prgr\_attr\_vals}$.

The type names $sa_1$, ..., $pap$ refer to the types denoted by the corresponding types name $nsa_1$, ..., $npap$.

value
104 $\text{stat\_attr\_vals}: E \rightarrow SA_1 \times SA_2 \times \ldots \times SAs$
104 $\text{stat\_attr\_vals}(e) \equiv$
104 $\text{let } [nsa_1, nsa_2, \ldots, nsa_s] = \text{stat\_attr\_types(e) in}$
104 $(\text{attr\_sa}_1(e), \text{attr\_sa}_2(e), \ldots, \text{attr\_sa}_s(e)) \text{ end}$
105 $\text{moni\_attr\_vals}: E \rightarrow MA_1 \times MA_2 \times \ldots \times MAm$
105 $\text{moni\_attr\_vals}(e) \equiv$

81 By using the term *meta-linguistically* here we shall indicate that we go outside what is computable – and thus appeal to the reader’s forbearance.
5.4 Attributes

\[ \text{let } \{\text{nma}_1, \text{nma}_2, \ldots, \text{nma}_m\} = \text{moni_attr_types(e)} \text{ in} \]
\[ \text{(attr}_{\text{nma}_1}(e), \text{attr}_{\text{nma}_2}(e), \ldots, \text{attr}_{\text{nma}_m}(e)) \text{ end} \]

\[ \text{prgr_attr_vals}: E \to \text{PA}_1 \times \text{PA}_2 \times \ldots \times \text{PA}_p \]
\[ \text{prgr_attr_vals}(e) \equiv \]
\[ \text{let } \{\text{npa}_1, \text{npa}_2, \ldots, \text{npa}_p\} = \text{prgr_attr_types(e)} \text{ in} \]
\[ \text{(attr}_{\text{npa}_1}(e), \text{attr}_{\text{npa}_2}(e), \ldots, \text{attr}_{\text{npa}_p}(e)) \text{ end} \]

The “ordering” of type values, \((\text{attr}_{\text{sa}_1}(e), \ldots, \text{attr}_{\text{sa}_s}(e)), (\text{attr}_{\text{ma}_1}(e), \ldots, \text{attr}_{\text{ma}_m}(e))\), etcetera, is arbitrary.

5.4.3 Operations on Monitorable Attributes of Parts

We remind the reader of the notions of states in general, Sect. 4.7 and updateable states, Sect. 4.7.2 on page 53. For every domain description there possibly is an updateable state. The is such a state if there is at least one part with at least one monitorable attribute. Below, as in Sect. 4.7.2, we refer to the updateable states as \(\sigma\).

Given a part, \(p\), with attribute \(A\), the simple operation \(\text{attr}_{A}(p)\) thus yields the value of attribute \(A\) for that part. But what if, what we have is just the global state \(\sigma\), of the set of all monitorable parts of a given universe-of-discourse, \(\text{uod}\), the unique identifier, \(\text{uid}_{\text{P}}(p)\), of a part of \(\sigma\), and the name, \(\eta_{A}\), of an attribute of \(p\)? Then how do we ascertain the attribute value for \(A\) of \(p\), and, for biddable attributes \(A\), “update” \(p\), in \(\sigma\), to some \(A\) value? Here is how we express these two issues.

5.4.3.1 Evaluation of Monitorable Attributes

\[ \text{let } \pi: \text{Pl} \text{ be the unique identifier of any part, } p, \text{ with monitorable attributes, let } A \text{ be a monitorable attribute of } p, \text{ and let } \eta_{A} \text{ be the name of attribute } A. \]
\[ \text{Evaluation of the [current] attribute } A \text{ value of } p \text{ is defined by function } \text{read}_{A}\text{from}_{P} - \]
\[ \text{retr}_{\text{part}}(\pi) \text{ is defined in Sect. 5.2.5.1 on page 64.} \]

\[ \text{read}_{A}(\pi, \eta_{A}) \equiv \text{attr}_{A}(\text{retr}_{\text{part}}(\pi)) \]

5.4.3.2 Update of Biddable Attributes

\[ \text{The update of a monitorable attribute } A, \text{ with attribute name } \eta_{A} \text{ of part } p, \text{ identified by } \pi, \]
\[ \text{to a new value writes to the global part state } \sigma. \]
\[ \text{Part } p \text{ is retrieved from the global state.} \]
\[ \text{A new part, } p' \text{ is formed such that } p' \text{ is like part } p: \]
\[ \text{a same unique identifier,} \]
\[ \text{b same mereology,} \]
\[ \text{c same attributes values,} \]
\[ \text{d except for } A. \]
That new $p'$ replaces $p$ in $\sigma$.

```
value
```

```
107. $\sigma$, a:A, pi:Pl, $\eta A:\eta T$
108. update_P with A: Pl $\times$ A $\times$ $\eta T \rightarrow$ write $\sigma$
109. update_P with A(pi,a,$\eta A$) $\equiv$
110. let $p = \text{retr_part(pi)}$ in
111. let $p':P \cdot$
111a. uid_P(p')=pi
111b. $\land$ mereo_P(p)=mereo_P(p')
111c. $\land$ $\forall \eta A$ in analyse_attribute_type_names(p) \{ $\eta A$\}
111d. $\land$ attr_A(p)=attr_A(p')
112. $\sigma := \sigma \setminus \{p\} \cup \{p'\}$
113. end end
```

5.5 Intentional Pull

In the next section we shall encircle the ‘intention’ concept by extensively quoting from Kai Sørlander’s Philosophy [108, 109, 110, 111].

5.5.1 Issues Leading Up to Intentionality

5.5.1.1 Causality of Purpose

“If there is to be the possibility of language and meaning then there must exist primary entities which are not entirely encapsulated within the physical conditions; that they are stable and can influence one another. This is only possible if such primary entities are subject to a supplementary causality directed at the future: a causality of purpose.”

5.5.1.2 Living Species

“These primary entities are here called living species. What can be deduced about them? They are characterised by causality of purpose: they have some form they can be developed to reach; and which they must be causally determined to maintain; this development and maintenance must occur in an exchange of matter with an environment. It must be possible that living species occur in one of two forms: one form which is characterised by development, form and exchange, and another form which, additionally, can be characterised by the ability to purposeful movements. The first we call plants, the second we call animals.”

5.5.1.3 Animate Entities

“For an animal to purposefully move around there must be “additional conditions” for such self-movements to be in accordance with the principle of causality: they must have sensory organs sensing among others the immediate purpose of its movement; they must have means
of motion so that it can move; and they must have instincts, incentives and feelings as causal conditions that what it senses can drive it to movements. And all of this in accordance with the laws of physics.”

5.5.1.4 Animals

“To possess these three kinds of “additional conditions”, must be built from special units which have an inner relation to their function as a whole; Their purposefulness must be built into their physical building units, that is, as we can now say, their genomes. That is, animals are built from genomes which give them the inner determination to such building blocks for instincts, incentives and feelings. Similar kinds of deduction can be carried out with respect to plants. Transcendentally one can deduce basic principles of evolution but not its details.”

5.5.1.5 Humans – Consciousness and Learning

“The existence of animals is a necessary condition for there being language and meaning in any world. That there can be language means that animals are capable of developing language. And this must presuppose that animals can learn from their experience. To learn implies that animals can feel pleasure and distaste and can learn. One can therefore deduce that animals must possess such building blocks whose inner determination is a basis for learning and consciousness.”

“Animals with higher social interaction uses signs, eventually developing a language. These languages adhere to the same system of defined concepts which are a prerequisite for any description of any world: namely the system that philosophy lays bare from a basis of transcendental deductions and the principle of contradiction and its implicit meaning theory. A human is an animal which has a language.”

5.5.1.6 Knowledge

“Humans must be conscious of having knowledge of its concrete situation, and as such that humans can have knowledge about what they feel and eventually that humans can know whether what they feel is true or false. Consequently a human can describe his situation correctly.”

5.5.1.7 Responsibility

“In this way one can deduce that humans can thus have memory and hence can have responsibility, be responsible. Further deductions lead us into ethics.”

We shall not further develop the theme of living species: plants and animals, thus excluding, most notably humans, in this chapter. We claim that the present chapter, due to its foundation in Kai Sørlander’s Philosophy, provides a firm foundation within which we, or others, can further develop this theme: analysis & description of living species.
5.5.2 Intentionality

*Intentionality* as a philosophical concept is defined by the Stanford Encyclopedia of Philosophy\(^2\) as “the power of minds to be about, to represent, or to stand for, things, properties and states of affairs.”

5.5.2.1 Intentional Pull

Two or more artefactual parts of different sorts, but with overlapping sets of intents may exert an intentional “pull” on one another. This intentional “pull” may take many forms. Let \( p_x : X \) and \( p_y : Y \) be two parts of different sorts \((X,Y)\), and with common intent, \( i \). Manifestations of these, their common intent must somehow be subject to constraints, and these must be expressed predicatively.

**Example 60. Double Bookkeeping:** A classical example of intentional pull is found in double bookkeeping which states that every financial transaction has equal and opposite effects in at least two different accounts. It is used to satisfy the accounting equation: 
\[
\text{Assets} = \text{Liabilities} + \text{Equity}.
\]
The intentional pull is then reflected in commensurate postings, for example: either in both debit and passive entries or in both credit and passive entries.

When a compound artefact is modelled as put together with a number of distinct sort endurants then it does have an intentionality and the components’ individual intentionalities does, i.e., shall relate to that. The composite road transport system has intentionality of the road serving the automobile part, and the automobiles have the intent of being served by the roads, across “a divide”, and vice versa, the roads of serving the automobiles.

Natural endurants, for example, rivers, lakes, seas\(^3\) and oceans become, in a way, artefacts when mankind use them for transport; natural gas becomes an artefact when drilled for, exploited and piped; and harbours make no sense without artefactual boats sailing on the natural water.

5.5.2.2 The Type Intent

This, perhaps vague, concept of intentionality has yet to be developed into something of a theory. Despite that this is yet to be done, we shall proceed to define an *intentionality analysis function*. First we postulate a set of *intent designators*. An *intent designator* is really a further undefined quantity. But let us, for the moment, think of them as simple character strings, that is, literals, for example "transport", "eating", "entertainment", etc.

\[ \text{type Intent} \]

5.5.2.3 Intentionalities

**Observer Function Prompt 10** Analyse_intentionality:
The domain analyser analyses an endurant as to the finite number of intents, zero or more,


\(^3\) Seas are smaller than oceans and are usually located where the land and ocean meet. Typically, seas are partially enclosed by land. The Sargasso Sea is an exception. It is defined only by ocean currents [ocean-service.noaa.gov/facts/oceansea.html].
5.5 Intentional Pull

with which the analyser judges the endurant can be associated. The method provides the
domain analysis prompt:

- analyse\textunderscore intentionality directs the domain analyser to observe a set of intents.

value analyse\textunderscore intentionality(e) = [i_1,i_2,...,i_n] \subseteq \text{Intent}

Example 61. Intentional Pull: Road Transport:

We simplify the link, hub and automobile histories — aiming at just showing an essence of
the intentional pull concept.

113 With links, hubs and automobiles we can associate history attributes:

a link history attributes time-stamped records, as an ordered list, the presence of auto-
mobiles;
b hub history attributes time-stamped records, as an ordered list, the presence of auto-
mobiles; and
c automobile history attributes time-stamped records, as an ordered list, their visits to
links and hubs.

\begin{align*}
\text{type} & \quad \text{value} \\
113a. \ LHist & = A_l \rightarrow \text{TIME}^r & 113a. \ \text{attr}\_LHist: L \rightarrow \text{LHist} \\
113b. \ HHist & = A_h \rightarrow \text{TIME}^r & 113b. \ \text{attr}\_HHist: H \rightarrow \text{HHist} \\
113c. \ AHist & = (L|H) \rightarrow \text{TIME}^r & 113c. \ \text{attr}\_AHist: A \rightarrow \text{AHist}
\end{align*}

5.5.2.4 Wellformedness of Event Histories

Some observations must be made with respect to the above modelling of time-stamped event
histories.

114 Each $\tau_\ell: \text{TIME}^r$ is an indefinite list. We have not expressed any criteria for the recording
of events: all the time, continuously ! (?)

115 Each list of times, $\tau_\ell: \text{TIME}^r$, is here to be in decreasing, continuous order of times.

116 Time intervals from when an automobile enters a link (a hub) till it first time leaves that
link (hub) must not overlap with other such time intervals for that automobile.

117 If an automobile leaves a link (a hub), at time $\tau$, then it may enter a hub (resp. a link) and
then that must be at time $\tau'$ where $\tau'$ is some infinitesimal, sampling time interval, quantity
larger that $\tau$. Again we refrain here from speculating on the issue of sampling !

118 Altogether, ensembles of link and hub event histories for any given automobile define
routes that automobiles travel across the road net. Such routes must be in the set of
routes defined by the road net.

As You can see, there is enough of interesting modelling issues to tackle !

5.5.2.5 Formulation of an Intentional Pull

119 An intentional pull of any road transport system, $rts$, is then if:

a for any automobile, $a$, of $rts$, on a link, $\ell$ (hub, $h$), at time $\tau$,
b then that link, $\ell$, (hub $h$) “records” automobile $a$ at that time.

120 and:
c for any link, ℓ (hub, h) being visited by an automobile, a, at time τ,
d then that automobile, a, is visiting that link, ℓ (hub, h), at that time.

axiom
119a. ∀ a:A • a ∈ as ⇒
119a. let ahist = attr(AHist(a)) in
119a. ∀ ui:(LI | HI) • ui ∈ dom ahist ⇒
119b. ∀ τ:TIME • τ ∈ elems ahist(ui) ⇒
119b. let hist = is LI(ui) → attr LHist(retr LI(ui))(σ),
119b. → attr HHist(retr H(ui))(σ) in
119b. τ ∈ elems hist(uid A(a)) end end
120. ∧
120c. ∀ u:(L | H) • u ∈ ls ∪ hs ⇒
120c. let uhist = attr(L | H)Hist(u) in
120d. ∀ ai:AI • ai ∈ dom uhist ⇒
120d. ∀ τ:TIME • τ ∈ elems uhist(ai) ⇒
120d. let ahist = attr(AHist(retr A(ai))(σ)) in
120d. τ ∈ elems uhist(ai) end end

Please note, that intents are not [thought of as] attributes. We consider intents to be a fourth, a comprehensive internal quality of endurants. They, so to speak, govern relations between the three other internal quality of endurants: the unique identifiers, the mereologies and the attributes. That is, they predicate them, “arrange” their comprehensiveness. Much more should be said about intentionality. It is a truly, I believe, worthy research topic of its own.

Example 62. Aspects of Comprehensiveness of Internal Qualities: Let us illustrate the issues “at play” here.

- Consider a road transport system uod.
  - Applying analyse_intentionality(uod) may yield the set {"transport", ...}.
- Consider a financial service industry, fss.
  - Applying analyse_intentionality(fss) may yield the set {"interest on deposit", ...}.
- Consider a health care system, hcs.
  - Applying analyse_intentionality(hcs) may yield the set {"cure diseases", ...}.

What these analyses of intentionality yields, with respect to expressing intentional pull, is entirely of the discretion of the domain analysis & description.

We bring the above example, Example 62, to indicate, as the name of the example reveals, “Aspects of Comprehensiveness of Internal Qualities”. That the various components of artefactual systems relate in – further to be explored – ways. In this respect, performing domain analysis & description is not only an engineering pursuit, but also one of research. We leave it to the readers to pursue this research aspect of domain analysis & description.

5.5.3 Artefacts

Humans create artefacts – for a reason, to serve a purpose, that is, with intent. Artefacts are like parts. They satisfy the laws of physics – and serve a purpose, fulfill an intent.
5.5 Intentional Pull

5.5.4 Assignment of Attributes

So what can we deduce from the above, almost three pages?

The attributes of natural parts and natural fluids are generally of such concrete types – expressible as some real with a dimension of the International System of Units: https://physics.nist.gov/cuu/Units/units.html. Attribute values usually enter into differential equations and integrals, that is, classical calculus.

The attributes of humans, besides those of parts, significantly includes one of a usually non-empty set of intents. In directing the creation of artefacts humans create these with an intent.

Example 63. Intentional Pull: General Transport: These are examples of human intents: they create roads and automobiles with the intent of transport, they create houses with the intents of living, offices, production, etc., and they create pipelines with the intent of oil or gas transport.

Human attribute values usually enter into modal logic expressions.

5.5.5 Galois Connections

Galois Theory was first developed by Évariste Galois [1811-1832] around 1830. Galois theory emphasizes a notion of Galois connections. We refer to standard textbooks on Galois Theory, e.g., [114, 2009].

5.5.5.1 Galois Theory: An Ultra-brief Characterisation

To us, an essence of Galois connections can be illustrated as follows:

- Let us observe properties of a number of endurants, say in the form of attribute types.
- Let the function $F$ map sets of entities to the set of common attributes.
- Let the function $G$ map sets of attributes to sets of entities that all have these attributes.
- $(F, G)$ is a Galois connection
  - if, when including more entities, the common attributes remain the same or fewer,
  - if when including more attributes, the set of entities remain the same or fewer.
- $(F, G)$ is monotonously decreasing.

Example 64. LEGO Blocks: We have

- There is a collection of LEGO® blocks.
- From this collection, $A$, we identify the red square blocks, $e$.
- That is $F(A)$ is $B = \{ \text{attr\_Color}(e) = \text{red}, \text{attr\_Form}(e) = \text{square} \}$.
- We now add all the blue square blocks.

---

84 Basic units are meter, kilogram, second, Ampere, Kelvin, mole, and candela. Some derived units are: Newton: kg·m·s$^{-2}$, Weber: kg·m$^2$·s$^{-2}$·A$^{-1}$, etc.

85 en.wikipedia.org/wiki/Galois_theory

86 The following is an edited version of an explanation kindly provided by Asger Eir, e-mail, June 5, 2020 [61, 62, 45].
• And obtain $A'$.
• Now the common properties are their squareness: $F(A')$ is $B' = \{\text{attr}\_\text{Form(e)}=\text{square}\}$.
• More blocks as argument to $F$ yields fewer or the same number of properties.
• The more entities we observe, the fewer common attributes they possess.

Example 65  Civil Engineering: Consultants and Contractors:  Less playful, perhaps more seriously, and certainly more relevant to our endeavour, is this next example.

• Let $X$ be the set of civil engineering, i.e., building, consultants, i.e., those who, like architects and structural engineers design buildings – of whatever kind.
• Let $Y$ be the set of building contractors, i.e., those firms who actually implement, i.e., build to, those designs.
• Now a subset, $X_{bridges}$ of $X$, contain exactly those consultants who specialise in the design of bridges, with a subset, $Y_{bridges}$, of $Y$ capable of building bridges.
• If we change to a subset, $X_{bridges,tunnels}$ of $X$, allowing the design of both bridges and tunnels, then we obtain a corresponding subset, $Y_{bridges,tunnels}$, of $Y$.
• So when
  \[\diamond \] we enlarge the number of properties from ‘bridges’ to ‘bridges and tunnels’,
  \[\diamond \] we reduce, most likely, the number of contractors able to fulfill such properties,
  \[\diamond \] and vice versa,

• then we have a Galois Connection\(^{88}\).

5.5.5.2 Galois Connections and Intentionality – A Possible Research Topic?

We have a hunch\(^{89}\)! Namely that there are some sort of Galois Connections with respect to intentionality. We leave to the interested reader to pursue this line of inquiry.

5.5.6 Discovering Intentional Pulls

The analysis and description of a domain’s external qualities and the internal qualities of unique identifiers, mereologies and attributes can be pursued systematically – endurant sort by sort. Not so with the discovery of a domain’s possible intentional pulls. Basically “what is going on” here is that the domain analyser cum describer considers pairs, triples or more part “independent\(^{90}\)” endurants and reflects on whether they stand in an intentional pull relation to one another. We refer to Sects. 5.5.2.2 – 5.5.2.3.

5.5.6.1 Identifying Intents

\(^{87}\) The E-mail, June 5, 2020, from Asger Eir
\(^{88}\) This was, more formally, shown Dr. Asger Eir’s PhD thesis [61].
\(^{89}\) Hunch: a feeling or guess based on intuition rather than fact.
\(^{90}\) By “independent” we shall here mean that these endurants are not ‘derived’ from one-another!
5.6 A Domain Discovery Procedure, II

5.5.6.2 Searching for Intentional Pulls

TO BE WRITTEN

5.5.6.3 Describing Intentional Pulls

TO BE WRITTEN

5.6 A Domain Discovery Procedure, II

This section is based on Sect. 5.8 of [41, Pages 146–147]. We continue from Sect. 4.8.

5.6.1 The Process

We shall again emphasize some aspects of the domain analysis & description method. A method procedures is that of exhaustively analyse & describe all internal qualities of the domain under scrutiny. A method technique implied here is that sketched below. The method tools are here all the analysis and description prompts covered so far.

Please be reminded of Discovery Schema 0’s declaration of Notice Board variables (Page 54). In this section we collect (i) the description of unique identifiers of all parts of the state; (ii) the description of mereologies of all parts of the state; and (iii) the description of attributes of all parts of the state. (iii) We finally gather these into the discover internal endurant qualities procedures.

An Endurant Internal Qualities Domain Analysis and Description Process

value
discover_unique_ids: Unit → Unit
discover_unique_ids() ⇨
  for ∀ v ∈ gen do
txt := txt + [type_name(v)→txt(type_name(v))ட(describe_unique_identifier(v))]
end
discover_mereologies: Unit → Unit
discover_mereologies() ⇨
  for ∀ v ∈ gen do
txt := txt + [type_name(v)→txt(type_name(v))ட(describe_mereology(v))]
end
discover_attributes: Unit → Unit
discover_attributes() ⇨
  for ∀ v ∈ gen do
txt := txt + [type_name(v)→txt(type_name(v))ṭ(describe_attributes(v))]
end
discover_intentional_pulls: Unit → Unit
discover_intentional_pulls() ⇨
  for ∀ (v',v'') ∈ gen do
txt := txt + [type_name(v')→txt(type_name(v'))ṭ(describe_intentional_pull(v'))]
  + [type_name(v'')→txt(type_name(v''))ṭ(describe_intentional_pull(v''))]
end
describe_intentional_pul: Unit → ...
describe_intentional_pull() ⇨ ...

value
discover_endurant_qualities: Unit → Unit
discovery_internal_qualities() ≡
  discovery_uids();
  axiom [ all parts have unique identifiers ]
  discovery_mereologies();
  axiom [ all unique identifiers are mentioned in sum total of ]
    axiom [ all mereologies and no isolated proper sets of parts ]
  discovery_attributes();
  axiom [ sum total of all attributes span all parts of the state ]
  discovery_intentional_pulls();

We shall comment on the axioms in the next section.

5.6.2 A Suggested Analysis & Description Approach, II

Figure 4.3 on page 49 possibly hints at an analysis & description order in which not only the external qualities of endurants are analysed & described, but also their internal qualities of unique identifiers, mereologies and attributes.

In Sect. 4.8 on page 53 we were concerned with the analysis & description order of endurants. We now follow up on the issue of (in Sect. 4.5.1.3 on page 48) on how compounds are treated: namely as both a “root” parts and as a composite of two or more “sibling” parts and/or fluids. The taxonomy of the road transport system domain, cf. Fig. 4.3 on page 49 and Example 35 on page 44, thus gives rise to many different analysis & description traversals. Figure 5.3 illustrates one such order.

Fig. 5.3 A Breadth-First, Top-Down Traversal

Again, it is up to the domain engineer cum scientist to decide. If the domain analyser cum describer decides to not endow a compound “root” with internal qualities, then an ‘internal qualities’ traversal will not have to neither analyse nor describe those qualities.
5.7 Summary

Internal Qualities Predicates and Functions

<table>
<thead>
<tr>
<th>#</th>
<th>Name</th>
<th>Introduced</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Analysis Predicates</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>is_manifest</td>
<td>page 59</td>
</tr>
<tr>
<td>17</td>
<td>is_structure</td>
<td>page 59</td>
</tr>
<tr>
<td>2</td>
<td>Attribute Analysis Predicates</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>is_static_attribute</td>
<td>page 72</td>
</tr>
<tr>
<td>2</td>
<td>is_dynamic_attribute</td>
<td>page 72</td>
</tr>
<tr>
<td>3</td>
<td>is_inert_attribute</td>
<td>page 72</td>
</tr>
<tr>
<td>4</td>
<td>is_reactive_attribute</td>
<td>page 72</td>
</tr>
<tr>
<td>5</td>
<td>is_active_attribute</td>
<td>page 72</td>
</tr>
<tr>
<td>6</td>
<td>is_autonomous_attribute</td>
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</tr>
<tr>
<td>7</td>
<td>is_biddable_attribute</td>
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</tr>
<tr>
<td>8</td>
<td>is_programmable_attribute</td>
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</tr>
<tr>
<td>9</td>
<td>is_monitorable_only_attribute</td>
<td>page 74</td>
</tr>
<tr>
<td>3</td>
<td>Analysis Functions</td>
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</tr>
<tr>
<td>106</td>
<td>analyse_unique_identifiers</td>
<td>page 62</td>
</tr>
<tr>
<td>107</td>
<td>calculate_unique_identifiers</td>
<td>page 62</td>
</tr>
<tr>
<td>6</td>
<td>sta_attr_types</td>
<td>page 76</td>
</tr>
<tr>
<td>7</td>
<td>mon_attr_types</td>
<td>page 76</td>
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<tr>
<td>8</td>
<td>pro_attr_types</td>
<td>page 77</td>
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<tr>
<td>9</td>
<td>retr_part</td>
<td>page 64</td>
</tr>
<tr>
<td>108</td>
<td>read_A_from_P</td>
<td>page 79</td>
</tr>
<tr>
<td>109</td>
<td>update_P_with_A</td>
<td>page 79</td>
</tr>
<tr>
<td>5</td>
<td>Description Functions</td>
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</tr>
<tr>
<td>5</td>
<td>describe_unique_identifier</td>
<td>page 61</td>
</tr>
<tr>
<td>6</td>
<td>describe_mereology</td>
<td>page 65</td>
</tr>
<tr>
<td>7</td>
<td>describe_attributes</td>
<td>page 71</td>
</tr>
<tr>
<td>3</td>
<td>Domain Discovery</td>
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<td>discover_uids</td>
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</tr>
<tr>
<td>6</td>
<td>discover_mereologies</td>
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<tr>
<td>7</td>
<td>discover_attributes</td>
<td>page 87</td>
</tr>
<tr>
<td>5</td>
<td>discover_internal_qualities</td>
<td>page 87</td>
</tr>
</tbody>
</table>
This chapter is a rather “drastic” reformulation and simplification of [41, Chapter 7, i.e., pages 159–196]. Besides, Sect. 6.5 is new.

In this chapter we transcendentally “morph” manifest parts into behaviours, that is: endurants into perdurants. We analyse that notion and its constituent notions of actors, channels and communication, actions and behaviours. We shall investigate the, as we shall call them, perdurants of domains. That is state and time-evolving domain phenomena. The outcome of this chapter is that the reader will be able to model the perdurants of domains. Not just for a particular domain instance, but a possibly infinite set of domain instances.

6.1 Part Behaviours – An Analysis

We remind the reader of Sect. 2.1.2 on page 9.
6.1.1 Behaviour Definition Analysis

Parts co-exist; they do so endurantly as well as perdurantly: endure and perdure.

Part perdurants, i.e., behaviours, interact with their surroundings, that is, with other behaviours. This is true for both natural and man-made parts. The present domain modelling method is mainly focused on man-made parts, that is artefacts. So our next analysis will take its clues from artefactual parts.

We can, roughly, analyse part behaviours into three kinds.

- **Proactive Behaviours:** Behaviour $B_i$ offers to synchronise and communicate values – internal non-deterministically with either of a definite number of distinct part sort behaviours $B_a, B_b, ... , B_c$:

$$B(i)(args) \equiv 
(\ldots \text{ch}[i,a] \mid a_\text{val} ; \ldots ; B(i)(args')) \\
\lor (\ldots \text{ch}[i,b] \mid b_\text{val} ; \ldots ; B(i)(args'')) \\
\lor \ldots \\
\lor (\ldots \text{ch}[i,c] \mid c_\text{val} ; \ldots ; B(i)(args'''))$$

The tail-recursive invocation of $B_i$ indicates a possible “update” of behaviour $B_i$ arguments. More on this later.

- **Responsive Behaviours:** Behaviour $B_i$ external non-deterministically expresses willingness to synchronisation with and accept values from either of a definite number of distinct part sort behaviours $B_a, B_b, ... , B_c$:

$$B(i)(args) \equiv 
\begin{aligned}
(\ldots \text{let} av = \text{ch}[i,a] \ ? \ in \ \ldots \ B(i)(args') \ \text{end}) \\
\lor (\ldots \text{let} bv = \text{ch}[i,b] \ ? \ in \ \ldots ; B(i)(args'') \ \text{end}) \\
\lor \ldots \\
\lor (\ldots \text{let} cv = \text{ch}[i,c] \ ? \ in \ \ldots ; B(i)(args''') \ \text{end})
\end{aligned}$$

- **Mixed Behaviours:** Or behaviours, more generally, “are” an internal non-deterministic “mix” of the above:

$$B(i)(args) \equiv 
(\ldots \text{ch}[i,a] \mid a_\text{val} ; \ldots ; B(i)(args')) \\
\lor (\ldots \text{ch}[i,b] \mid b_\text{val} ; \ldots ; B(i)(args'')) \\
\lor \ldots \\
\lor (\ldots \text{ch}[i,c] \mid c_\text{val} ; \ldots ; B(i)(args''')) \\
\lor (\ldots \text{let} av = \text{ch}[i,a] \ ? \ in \ \ldots ; B(i)(args') \ \text{end}) \\
\lor (\ldots \text{let} bv = \text{ch}[i,b] \ ? \ in \ \ldots ; B(i)(args'') \ \text{end}) \\
\lor \ldots \\
\lor (\ldots \text{let} cv = \text{ch}[i,c] \ ? \ in \ \ldots ; B(i)(args''') \ \text{end})$$

The “bodies” of the $B_i$ behaviour definitions, i.e., “…” , may contain interactions with [yet other] behaviours. Schematically for example:

```plaintext
ch[i,x] ! x_\text{val} \\
\{ ch[i,z] ! z_\text{val} ; \{ z:z_1,z_2,...,zm \} \} \\
\text{let} \ yv = \text{ch}[i,y] \ ? \ in \ \ldots \ \text{end} \\
\text{let} \ zv = \[] \{ ch[i,z] ? | z:z_1,z_2,...,zm \} \ \text{in} \ \ldots \ \text{end}
```

Etcetera. The full force of CSP with RSL is at play!
6.1.2 Channel Analysis

This is the first of two treatments of the concept of channels; the present treatment is informal, motivational, the second treatment, Sect. 6.2 (right next!), is more formal.

The CSP concept of channel is to be our way of expressing the “medium” in which behaviours interact. Channels is thus an abstract concept. Please do not think of it as a physical, an IT (information technology) device. As an abstract concept it is defined in terms of, roughly, the laws, the semantics, of CSP [78]. We write ‘roughly’ since the CSP we are speaking of, is “embedded” in RSL.

6.2 Domain Channel Description

We simplify the general treatment of channel declarations. Basically all we can say, for any domain, is that any two distinct part behaviours may need to communicate. Therefore we declare a vector of channels indexed by sets of two distinct part identifiers.

\[
\begin{align*}
\text{value} & \quad \text{discover\_channels}: \text{Unit} \rightarrow \text{Unit} \\
& \quad \text{discover\_channels}() \equiv \\
& \quad \text{channel} \left\{ \text{ch}[\text{ij,ik}] \mid \text{ij,ik:UI} \cdot \{|\text{ij,ik}| \subseteq \text{uid}, \text{ij} \neq \text{ik}\} \right\} \subseteq \text{M} \ 
\end{align*}
\]

Initially we shall leave the type of messages over channels further undefined. As we, laboriously, work through the definition of behaviours, we shall be able to make \( \text{M} \) precise. all\_uniq\_ids was defined in Sect. 5.2.4 on page 62.

6.3 Behaviour Definition Description

Behaviours have to be described. Behaviour definitions are in the form of function definitions and are here expressed in RSL relying, very much, on its CSP component. Behaviour definitions describe the type of the arguments the function, i.e., the behaviour, for which it is defined, that is, which kind of values it accepts. Behaviour definitions further describe

Thus there are two elements to a behaviour definition: the behaviour signature and the behaviour body definitions.

6.3.1 Behaviour Signatures

6.3.1.1 General

Function, \( F \), signatures consists of two textual elements: the function name and the function type:

\[
\begin{align*}
\text{value} & \quad F: A \rightarrow B, \text{ or } F: a:A \rightarrow B \\
\end{align*}
\]

where \( A \) and \( B \) are the types of function (“input”) arguments, respectively function (“output”) values for such arguments. The first form \( F: A \rightarrow B \) is what is normally referred to as the form for function signatures. The second form: \( F: a:A \rightarrow B \) “anticipates” the general for for function \( F \) invocation: \( F(a) \).
6.3.1.2 Domain Behaviour Signatures

A schematic form of part \((p)\) behaviour signatures is:

\[
\begin{align*}
\text{b: bi:BI} & \rightarrow \text{me:Mer} \\
& \rightarrow \text{svl:StaV}^* \\
& \rightarrow \text{mvl:MonV}^* \\
& \rightarrow \text{prgl:PrgV}^* \\
\text{channels} & \text{ Unit}
\end{align*}
\]

We shall motivate the general form of part behaviour, \(B\), signatures, “step-by-step”:

\(\alpha\). \(b\) the [chosen] name of part \(p\) behaviours.
\(\beta\). \(U \rightarrow V \rightarrow \ldots \rightarrow W \rightarrow Z\): The function signature is expressed in the Schönfinkel/Curry\(^2\) style – corresponding to the invocation form \(F(u)(v)\ldots(w)\)
\(\gamma\). \(\text{bi:BI}\): a general value and the type of part \(p\) unique identifier

\(\delta\). \(\text{me:Mer}\): a general value and the type of part \(p\) mereology

\(\epsilon\). \(\text{svl:StaV}^*\): a general (possibly empty) list of values and types of part \(p\)’s (possibly empty) list of static attributes

\(\zeta\). \(\text{mvl:MonV}^*\): a general list of names of types of part \(p\)’s (possibly empty) list of monitorable attributes

\(\eta\). \(\text{prgl:PrgV}^*\): a general list of values and types of part \(p\)’s (possibly empty) list of programmable attributes

\(\theta\). \(\text{channels}\): are usually of the form: \(\{\text{ch}[i,j][k,i,j] \in \text{I}(\text{me})\}\) and express the subset of channels over which behaviour \(B\)s interact with other behaviors

\(\iota\). \(\text{Unit}\): designates the single value ()

In detail:

\(\alpha\). \textbf{Behaviour name}: In each domain description there are many sorts, \(B\), of parts. For each sort there is a generic behaviour, whose name, here \(b\), is chosen to suitably reflect \(B\).

\(\beta\). \textbf{Currying} is here used in the pragmatic sense of grouping “same kind of arguments”, i.e., separating these from one-another, by means of the \(\rightarrow\)s.

\(\gamma\). The \textbf{unique identifier} of part sort \(B\) is here chosen to be \(BI\). Its value is a constant.

\(\delta\). The \textbf{mereology} is a usually constant. For same part sorts it may be a variable.

\(\epsilon\). \textbf{Static attribute values} are constants. The use of static attribute values in behaviour body definitions is expressed by an identifier of the \(\text{svl}\) list of identifiers.

\(\zeta\). \textbf{Monitorable attribute values} are generally, ascertainable, i.e., readable, cf. Sect. 5.4.3.1 on page 79. Some are \textit{biddable}, can be changed by \(a\), or the behaviour, cf. Sect. 5.4.3.2 on page 79, but there is no guarantee, as for programmable attributes, that they remain fixed.

The use of \(a\)[ny] monitorable attribute value in behaviour body definitions is expressed by a \texttt{read_A_from_P(mv,bi)} where \(mv\) is an identifier of the \(\text{mvl}\) list of identifiers and \(bi\) is the unique part identifier of the behaviour definition in which the \texttt{read} occurs.

The update of a biddable attribute value in behaviour body definitions is expressed by a \texttt{update_P_with_A(bi,mv,a)}.

---

\(92\) Moses Schönfinkel (1888–1942) was a Russian logician and mathematician accredited with having invented combinatory logic [https://en.wikipedia.org/wiki/Moses_Schönfinkel]. Haskell B. Curry (1900–1982) was an American mathematician and logician known for his work in combinatory logic [https://en.wikipedia.org/wiki/Haskell_Curry]
6.3 Behaviour Definition Description

η. **Programmable attribute values** are just that. They vary as specified, i.e., “programmed”, by the behaviour body definition. Tail-recursive invocations of behaviour $B_i$ “replace” relevant programmable attribute argument list elements with “new” values.

θ. **channels**: $I(me)$ expresses a set of unique part identifiers different from $b_i$, hence of behaviours, with which behaviour $b(i)$ interacts.

ι. The **Unit** of the behaviour signature is a short-hand for the behaviour either **reading** the value of a monitorable attribute, hence global state $\sigma$, or performing a **write**, i.e., an **update**, on $\sigma$.

6.3.1.3 Action Signatures

Actions come in any forms:

121 Some take no arguments, say `action_a()`, but read the global state component $\sigma$, and
122 others also take no arguments, say `action_b()`, but update the global state component $\sigma$.
123 Some take an argument, say, `action_c(c)`, but do not “touch” a global state component, while others both take an argument and deliver a value, say `action_d(d)` and also do not “touch” a global state component.
125 Et cetera!

`type A, B, C, D,...
value
121. action_a: Unit $\rightarrow$ read $\sigma$ A
122. action_b: Unit $\rightarrow$ write $\sigma$ B
123. action_c: C $\rightarrow$ Unit
124. action_d: D $\rightarrow$ E Unit
125. ...

An example of 123 are the CSP output: `ch[...]|c`, and an example of 124 are the CSP input: `let e = ch[...]? in ... end`.

6.3.2 Behaviour Invocation

The general form of behaviour invocation is shown below. The invocation follows the “Currying” of the behaviour type signature. [Normally one would write all this on one line: $b(i)(m)(s)(m)(p) \equiv$.]

```
behaviour_name
  (unique_identifier)
  (mereology)
  (static_values)
  (monitorable_attribute_names)
  (programmable_variables) \equiv
...
body ...
```

When first “invoked”, that is, transcendentally deduced, i.e., “morphed”, from a manifest part, $p$, the invocation looks like:

```
value
discover_.behaviour_signature: P $\rightarrow$ RSL-Text
discover_.behaviour_signature(p) \equiv
```
6.3.3 Behaviour Definition Bodies

We remind the reader of Sect. 6.1.1 on page 92. The general, “mixed”, form of behaviour definitions was given as:

\[ B(i)(args) \equiv \]
\[ \left( \ldots \right) \]
\[ \left( ... \ ch\{i,b\} \ ! \ b_{\text{val}} \ ; \ldots ; \ B(i)(args') \left( ... \right) \right) \]
\[ \left( ... \right) \]
\[ \left( ... \right) \]
\[ \left( ... \right) \]
\[ \left( ... \right) \]
\[ \left( ... \right) \]
\[ \left( ... \right) \]
\[ \left( ... \right) \]
\[ \left( ... \right) \]
\[ \left( \ldots \right) \]

We can express the same by separating the alternatives into invocations of separately defined behaviours.

\[ B(i)(args) \equiv \]
\[ \left( ... \right) \]
\[ \left( Bin_j(i)(args) \left( ... \right) \right) \]
\[ \left( ... \right) \]
\[ \left( ... \right) \]
\[ \left( Bx_n(i)(args) \left( ... \right) \right) \]

where the internal don-deterministically invoked behaviours \( Bin_j(i)(args) \) and the external don-deterministically invoked behaviours \( Bin_k(i)(args) \) are then separately defined:

\[ Bin_j(i)(args) \equiv \left( ... Bin_j(i)(args') \right) \]
\[ Bx_n(i)(args) \equiv \left( ... Bx_n(i)(args'') \right) \]

6.3.4 Discover Behaviour Definition Bodies

In other words, for current lack of a more definitive methodology for “discovering” the bodies of behaviour definitions we resort to “…”!
6.3 Behaviour Definition Description

value

discover behaviour definition: \( P \rightarrow \text{RSL-Text} \)

discover behaviour definition(p) \( \equiv \ldots \)

discover behaviour definitions: Unit \( \rightarrow \text{RSL-Text} \)

discover behaviour definitions() \( \equiv \{ \text{discover behaviour definition(p)} \mid p \in \sigma \land \text{is manifest(p)} \} \)

Example 67. Automobile Behaviour: We remind the reader of the main, running example of this primer, the of the road transport system Example
d

Signatures

126 automobile:

a there is the usual “triplet” of arguments: unique identifier, mereology and static attributes;
b then there are two programmable attributes: the automobile position (cf. Item 99 on page 75), and the automobile history (cf. Item 113c on page 83);
c and finally there are the input/output channel references allowing communication between the automobile and the hub and link behaviours.

127 Similar for

a link and
b hub behaviours.

We omit the modelling of monitorable attributes (...).

Definitions: Automobile at a Hub

128 We abstract automobile behaviour at a Hub (hi).

a Either the automobile remains in the hub,
b or, internally non-deterministically,
c leaves the hub entering a link,
d or, internally non-deterministically,
e stops.

128 \text{automobile}(ai)(aai,uis)(...) (apos: atH(fli,hi,tli),ahist) \equiv \\
128a \text{automobile\_remains\_in\_hub}(ai)(aai,uis)(...) (apos: atH(fli,hi,tli),ahist) \\
128b \lfloor \text{let} \tau = \text{record\_TIME()} \text{ in} \\
128c \text{ch}[ai,hi]!\tau; \\
128d \text{automobile\_leaving\_hub}(ai)(aai,uis)(...) (apos: atH(fli,hi,tli),ahist) \\
128e \text{automobile\_stop}(ai)(aai,uis)(...) (apos: atH(fli,hi,tli),ahist) \\

129 [128a] The automobile remains in the hub:
   a the automobile remains at that hub, “idling”, 
   b informing (”first”) the hub behaviour.

129 \text{automobile\_remains\_in\_hub}(ai)(aai,uis)(...) (apos: atH(fli,hi,tli),ahist) \equiv \\
129a \text{let} \tau = \text{record\_TIME()} \text{ in} \\
129b \text{ch}[ai,hi]!\tau; \\
129c \text{automobile}(ai)(aai,uis)(...) (apos, upd\_hist(\tau,hi)(ahist)) \\
129d \text{end} \\
129a \text{upd\_hist}(\tau,i)(hist) \equiv hist \uplus [i \mapsto \langle \tau \rangle \ast hist(i)]

130 [128c] The automobile leaves the hub entering a link:
   a tli, whose “next” hub, identified by thi, is obtained from the mereology of the link 
   identified by tli; 
   b informs the hub it is leaving and the link it is entering, 
   c “whereupon” the vehicle resumes (i.e., “while at the same time” resuming) the vehicle 
   behaviour positioned at the very beginning (0) of that link.

130 \text{automobile\_leaving\_hub}(ai)(aai,uis)(...) (apos: atH(fli,hi,tli),ahist) \equiv \\
130a \text{let} ((fhi,thi),ais) = \text{mereo\_L retr\_L(tli)(\sigma)) in} \\
130b \text{assert: fhi=hi} \\
130c \text{ch}[ai,hi]!\tau || \text{ch}[ai,tli]!\tau; \\
130d \text{automobile}(ai)(aai,uis)(...) \\
130e \text{onL(tli,(hi,thi),0),upd\_hist(\tau,tli)(upd\_hist(\tau,hi)(ahist))) end}

131 [128e] Or the automobile “disappears — off the radar”!

131 \text{automobile\_stop}(ai)(aai,uis),(...) (apos: atH(fli,hi,tli),ahist) \equiv \text{stop}

Similar behaviour definitions can be given for automobiles on a link, for links and for hubs.
Together they must reflect, amongst other things: the time continuity of automobile flow,
that automobiles follow routes, that automobiles, links and hubs together adhere to the
intentional pull expressed earlier, et cetera. A specification of these aspects must be proved
to adhere to these properties.

6.4 Domain Behaviour Initialisation

For every manifest part it must be described how its behaviour is initialised.
Example 68. The Road Transport System Initialisation: We “wrap up” the main example of this primer: We omit treatment of monitorable attributes.

Let us refer to the system initialisation as an action. All links are initialised, all hubs are initialised, all automobiles are initialised, etc.

value

\[
\text{rts\_initialisation: Unit } \rightarrow \text{ Unit}
\]

\[
\text{rts\_initialisation() } \equiv
\]

\[
\| \{ \text{link}(\text{uid}_L(l))(\text{mero}_L(l))(\text{attr}_\text{LEN}(l),\text{attr}_\text{Traffic}(l),\text{attr}_\text{L}(l))| l:L \in \text{ls} \}
\]

\[
\| \{ \text{hub}(\text{uid}_H(l))(\text{mero}_H(l))(\text{attr}_\text{H\_Traffic}(l),\text{attr}_\text{H}(l))| h:H \in \text{hs} \}
\]

\[
\| \{ \text{automobile}(\text{uid}_A(a))(\text{mero}_A(a))(\text{attr}_\text{RegNo}(a))(\text{attr}_\text{APos}(a))| a:A \in \text{as} \}
\]

\[
\| \ ...
\]

We have here omitted possible monitorable attributes. We refer to ls: Item 45 on page 53, hs: Item 46 on page 53, and as: Item 47 on page 53.

6.5 Discrete Dynamic Domains

Up till now our analysis & description of a domain, has, in a sense, been static: in analysing a domain we considered its entities to be of a definite number. In this section we shall consider the case where the number of entities change: where new entities are created and existing entities are destroyed, that is: where new parts, and hence behaviours, arise, and existing parts, and hence behaviours, cease to exist.

6.5.1 Create and Destroy Entities

In the domain we can expect that its behaviours create and destroy entities.

Example 69. Creation and Destruction of Entities: In the road transport domain new hubs, links and automobiles may be inserted into the road net, and existing links, hubs and automobiles may be removed from the road net. In a container terminal domain [15, 36] new containers are introduced, old are discarded; new container vessels are introduced, old are discarded; new ship-to-shore cranes are introduced, old are discarded; et cetera. In a retailer domain [39] new customers are introduced, old are discarded; new retailers are introduced, old are discarded; new merchandise is introduced, old is discarded; et cetera. In a financial system domain new customers are introduced, old are discarded; new banks are introduced, old are discarded; new brokers are introduced, old are discarded; et cetera.

The issue here is: When hubs and links are inserted or removed the mereologies of “neighbouring” road elements change, and so does the mereology of automobiles. When automobiles are inserted or removed The mereology of road elements have to be changed to take account of the insertions and removals, and so does the mereology of automobiles. And,
some domain laws must be re-expressed: The domain part state, $\sigma$, must be updated\textsuperscript{94}, and so must the unique identifier state, $uid_\sigma$\textsuperscript{95}.

\textbf{6.5.1.1 Create Entities}

It is taken for granted here that there are behaviours, one or more, which take the initiative to and carry out the creation of specific entities. Let us refer to such a behaviour as the “creator”. To create an entity implies the following three major steps [A.–C.] the step wise creation of the part and initialisation of the transduced behaviour, and [D.] the adjustment of all such part behaviours that might have their mereologies and attributes updated to accept such requests from creators.

A. To decide on the part sort – in order to create that part – that is

- to obtain a unique identifier – one hitherto not used;
- to obtain a mereology, one
  - according to the general mereology for parts of that sort,
  - and how the part specifically is to “fit” into its surroundings;
- to obtain an appropriate set of attributes:
  - again according to the attribute types for that part sort
  - and, more specifically, choosing initial attribute values.
- This part is then “joined” to the global part state, $\sigma$\textsuperscript{96} and
- its unique identifier “joined” to the global unique identifier state, $uid_\sigma$\textsuperscript{97}.

B. Then to transcendentally deduce that part into a behaviour:

- initialised (according to Sect. 6.3.1) with
  - the unique identifier,
  - the mereology, and
  - the attribute values
- This behaviour is then invoked and “joined” to the set of current behaviours, cf. Sect. 6.4 on page 98 – i.e., just above!

C. Then, finally, to “adjust” the mereologies of topologically or conceptually related parts,

- that is, for each of these parts to update:
- their mereology and possibly some
- state and state space
- arguments of their corresponding behaviours.

D. The update of the mereologies of already “running” behaviours requires the following:

- that, potentially all, behaviours offers to accept
- mereology update requests from the “creator” behaviour.

The latter means, practically speaking, that each part/behaviour which may be subject to mereology changes externally non-deterministically expresses an offer to accept such a change.

\textsuperscript{94} Cf. Sect. 4.7.2 on page 53
\textsuperscript{95} Cf. Sect. 5.2.4 on page 62
\textsuperscript{96} Cf. Sect. 4.7.2 on page 53
\textsuperscript{97} Cf. Sect. 5.2.4 on page 62
6.5 Discrete Dynamic Domains

Example 70. Road Net Administrator: We introduce the road net behaviour – based on the road net composite part, RN.

137 The road net has a programmable attribute: a road net (development & maintenance) graph. The road net graph consists of a quadruple: a map that for each hub identifier records “all” the information that the road net administrator deems necessary for the maintenance and development of road net hubs; a map that for each link identifier records “all” the information that the road net administrator deems necessary for the maintenance and development of road net links; and a map from the hub identifiers to the set of identifiers of the links it is connected to, and the set of all automobile identifiers.

138 This graph is commensurate with the actual topology of the road net.

\[
G = (\text{HI} \rightarrow \text{H}_\text{Info}) \times (\text{LI} \rightarrow \text{L}_\text{Info}) \times (\text{HI} \rightarrow \text{LI}-\text{set}) \times \text{AI}-\text{set}
\]

value

\[
\text{attr}_G : \text{RN} \rightarrow G
\]

axiom

137. \( \forall \) (hi_info, li_info, map, ais):G •
137. \( \text{dom} \) map = \( \text{dom} \) hi_info = his \& \( \text{rng} \) map = \( \text{dom} \) li_info = lis \&
138. \( \forall \) hi:HI • hi \( \in \) \( \text{dom} \) hi_info \( \Rightarrow \)
138. let h:H • h \( \in \) \( \sigma \) \& uid_H(h)=hi in
138. let (lis’,...) = mereo_H(h) in lis’ = map(hi)
138. ais \( \subseteq \) ais \&...
138. end end

Please note the fundamental difference between the road net (development & maintenance) graph and the road net. The latter pretends to be “the real thing”. The former is “just” an abstraction thereof!

139 The road net mereology (“bypasses”) the hub and link aggregates, and comprises a set of hub identifiers and a set of link identifiers – of the road net.

\[
\text{H}_\text{Mer} = \text{AI}-\text{set} \times \text{LI}-\text{set}
\]

\[
\text{mereo}_\text{RN} : \text{RN} \rightarrow \text{RNMer}
\]

axiom

139. \( \forall \) rts:RTS • let (_,lis) = mereo_H(obs_RN(rts)) in lis \( \subseteq \) lis end

value

140 The road net [administrator] behaviour,
141 amongst other activities (…)
142 internal non-deterministically decides upon

- a either a hub insertion,
- b or a link insertion,
- c or a hub removal,
- d or a link removal;

These four sub-behaviours each resume being the road net behaviour.

---

58 The presentation of the road net Behaviour, rn, is simplified.
99 We presently abstract from what this information is.
100 See footnote 99.
101 This is a repeat of the hub mereology given in Item 74 on page 66.
value
140. \( \text{rn}: \text{RNI} \rightarrow \text{RNMer} \rightarrow \text{G} \rightarrow \text{in.out}([\text{ch}[i,j][i,j]\subseteq \text{uid}_o]) \)
140. \( \text{rn(rni)(rnmer)}(g) \equiv \)
141. ... 
142a. \( \exists \text{insert}_{\text{hub}}(g)(\text{rni})(\text{rnmer}) \)
142b. \( \exists \text{insert}_{\text{link}}(g)(\text{rni})(\text{rnmer}) \)
142c. \( \exists \text{remove}_{\text{hub}}(g)(\text{rni})(\text{rnmer}) \)
142d. \( \exists \text{remove}_{\text{link}}(g)(\text{rni})(\text{rnmer}) \)

Details on the insert and remove actions are given below.

143 These road net sub-behaviours require information about:

a a hub to be inserted: its initial state, state space and [empty] traffic history, or
b a link to be inserted: its length, initial state, state space and [empty] traffic history, or
c a hub to be removed: its unique identifier, or
d a link to be removed: its unique identifier.

type
143. \( \text{Info} == \text{nHInfo} | \text{nLInfo} | \text{oHInfo} | \text{oLInfo} \)
143. \( \text{nHInfo} :: \text{H} \times \text{H} \times \text{H} \times \text{H}_\text{Traffic} \)
143. \( \text{nLInfo} :: \text{LEN} \times \text{L} \times \text{L} \times \text{L} \times \text{L}_\text{Traffic} \)
143. \( \text{oHInfo} :: \text{HI} \)
143. \( \text{oLInfo} :: \text{LI} \).

Example 71. Road Net Development: Hub Insertion: Road net development alternates between design, based on the road net (development & maintenance) graph, and actual, "real life", construction taking place in the real surroundings of the road net.

144 If a hub insertion then the road net behaviour, based on the hub and link information and the road net layout in the road net (development & maintenance) graph selects:

a an initial mereology for the hub, \( h_{\text{mer}} \),
b an initial hub state, \( h_{\sigma} \), and
c an initial hub state space, \( h_{\omega} \), and
d an initial, i.e., empty hub traffic history;

145 updates its road net (development & maintenance) graph with information about the new hub,

146 and results in a suitable grouping of these.

value
144. \( \text{design}_{\text{new}_{\text{hub}}}: \text{G} \rightarrow (\text{nHInfo} \times \text{G}) \)
144. \( \text{design}_{\text{new}_{\text{hub}}}(g) \equiv \)
144a. \( \text{let } h_{\text{mer}}:\text{H} = \text{M}_{\text{ih}}(g), \)
144b. \( h_{\sigma}:\text{H} = \text{S}_{\text{ih}}(g), \)
144c. \( h_{\omega}:\text{H} = \text{O}_{\text{ih}}(g), \)
144d. \( h_{\text{traffic}} = [], \)
145. \( g' = \text{MSO}_{\text{ih}}(g) \) in
146. \((h_{\text{mer}},h_{\sigma},h_{\omega},h_{\text{traffic}}),g') \) end

We leave open, in Items 144a–144c, as to what the initial hub mereology, state and state space should be initialised, i.e., the \( \text{M}_{\text{ih}},\text{S}_{\text{ih}},\text{O}_{\text{ih}} \) and \( \text{MSO}_{\text{ih}} \) functions.
6.5 Discrete Dynamic Domains

147. To insert a new hub the road net administrator
   a first designs the new hub,
   b then selects a hub part
   c which satisfies the design,
   whereupon it updates the global states
   d of parts $\sigma$,
   e of unique identifiers, and
   f of hub identifiers –
   in parallel, and in parallel with
148. initiating a new hub behaviour
149. and resuming being the road net behaviour.

147. insert_hub: $G \times RNI \times RNMer \rightarrow \text{Unit}$
147. insert_hub(g,rni,rnmer) ≡
147a. let \((h_{\text{mer}}, h_{\sigma}, h_{\omega}, h_{\text{traffic}}), g') = \text{design\_new\_hub}(g)\) in
147b. let $h: H \cdot h \notin \sigma \cdot$
147c. mereo$_H(h) = h_{\text{mer}} \land h_{\sigma} = \text{attr}_H \Sigma(h) \land$
147c. $h_{\omega} = \text{attr}_H \Omega(h) \land h_{\text{traffic}} = \text{attr}_H \text{Traffic}(h)$ in
147d. $\sigma := \sigma \cup \{h\}$
147e. $\parallel \text{uid}_\sigma := \text{uid}_\sigma \cup \{\text{uid}_H(h)\}$
147f. $\parallel \text{his} := \text{his} \cup \{\text{uid}_H(h)\}$
148. $\parallel \text{hub}(\text{uid}_H(h))(\text{attr}_H \Sigma(h), \text{attr}_H \Omega(h), \text{attr}_H \text{Traffic}(h))$
149. $\parallel \text{rn}(\text{rni})(\text{rnmer})(g')$
147. end end .

Example 72. Road Net Development: Link Insertion:

150. If a link insertion then the road net behaviour based on the hub and link information and
151. the road net layout in the road net (development & maintenance) graph selects
   a the mereology for the link, $h_{\text{mer}}$\textsuperscript{102},
   b the (static) length (attribute),
   c an initial link state, $l_{\sigma}$,
   d an initial link state space $l_{\omega}$, and
   e and initial, i.e., empty, link traffic history;
151. updates its road net (development & maintenance) graph with information about the new
152. link, and results in a suitable grouping of these.

val
150. design\_new\_link: $G \rightarrow (n\text{LInfo} \times G)$
150. design\_new\_link(g) ≡
150a. let $l_{\text{mer}}: L\text{Mer} = M\text{er}_l(g)$,
150b. $l_e: L\text{EN} = L\text{en}_l(g)$,
150c. $l_{\sigma}: L\Sigma = S\text{igma}_l(g)$,
150d. $l_{\omega}: L\Omega = O\text{mega}_l(g)$,
150e. $l_{\text{his}}: L\text{Hist} = []$
151. $g': G = M\text{LSO}^l_l(g)\text{ in}$
152. $(l_{\text{mer}}, l_e, l_{\sigma}, l_{\omega}, l_{\text{his}}), g'$ end

\textsuperscript{102} that is, the two existing hub identifiers between whose hubs the new link is to be inserted
We leave open, in Items 150a–150d, as to what the initial link mereology, state and state space should be initialised.

To insert a new link the road net administrator

- first designs the new link,
- then selects a link part
- which satisfies the design,
- whereupon it updates the global states
- of parts, $\sigma$,
- of unique part identifiers, and
- of link identifiers –

in parallel, and in parallel with

initiating a new link behaviour and

updating the mereologies and possibly the state and the state space attributes of the connected hubs.

We leave undefined the mereology and the state $\sigma$ and state space $\omega$ update functions.

### 6.5.1.2 Destroy Entities

The introduction to Sect. 6.5.1.1 on page 100 on the creation of entities outlined a number of creation issues $\{A, B, C, D\}$. For the destruction of entities description matters are a bit simpler. It is, almost, simply a matter of designating, by its unique identifier, the entity: part and behaviour to be destroyed. Almost! The mereology of the destroyed entity must be such that the destruction does not leave “dangling” references!

**Example 73. Road Net Development: Hub Removal:**

If a hub removal then the road net design\_remove\_hub behaviour, based on the road net (development & maintenance) graph, calculates the unique hub identifier of the “isolated” hub to be removed – that is, is not connected to any links,

updates the road net (development & maintenance) graph, and

results in a pair of these.

We leave undefined the mereology and the state $\sigma$ and state space $\omega$ update functions.
6.5 Discrete Dynamic Domains

156. \( \text{let } h_1 : H \cdot h_1 \in \text{his} \land \text{let } (\_\_\_\text{lis}) = \text{mero}_H(\text{retr}_\text{part}(hi)) \text{ in lis} = \{\} \text{ end in} \)
157. \( \text{let } g' = M_{hi}(hi,g) \text{ in} \)
158. \( (hi,g') \text{ end end} \)

159. To remove a hub the road net administrator

\[ \begin{align*} &\text{a first designs which old hub is to be removed} \\
&\text{b then removes the designated hub,} \\
&\text{c whereupon it updates the global states} \\
&\text{d of parts } \sigma, \\
&\text{e of unique identifiers, and} \\
&\text{f of hub identifiers --} \\
&\text{in parallel, and in parallel with} \\
&\text{g stopping the old hub behaviour} \\
&\text{h and resuming being a road net behaviour.} \\
\end{align*} \]

value
159. remove_hub: G \to RNI \to RNMer \to Unit
159. remove_hub(g)(rni)(rnmer) \equiv
159a. \( \text{let } (hi,g') = \text{design}_\text{remove}_\text{hub}(g) \text{ in} \)
159b. \( \text{let } h : H \cdot \text{uid}_H(h) = hi \land ... \text{ in} \)
159c. \( \sigma := \sigma \setminus \{ h \} \)
159d. \( || \text{uid}_d := \text{uid}_d \setminus \{ hi \} || \)
159e. \( || \text{his} := \text{his} \setminus \{ hi \} || \)
159f. \( || \text{ch}[\text{rni,hi}] ! \text{mkStop()} \)
160. \( || \text{rn}(\text{rni})(\text{rnmer})(g') \)
159. \text{end end} \)

6.5.2 Adjustment of Creatable and Destructable Behaviours

When an entity is created or destroyed its creation, respectively destruction affects the mereo-
logically related parts and their behaviours. their mereology and possibly their programmable
state attributes need be adjusted. And when entities are destroyed their behaviours are
stopped! These entities are “informed” so by the creator/destructor entity – as was shown
in Examples 71–73. The next example will illustrate how such “affected” entities handle such
creator/destructor communication.

Example 74. Hub Adjustments: We have not yet illustrated hub (nor link) behaviours. Now
we have to!

162 The mereology of a hub is a triple: the identification of the set of auromibles that may enter
the hub, the identification of the set of links that connect to the hub, and the identification
of the road net.
163 The hub behaviour external non-deterministically ([]) alternates between
164 doing “own work”,
165 or accepting a stop “command” from the road net administrator, or
166 or accepting mereology & state update information,
167 or other.
type
162. \( \text{HMer} = \text{AI-set} \times \text{LI-set} \times \text{RNI} \)

value
162. \( \text{mereo}_H : H \rightarrow \text{HMer} \)
163. \( \text{hub} : \text{hi:H} \rightarrow (\text{auis, lis, rni}) : \text{HMer} \rightarrow h_\omega : H_\Omega \rightarrow (h_\sigma : H_\Sigma \times \text{ht: HTraffic}) \rightarrow \)
163. \( \{ \text{ch[hi,ui]} | \text{ui:(RNI)} \cdot \text{ui=} \text{rni} \lor \text{ui} \in \text{auis} \} \) Unit
163. \( \text{hub}(\text{hi})(\text{hm}:(\text{auis, lis, rni}))(h_\omega)(h_\sigma, \text{ht}) \equiv \)
164. ... 
165. \( \lceil \ l e t \ \text{mkStop}() = \text{ch[hi, rni]} \ ? \ \text{in} \ \text{stop} \ \text{end} \)
166. \( \lceil \ l e t \ \text{mkUpdH}(\text{hm}', h_\sigma', h_\sigma') = \text{ch[} \text{rni, hi]} \ ? \ \text{in} \)
166. \( \text{hub}(\text{hi})(\text{hm'})(h_\omega')(h_\sigma', \text{ht}) \) end
167. ... 

Observe from formula Item 165 that the hub behaviour ends, whereas “from” Item 166 it tail recurses!

6.5.3 Summary on Creatable & Destructable Entities

We have sketched how we may model the dynamics of creating and destroying entities. It is, but a sketch. We should wish for a more methodological account. So, that is what we are working on – amongst other issues – at the moment.

6.6 Domain Engineering: Description and Construction

There are two meanings to the term ‘Domain Engineering’.

- the construction of descriptions of domains, and
- the construction of domains.

Most sections of Chapters 4–6 are “devoted” to the former; the previous section, Sect. 6.5 to the latter.

6.7 Domain Laws

The\(^{103}\) issue of domain laws seems to be crucial. Inklings of domain laws have been hinted at: (i) intentional pulls, Sect. 5.5 and (ii) Galois Connections (?), Sect. 5.5.5.

More to come

6.8 A Domain Discovery Procedure, III

The predecessors of this section are Sects. 4.8.2 on page 54 and 5.6 on page 87.

\(^{103}\) This section is currently under consideration.
6.8.1 Review of the Endurant Analysis and Description Process

The discover... functions below were defined in Sects. 4.8.2 on page 54 and 5.6 on page 87.

```plaintext
value
   endurant_analysis_and_description: Unit → Unit
   endurant_analysis_and_description() ≡
       discover_sorts(); [Page 54]
       discover_internal_endurant_qualities() [Page 87]
```

We are now to define a perdurant_analysis_and_description procedure – to follow the above endurant_analysis_and_description procedure.

6.8.2 A Domain Discovery Process, III

We define the perdurant_analysis_and_description procedure in the reverse order of that of Sect. 5.6 on page 87, first the full procedure, then its sub-procedures.

```plaintext
value
   perdurant_analysis_and_description: Unit → Unit
   perdurant_analysis_and_description() ≡
       discover_state();  axiom ... [ Note (a) ]
       discover_channels();  axiom ... [ Note (b) ]
       discover_behaviour_signatures();  axiom ... [ Note (c) ]
       discover_behaviour_definitions();  axiom ... [ Note (d) ]
       discover_initial_system()  axiom ... [ Note (e) ]
```

Notes:

- (a) **The States**: \( \sigma \) and \( \mu_e \). We refer to Sect. 4.7.2 on page 53 and Sect. 5.2.4 on page 62. The state calculation, as shown on Page 52, must be replicated, i.e., re-discovered, in any separate domain analysis & description. The purpose of the state, i.e., \( \sigma \), is to formulate appropriate axiomatic constraints and domain laws.

- (b) **The Channels**: We refer to Sects. 6.1.2 on page 93 and 6.2 on page 93. Thus we indiscriminately declare a channel for each pair of distinct unique part identifiers whether the corresponding pair of part behaviours, if at all invoked, communicate or not.

- (c) **Behaviour Signatures**: We refer to Sect. 6.3.1.2 on page 94. We find it more productive to first settle on the signatures of all behaviours – careful thinking has to go into that – before tackling the far more time-consuming work on defining the behaviours.

- (d) **Behaviour Definitions**: We refer to Sect. 6.3.3 on page 96.

- (e) **The Running System**: We refer to Sect. 6.4 on page 98.
6.9 Summary

Perdurants: Analysis & Description

- **Domain Discovery**: The procedures being described here, informally, guides the domain analyser cum describer to do the job!

We have basically finished our listings of the procedural steps of the domain engineering methodology of this *primer*!

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Chapter 7
Closing

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7.1 Summary

7.2 Domain Facets

There are other, additional methodological domain modelling steps. In [41, Chapter 8, Pages 205–240] we cover the notion of domain facets. By a domain facet we shall understand one amongst a finite set of generic ways of analysing a domain: a view of the domain, such that the different facets cover conceptually different views, and such that these views together cover the domain. We there list intrinsics, support technologies, rules & regulations, scripts, license languages, management & organisation, and human behaviour, as such facets. The referenced chapter ([41, Chapter 8, Pages 205–240]) is traditional, programming methodological, in the sense that there is no [semi-]formal calculi involved, as in this primer’s Chapters 4–5, I could wish for that!

7.3 Requirements Engineering

Domain modelling, to repeat, can be pursued for two different, but related, reasons. (i) simply, without any concern for, or idea of possible software, in order to “just” understand a domain, or (ii) for reasons of subsequent software development. In the later case a step of requirements engineering need be pursued. [41, Chapter 9, Pages 243–298] covers a notion of requirements engineering. In that chapter we show three stages of requirements development: (α) domain
requirements, (β) interface requirements, and (γ) machine requirements. But first a definition of the term ‘machine’. By machine we shall understand a, or the, combination of hardware and software that is the target for, or result of the required computing systems development. By a requirements we shall understand (cf., IEEE Standard 610.12): “A condition or capability needed by a user to solve a problem or achieve an objective.”  ■ By a domain requirements we shall understand those requirements which can be expressed solely using terms of the domain.  ■ By an interface requirements we shall understand those requirements which can be expressed only using technical terms of both the domain and the machine.  ■ By a machine requirements we shall understand those requirements which, in principle, can be expressed solely using terms of the machine.

The domain requirements stage of requirements development starts with a basis in the domain engineering’s domain description. It is, so-to-speak, a first step in the development of a requirements prescription.104 From there follows, according to [41, Chapter 9] a number of (five) steps: (1.) projection: By projection is meant a subset of the domain description, one which projects out all those endurants: parts and fluids, as well as perdurants: actions, events and behaviours that the stakeholders do not wish represented or relied upon by the machine.  ■ (2.) instantiation: By instantiation we mean a refinement of the partial domain requirements prescription (resulting from the projection step) in which the refinements aim at rendering more concrete, more specific the endurants: parts and fluids, as well as the perdurants: actions, events and behaviours of the domain requirements prescription.  ■ (3.) determination: By determination we mean a refinement of the partial domain requirements prescription, resulting from the instantiation step, in which the refinements aim at rendering less non-determinate, more determinate the endurants: parts and fluids, as well as the perdurants: functions, events and behaviours of the partial domain requirements prescription.  ■ (4.) extension: By extension we understand the introduction of endurants and perdurants that were not feasible in the original domain, but for which, with computing and communication, and with new, emerging technologies, for example, sensors, actuators and satellites, there is the possibility of feasible implementations, hence the requirements, that what is introduced becomes part of the unfolding requirements prescription.  ■ (5.) fitting: By requirements fitting we mean a harmonisation of two or more domain requirements that have overlapping (shared) not always consistent parts and which results in n partial domain requirement, and m shared domain requirement, that “fit into” two or more of the partial domain requirements.

[41, Chapter 9] then goes on to outline interface and machine requirements steps. So domain engineering is a sound basis, we claim, for software development.

How that basis harmonises with the approaches taken by Axel van Lamsweerde [88] and Michael A. Jackson [85] is, really, a worthwhile study in-and-by itself!

7.4 Possible [PhD] Research Topics

I list here a number of possible (PhD) research topics:

---

104 The “passage” from domain description to requirements prescription marks a transcendental deduction. Domain descriptions designate that which is being described. Requirements prescriptions designate what is intended to be implemented by computing. Please note the distinction: At the end of the development of a domain description we have just that: a domain description. At the beginning of the development of a requirements prescription we consider the domain description to be the initial requirements prescription: “Thus, seemingly bewildering, in one instance a document is considered a domain description, in the next instance, without that document having been textually changed, it is now considered a requirements prescription. The transition from domain description to requirements prescription also marks a transition from “no-design mode” description to “design-mode” prescription.
7.5 What Next?

1 **Intentional Pull:** This topic is not treated to the depth it deserves in this primer. Try think of intentional pulls in several domains: (i) money flow in financial institutions (while domain modelling a fair selection of such: banks, credit card companies, brokers, stock exchanges [26], etc.); (ii) railway systems (study, for example, [50, 47, 11, 96, 44, 115, 101, 10, 46]); and (iii) container terminals (see [36]).

2 **Discrete vs. Continuous Endurants and Perdurants:** Take the example of (oil, gas, water) pipelines. See draft report [28, Pipelines – a Domain, 2013]. Try model the dynamic flow of liquid in pipes, valves, pumps, etc., that is “mix”, as may be expected, differential equations with RSL formulas. Some have tried. No real progress seems attained. Se however [121, 122]. The pipeline example should illustrate the use of monitorable attributes, their “reading” and their “biddable updates”.

3 **Towards a Calculus of Perdurants:** This primer has unveiled the beginnings of a Calculus of Endurants. (Yet, its real “calculus-orientation” has yet to emerge: its laws, etc.) Sect. 6.3.3 hints at what I have in mind. A systematic analysis which aims at uncovering a fixed number of behaviour patterns such as sketched in Sect. 6.3.3.

4 **Modelling Human Interaction:** The “running example”, summarised in Appendix A, illustrated a road net “populated” with automobiles driving “hither & dither”. The current primer has not treated the interaction between humans and man-made artifacts, like, for example, drivers and their automobiles. You are to model, for example, such human actions as starting an automobile, accelerating, braking, turning left, turning right, and stopping. Doing so you will have to try out, experiment with the rôles of monitorable, including biddable automobile attributes. An aim, besides such a domain model, is to research method issues of modelling human interaction. Please disregard modelling issues of sentiments, feelings, etc.

5 **Transcendental Deduction:** In the philosophy of Kai Sørlander such as, for example, explained in Chapter 2, transcendental deduction is appealed to repeatedly. In this primer, as in [41], transcendental deduction is appealed to only once! Maybe research into possible calculi for perdurants, cf. Research Challenge 3, might yield some more examples of transcendental deductions.

6 **Formal Models of Domain Modelling Calculi:** In [30] I attempted a first formal model of the domain analysis & description calculi. With [41] and, especially, this primer as a background, perhaps a more thorough attempt should be made to bring the model of [30] up-to-date and complete!

7 **Kai Sørlander’s Philosophy:** We refer to Chapter 2. It is here strongly suggested that this research project be based on [112], Kai Sørlander’s most recent book.\(^\text{[105]}\) The challenge, in a sense, has two elements: (i) the identification of Sørlander’s use of transcendental deduction: painstakingly identifying all it uses, analysing each of these, studying whether one can characterise these uses into more than one common kind of deduction, or whether one might claim “classes of deductions”, not necessarily disjoint, but perhaps structured in some kind of taxonomy; and (ii) the analysis of this report’s presentation of Sørlander’s metaphysics.

---

\(^{[105]}\) All of Sørlander’s books [108, 109, 110, 111, 112, 1994–2022] are in Danish – so the researcher would either be able to read Danish, or, more preferably to me, to have a suitable (German, English, French, ...) translation at hand.
7.6 Acknowledgments

In [41, Preface/Acknowledgments, Page xiv] I acknowledged the very many who, over my professional life, has inspired me. In “rewriting” this primer from [41] I have, again, attempted to “capture” Kai Sørlander’s Philosophy, cf. Chapter 2. And again I wishes to deeply acknowledge that work and, hence, Kai Sørlander. Here I, additionally, wishes to acknowledge, with pleasure, Laura Kovacs, TU Wien. Laura invited me to lecture, in the fall of 2022\textsuperscript{106}, at TU Wien. This primer is the result of that invitation.

\textsuperscript{106} Well, an invitation for Covid-19 year 2021 had to be postponed!
Chapter 8

Bibliography

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8.1 Bibliographical Notes

Of the 30 citations given in Footnote 16, Pages 7–8 I have not read 20 of them, but have studied some of Kant’s, Russel’s, Wittgenstein’s and Popper’s writings. The dictionaries [3, 53, 81], as well as [90], have followed me for years.

8.2 References


54. Manuel Clavel, Francisco Durán, Steven Eker, Patrick Lincoln, Narciso Martí Oliet, José Meseguer, and Carolyn Talcott. Maude 2.6 Manual. Department of Computer Science, University of Illinois and Urbana-Champaign, Urbana-Champaign, Ill., USA, January 2011.


92. Theodore McCombs. Maude 2,0 Primer. Department of Computer Science, University of Illinois and Urbana-Champaign, Urbana-Champaign, Ill., USA, August 2003.

111. Kai Sørlander. Indføring i Filosofien [Introduction to The Philosophy]. Informations Forlag, Copenhagen, Denmark, 2016. 233 pages.
Appendix A
Road Transport

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A.1 The Road Transport Domain

Our universe of discourse in this chapter is the road transport domain.

A.1.1 Naming

type RTS

A.1.2 Rough Sketch

The road transport system that we have in mind consists of a road net and a set of vehicles such that the road net serves to convey vehicles. We consider the road net to consist of hubs, i.e., street intersections, or just street segment connection points, and links, i.e., street segments between adjacent hubs. We consider vehicles to additionally include departments of motor vehicles (DMVs), bus companies, each with zero, one or more buses, and vehicle associations, each with zero, one or more members who are owners of zero, one or more vehicles.

A.2 External Qualities

A Road Transport System, I – Manifest External Qualities: Our intention is that the manifest external qualities of a road transport system are those of its roads, their hubs, i.e., road (or street) intersections, and their links, i.e., the roads (streets) between hubs, and vehicles, i.e., automobiles – that ply the roads – the buses, trucks, private cars, bicycles, etc.

A.2.1 A Road Transport System, II – Abstract External Qualities

Examples of what could be considered abstract external qualities of a road transport domain are: the aggregate of all hubs and all links, the aggregate of all buses, say into bus companies, the aggregate of all DMVs, bus companies, buses and vehicle associations are. In presenting it here, as we are, we rely on your a priori understanding of these terms. But that is dangerous! The danger, if we do not painstakingly narrate and formalise what we mean by all these terms, then readers (software designers, etc.) may make erroneous assumptions.

1 This “rough” narrative fails to narrate what hubs, links, vehicles, DMVs, bus companies, buses and vehicle associations are. In presenting it here, as we are, we rely on your a priori understanding of these terms. But that is dangerous! The danger, if we do not painstakingly narrate and formalise what we mean by all these terms, then readers (software designers, etc.) may make erroneous assumptions.

2 We have highlighted certain endurant sort names – as they will re-appear in rather many upcoming examples.
the aggregate of all bus companies into public transport, and the aggregate of all vehicles
to a department of vehicles. Some of these aggregates may, at first be treated as abstract.
Subsequently, in our further analysis & description we may decide to consider some of them
as concretely manifested in, for example, actual departments of roads.

A.2.2 Transport System Structure

A transport system is modeled as structured into a road net structure and an automobile
structure. The road net structure is then structured as a pair: a structure of hubs and a
structure of links. These latter structures are then modeled as set of hubs, respectively links.

We could have modeled the road net structure as a composite part with unique identity,
mereology and attributes which could then serve to model a road net authority. And
we could have modeled the automobile structure as a composite part with unique identity,
mereology and attributes which could then serve to model a department of vehicles.

A.2.3 Atomic Road Transport Parts

From one point of view all of the following can be considered atomic parts: hubs, links\(^3\), and
automobiles.

A.2.4 Compound Road Transport Parts

A.2.4.1 The Composites

168 There is the universe of discourse, UoD, a road net, RN, and
It is structured into a fleet of vehicles, FV.

Both are structures. .................................................................

<table>
<thead>
<tr>
<th>type</th>
<th>value</th>
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<tr>
<td>168 UoD axiom</td>
<td>\forall uod:UoD \cdot is_structure(uod)</td>
</tr>
<tr>
<td>169 RN axiom</td>
<td>\forall rn:RN \cdot is_structure(rn)</td>
</tr>
<tr>
<td>170 FV axiom</td>
<td>\forall fv:FV \cdot is_structure(fv)</td>
</tr>
</tbody>
</table>

A.2.4.2 The Part Parts

171 The structure of hubs is a set, sH, of atomic hubs, H.
172 The structure of links is a set, sL, of atomic links, L.
173 The structure of buses is a set, sBC, of composite bus companies, BC.
174 The composite bus companies, BC, are sets of buses, sB.
175 The structure of private automobiles is a set, sA, of atomic automobiles, A.

\(^3\) Hub = street intersection; link = street segments with no intervening hubs.
type 171 H, sH = H-set axiom ∀ h:H • is_atomic(h) 172 L, sL = L-set axiom ∀ l:L • is_atomic(l) 173 BC, BCs = BC-set axiom ∀ bc:BC • is_composite(bc) 174 B, Bs = B-set axiom ∀ b:B • is_atomic(b) 175 A, sA = A-set axiom ∀ a:A • is_atomic(a)  
value 176 obs_sH: SH → sH 177 obs_sL: SL → sL 178 obs_sBC: SBC → BCs 179 obs Bs: BCs → Bs 180 obs_sA: SA → sA

A.2.5 The Transport System State

176 Let there be given a universe of discourse, rts. It is an example of a state. From that state we can calculate other states.
177 The set of all hubs, hs.
178 The set of all links, ls.
179 The set of all hubs and links, hls.
180 The set of all bus companies, bcs.
181 The set of all buses, bs.
182 The set of all private automobiles, as.
183 The set of all parts, ps.

value 176 rts:UoD [43] 177 hs:H-set ≡ H-set ≡ obs_sH(ops_sH(ops_RN(rts))) 178 ls:L-set ≡ L-set ≡ obs_sL(ops_sL(ops_RN(rts))) 179 hls:(H|L)-set ≡ hs∪ls
A.3 Internal Qualities

A.3.1 Unique Identifiers

184 We assign unique identifiers to all parts.
185 By a road identifier we shall mean a link or a hub identifier.
186 By a vehicle identifier we shall mean a bus or an automobile identifier.
187 Unique identifiers uniquely identify all parts.
   a All hubs have distinct unique identifiers.
   b All links have distinct identifiers.
   c All bus companies have distinct identifiers.
   d All buses of all bus companies have distinct identifiers.
   e All automobiles have distinct identifiers.
   f All parts have distinct identifiers.

\[ P = H \mid L \mid BC \mid B \mid A \]
\[ \varphi : H_{UI} \rightarrow H \mid L_{UI} \rightarrow L \mid BC_{UI} \rightarrow BC \mid B_{UI} \rightarrow B \mid A_{UI} \rightarrow A \]
\[ \varphi (ui) = \text{let } p : (H \mid L \mid BC \mid B \mid A) \rightarrow ps \text{ and } \text{uid}_p(p) = \text{ui} \text{ in } p \text{ end} \]

A.3.1.2 All Unique Identifiers of a Domain

We can calculate:
189 the set, \( h_{ui} \), of unique hub identifiers;
190 the set, \( l_{ui} \), of unique link identifiers;
191 the map, \( h_{ui}m \), from unique hub identifiers to the set of unique link identifiers of the links connected to the zero, one or more identified hubs,
192 the map, \( l_{ui}m \), from unique link identifiers to the set of unique hub identifiers of the two hubs connected to the identified link;
Uniqueness of Road Net Identifiers

We must express the following axioms:

- All hub identifiers are distinct.
- All link identifiers are distinct.
- All private automobile identifiers are distinct.
- All bus company identifiers are distinct.
- All bus identifiers are distinct.
- All company identifiers are distinct.

Axiom

- \( \text{card} h_s = \text{card} h_m \)
- \( \text{card} l_s = \text{card} l_m \)
- \( \text{card} bcs = \text{card} b_m \)
- \( \text{card} bs = \text{card} b_m \)
- \( \text{card} as = \text{card} a_m \)
- \( \text{card} [h_s \cup l_s \cup bc_m] = \text{card} a_m \)
- \( \text{card} h_s + \text{card} l_s + \text{card} bcs + \text{card} b_m + \text{card} a_m \).
A.3 Internal Qualities

A.3.2 Mereology

A.3.2.1 Mereology Types and Observers

206 The mereology of hubs is a pair: (i) the set of all bus and automobile identifiers, and (ii) the set of unique identifiers of the links that it is connected to and the set of all unique identifiers of all vehicles (buses and private automobiles).

207 The mereology of links is a pair: (i) the set of all bus and automobile identifiers, and (ii) the set of the two distinct hubs they are connected to.

208 The mereology of a bus company is a set the unique identifiers of the buses operated by that company.

209 The mereology of a bus is a pair: (i) the set of the single unique identifier of the bus company it is operating for, and (ii) the unique identifiers of all links and hubs.

210 The mereology of an automobile is the set of the unique identifiers of all links and hubs.

\[
\begin{array}{ll}
\text{type} & \text{value} \\
H_{\text{Mer}} & \text{mereo}_H: H \to H_{\text{Mer}} \\
L_{\text{Mer}} & \text{mereo}_L: L \to L_{\text{Mer}} \\
BC_{\text{Mer}} & \text{mereo}_{BC}: BC \to BC_{\text{Mer}} \\
B_{\text{Mer}} & \text{mereo}_B: B \to B_{\text{Mer}} \\
A_{\text{Mer}} & \text{mereo}_A: A \to A_{\text{Mer}} \\
\end{array}
\]

A.3.2.2 Invariance of Mereologies

For mereologies one can usually express some invariants. Such invariants express “law-like properties”, facts which are indisputable.

A.3.2.2.1 Invariance of Road Nets

The observed mereologies must express identifiers of the state of such for road nets:

\[
\begin{array}{ll}
\text{axiom} & \forall (\text{vuis},\text{luis}): H_{\text{Mer}} \cdot \text{luis} \subseteq \text{luis} \land \text{vuis} = \text{vuis} \\
\text{axiom} & \forall (\text{vuis},\text{huis}): L_{\text{Mer}} \cdot \text{vuis} = \text{vuis} \land \text{huis} \subseteq \text{huis} \land \text{cardhuis} = 2 \\
\text{axiom} & \forall \text{buis}: H_{\text{Mer}} \cdot \text{buis} = \text{buis} \\
\text{axiom} & \forall (\text{bcui},\text{ruis}): H_{\text{Mer}} \cdot \text{bcui} \in \text{bcui} \land \text{ruis} = \text{ruis} \\
\text{axiom} & \forall \text{ruis}: A_{\text{Mer}} \cdot \text{ruis} = \text{ruis} \\
\end{array}
\]

211 For all hubs, \( h \), and links, \( l \), in the same road net,
212 if the hub \( h \) connects to link \( l \) then link \( l \) connects to hub \( h \).

\[
\begin{array}{ll}
\text{axiom} & \forall h:H,L \cdot h \in h \land l \in l \\
\end{array}
\]

4 This is just another way of saying that the meaning of hub mereologies involves the unique identifiers of all the vehicles that might pass through the hub is of interest to it.

5 The link identifiers designate the links, zero, one or more, that a hub is connected to is of interest to both the hub and that these links is interested in the hub.

6 — that the bus might pass through

7 — that the automobile might pass through
213 For all links, $l$, and hubs, $h_a, h_b$, in the same road net, 
214 if the $l$ connects to hubs $h_a$ and $h_b$, then $h_a$ and $h_b$ both connects to link $l$.

\[ \forall h_a, h_b : H, l : L \cdot \{ h_a, h_b \} \subseteq ls \land l \in ls \Rightarrow \]

\[ \text{let } (_l\text{luis}) = \text{mereo}_H(h), (_l\text{huis}) = \text{mereo}_L(l) \]

\[ \text{in } uid_L(l) \text{luis} \equiv uid_H(h) \text{huis} \text{ end} \]

A.3.2.2.2 Possible Consequences of a Road Net Mereology

215 are there [isolated] units from which one can not “reach” other units?
216 does the net consist of two or more “disjoint” nets?
217 et cetera.

We leave it to the reader to narrate and formalise the above properly.

A.3.2.2.3 Fixed and Varying Mereology

Let us consider a road net. If hubs and links never change “affiliation”, that is: hubs are in fixed relation to zero one or more links, and links are in a fixed relation to exactly two hubs then the mereology is a fixed mereology. If, on the other hand hubs may be inserted into or removed from the net, and/or links may be removed from or inserted between any two existing hubs, then the mereology is a varying mereology.

A.3.3 Attributes

A.3.3.1 Hub Attributes

We treat some attributes of the hubs of a road net.

218 There is a hub state. It is a set of pairs, $(l_f, l_t)$, of link identifiers, where these link identifiers are in the mereology of the hub. The meaning of the hub state in which, e.g., $(l_f, l_t)$ is an element, is that the hub is open, “green”, for traffic from link $l_f$ to link $l_t$. If a hub state is empty then the hub is closed, i.e., “red” for traffic from any connected links to any other connected links.

219 There is a hub state space. It is a set of hub states. The current hub state must be in its state space. The meaning of the hub state space is that its states are all those the hub can attain.

220 Since we can think rationally about it, it can be described, hence we can model, as an attribute of hubs, a history of its traffic: the recording, per unique bus and automobile identifier, of the time ordered presence in the hub of these vehicles. Hub history is an event history.

\[ \text{type } \text{H}_r = (L\_Ul \times L\_Ul)\text{-set} \]

\[ \text{axiom} \]
A.3 Internal Qualities

\( \forall h : H \cdot \text{obs}_H\Sigma(h) \in \text{obs}_H\Omega(h) \)

type

\( H\Omega = H\Sigma\text{-set} \)

\( H\text{\textunderscore Traffic} = (A\text{\textunderscore UI}|B\text{\textunderscore UI}) \Rightarrow (\text{\textit{TIME}} \times \text{VPos})' \)

axiom

\( \forall h : H\text{\textunderscore Traffic}, u : (A\text{\textunderscore UI}|B\text{\textunderscore UI}) \cdot u \in \text{dom} h \Rightarrow \text{time\textunderscore ordered}(h(u)) \)

value

\( \text{attr}_H\Sigma : H \rightarrow H\Sigma \)

\( \text{attr}_H\Omega : H \rightarrow H\Omega \)

\( \text{attr}_H\text{\textunderscore Traffic} : H \rightarrow H\text{\textunderscore Traffic} \)

value

\( \text{time\textunderscore ordered} : (\text{\textit{TIME}} \times \text{VPos})' \rightarrow \text{Bool} \)

\( \text{time\textunderscore ordered}(tvpl) \equiv ... \)

In Item 220 on the preceding page we model the time-ordered sequence of traffic as a discrete sampling, i.e., \( \rightarrow \), rather than as a continuous function, \( \rightarrow \).

A.3.3.2 Invariance of Traffic States

221 The link identifiers of hub states must be in the set, \( l_{ui}s \), of the road net’s link identifiers.

axiom

\( \forall h : H \cdot h \in hs \Rightarrow \)

\( \text{let } h_{\sigma} = \text{attr}_H\Sigma(h) \text{ in} \)

\( \forall (l_{ui}, l_{ui}') : (L\text{\textunderscore UI} \times L\text{\textunderscore UI}) \cdot (l_{ui}, l_{ui}') \in h_{\sigma} \Rightarrow \{l_{ui}, l_{ui}'\} \subseteq l_{ui}s \text{ end} \)

A.3.3.3 Link Attributes

We show just a few attributes.

222 There is a link state. It is a set of pairs, \( (h_f, h_t) \), of distinct hub identifiers, where these hub identifiers are in the mereology of the link. The meaning of a link state in which \( (h_f, h_t) \) is an element is that the link is open, “green”, for traffic from hub \( h_f \) to hub \( h_t \). Link states can have either 0, 1 or 2 elements.

223 There is a link state space. It is a set of link states. The meaning of the link state space is that its states are all those the which the link can attain. The current link state must be in its state space. If a link state space is empty then the link is (permanently) closed. If it has one element then it is a one-way link. If a one-way link, \( l \), is imminent on a hub whose mereology designates that link, then the link is a “trap”, i.e., a “blind cul-de-sac”.

224 Since we can think rationally about it, it can be described, hence it can model, as an attribute of links a history of its traffic: the recording, per unique bus and automobile identifier, of the time ordered positions along the link (from one hub to the next) of these vehicles.

225 The hub identifiers of link states must be in the set, \( h_{ui}s \), of the road net’s hub identifiers.

type

\( L\Sigma = H\text{\textunderscore UI}\text{-set} \)

axiom
A.3.3.4 Bus Company Attributes

Bus companies operate a number of lines that service passenger transport along routes of the road net. Each line being serviced by a number of buses.

Bus companies create, maintain, revise and distribute [to the public (not modeled here), and to buses] bus time tables, not further defined.

A.3.3.5 Bus Attributes

We show just a few attributes.

Buses run routes, according to their line number, ln:LN, in the bus time table, btt:BusTimTbl obtained from their bus company, and and keep, as inert attributes, their segment of that time table.

Buses occupy positions on the road net:

- either at a hub identified by some \( h_{ui} \),
- or on a link, some fraction, \( f:\text{Fract} \), down an identified link, \( L_{ui} \), from one of its identified connecting hubs, \( fh_{ui} \), in the direction of the other identified hub, \( th_{ui} \).

Et cetera.
### A.3 Internal Qualities

229 \[ \text{BPos} == \text{atHub} \mid \text{onLink} \]
229a \[ \text{atHub} :: \text{h}_{ui} : \text{H}_{UI} \]
229b \[ \text{onLink} :: \text{fh}_{ui} : \text{H}_{UI} \times \text{l}_{ui} : \text{L}_{UI} \times \text{frac} : \text{Fract} \times \text{th}_{ui} : \text{H}_{UI} \]
229b \[ \text{Fract} = \text{Real}, \text{axiom frac} : \text{Fract} \cdot 0 < \text{frac} < 1 \]
230

value
228 \[ \text{attr}_\text{BusTimTbl}: \text{B} \rightarrow \text{BusTimTbl} \]
229 \[ \text{attr}_\text{BPos}: \text{B} \rightarrow \text{BPos} \]

#### A.3.3.6 Private Automobile Attributes

We illustrate but a few attributes:

231 Automobiles have static number plate registration numbers.
232 Automobiles have dynamic positions on the road net:

- [229a] either at a hub identified by some \( h_{ui} \),
- [229b] or on a link, some fraction, frac:Fract down an identified link, \( l_{ui} \), from one of its identified connecting hubs, \( fh_{ui} \), in the direction of the other identified hub, \( th_{ui} \).

**type**

231 \[ \text{RegNo} \]
232 \[ \text{APos} == \text{atHub} \mid \text{onLink} \]
229a \[ \text{atHub} :: \text{h}_{ui} : \text{H}_{UI} \]
229b \[ \text{onLink} :: \text{fh}_{ui} : \text{H}_{UI} \times \text{l}_{ui} : \text{L}_{UI} \times \text{frac} : \text{Fract} \times \text{th}_{ui} : \text{H}_{UI} \]
229b \[ \text{Fract} = \text{Real}, \text{axiom frac} : \text{Fract} \cdot 0 < \text{frac} < 1 \]

**value**

231 \[ \text{attr}_\text{RegNo}: \text{A} \rightarrow \text{RegNo} \]
232 \[ \text{attr}_\text{APos}: \text{A} \rightarrow \text{APos} \]

Obvious attributes that are not illustrated are those of velocity and acceleration, forward or backward movement, turning right, left or going straight, etc. The acceleration, deceleration, even velocity, or turning right, turning left, moving straight, or forward or backward are seen as command actions. As such they denote actions by the automobile — such as pressing the accelerator, or lifting accelerator pressure or braking, or turning the wheel in one direction or another, etc. As actions they have a kind of counterpart in the velocity, the acceleration, etc. attributes. Observe that bus companies each have their own distinct bus time table, and that these are modeled as programmable, Item 226 on the facing page, page 128. Observe then that buses each have their own distinct bus time table, and that these are model-led as inert, Item 228 on the preceding page, page 128. In Items 92 Pg. 74 and 96 Pg. 75, we illustrated an aspect of domain analysis & description that may seem, and at least some decades ago would have seemed, strange: namely that if we can think, hence speak, about it, then we can model it “as a fact” in the domain. The case in point is that we include among hub and link attributes their histories of the timed whereabouts of buses and automobiles.\(^8\)

---

\(^8\) In this day and age of road cameras and satellite surveillance these traffic recordings may not appear so strange: We now know, at least in principle, of technologies that can record approximations to the hub and link traffic attributes.
A.3.3.7 Intentionality

233 Seen from the point of view of an automobile there is its own traffic history, $A_{\text{Hist}}$, which is a (time ordered) sequence of timed automobile’s positions; 
234 seen from the point of view of a hub there is its own traffic history, $H_{\text{Traf}}$, which is a (time ordered) sequence of timed maps from automobile identities into automobile positions; and
235 seen from the point of view of a link there is its own traffic history, $L_{\text{Traf}}$, which is a (time ordered) sequence of timed maps from automobile identities into automobile positions.

The *intentional “pull”* of these manifestations is this:

236 The union, i.e. proper merge of all automobile traffic histories, $\text{AllA}_{\text{TH}}$, must now be identical to the same proper merge of all hub, $\text{AllH}_{\text{TH}}$, and all link traffic histories, $\text{AllL}_{\text{TH}}$.

```
type
233  A_{\text{Hi}} = (T \times \text{APos})^*
220  H_{\text{Traf}} = A_{\text{UI}} \mapsto (\text{TIME} \times \text{APos})^*
224  L_{\text{Traf}} = A_{\text{UI}} \mapsto (\text{TIME} \times \text{APos})^*
236  \text{AllA}_{\text{TH}} = \text{TIME} \mapsto m(A_{\text{UI}} \mapsto m(\text{APos}))
236  \text{AllHTH} = \text{TIME} \mapsto m(A_{\text{UI}} \mapsto m(\text{APos}))
236  \text{AllL}_{\text{TH}} = \text{TIME} \mapsto m(A_{\text{UI}} \mapsto m(\text{APos}))
axiom
236  let \text{allA} = \text{mrg\_AllA}(\{(a,\text{attr\_A\_Hi(a)})|a:A \cdot a \in a_s\}),
236  allH = \text{mrg\_AllHTH}(\{(\text{attr\_H\_Traf(h)})|h:H \cdot h \in h_s\}),
236  allL = \text{mrg\_AllLTH}(\{(\text{attr\_L\_Traf(l)})|l:L \cdot l \in l_s\}) \text{ in}
236  \text{allA} = \text{mrg\_HLT}(\text{allH,allL}) \text{ end}
```

We leave the definition of the four merge functions to the reader! We endow each automobile with its history of timed positions and each hub and link with their histories of timed automobile positions. These histories are facts! They are not something that is laboriously recorded, where such recordings may be imprecise or cumbersome. The facts are there, so we can (but may not necessarily) talk about these histories as facts. It is in that sense that the purpose (*transport*) for which man let automobiles, hubs and link be made with their *transport* intent are subject to an *intentional “pull”*. It can be no other way: if automobiles “record” their history, then hubs and links must together “record” identically the same history!.

**Intentional Pull – General Transport:** These are examples of human intents: they create roads and automobiles with the intent of *transport*, they create houses with the intents of living, offices, production, etc., and they create pipelines with the intent of oil or gas transport.

A.4 Perdurants

In this section we transcendentally “morph” parts into behaviours. We analyse that notion and its constituent notions of actors, channels and communication, actions and events.

The main transcendental deduction of this chapter is that of associating with each part a behaviour. This section shows the details of that association. Perdurants are understood in terms of a notion of state and a notion of time.

---

* or thought technologically in-feasible – at least some decades ago!
State Values versus State Variables: Item 183 on page 122 expresses the value of all parts of a road transport system:

183. \( ps: (UoB|H|L|BC|B|A)\text{-set} \equiv rts \cup hls \cup bcs \cup bs \cup as. \)

237 We now introduce the set of variables, one for each part value of the domain being modeled.

237. \{ variable \( vp: (UoB|H|L|BC|B|A) \mid vp \in ps \}\}

Buses and Bus Companies A bus company is like a “root” for its fleet of “sibling” buses. But a bus company may cease to exist without the buses therefore necessarily also ceasing to exist. They may continue to operate, probably illegally, without, possibly, a valid bus driving certificate. Or they may be passed on to either private owners or to other bus companies. We use this example as a reason for not endowing a “block structure” concept on behaviours.

A.4.1 Channels and Communication

A.4.1.1 Channel Message Types

We ascribe types to the messages offered on channels.

238 Hubs and links communicate, both ways, with one another, over channels, \( h\_ch \), whose indexes are determined by their mereologies.

239 Hubs send one kind of messages, links another.

240 Bus companies offer timed bus time tables to buses, one way.

241 Buses and automobiles offer their current, timed positions to the road element, hub or link they are on, one way.

242 This justifies the channel declaration which is calculated to be:

channel

\[
\{ h\_ch[hui,ui]:HL\_Msg \mid hui:HLUI, \text{ui}:LUI \in h_{ui}s \land \text{ui} \in lh_{ui}m(hui) \} \cup \{ h\_ch[hui,ui]:LH\_Msg \mid hui:HLUI, \text{ui}:LUI \in l_{ui}s \land \text{ui} \in lh_{ui}m(hui) \}
\]

We shall argue for bus company-to-bus channels based on the mereologies of those parts. Bus companies need communicate to all its buses, but not the buses of other bus companies. Buses of a bus company need communicate to their bus company, but not to other bus companies.

243 This justifies the channel declaration which is calculated to be:

channel

\[
\{ bc\_ch[bcui,ui]:BC\_B\_Msg \mid bcui:BCUI, \text{ui}:BUI \in bc_{ui}s \land \text{ui} \in bc_{ui}m(\text{ui}) \}
\]

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We shall argue for vehicle to road element channels based on the mereologies of those parts. Buses and automobiles need communicate to all hubs and all links.

This justifies the channel declaration which is calculated to be:

\[
\text{channel} \quad \{ \text{v}_r \text{ch}[\text{v}_{ui},r_{ui}] \mid \text{v}_{ui} : \text{V}_{UI},r_{ui} : \text{R}_{UI} \cdot \text{v}_{ui} \in v_{ui}s \land r_{ui} \in r_{ui}s \} : \text{V}_R \text{Msg}
\]

### A.4.2 Behaviours

#### A.4.2.1 Road Transport Behaviour Signatures

We first decide on names of behaviours. In the translation schemas we gave schematic names to behaviours of the form $M_P$. We now assign mnemonic names: from part names to names of transcendentally interpreted behaviours and then we assign signatures to these behaviours.

#### A.4.2.1.1 Hub Behaviour Signature

245 $\text{hub}_{hui}$:

- a there is the usual “triplet” of arguments: unique identifier, mereology and static attributes;
- b then there are the programmable attributes;
- c and finally there are the input/output channel references: first those allowing communication between hub and link behaviours,
- d and then those allowing communication between hub and vehicle (bus and automobile) behaviours.

\[
\text{value} \quad 245 \quad \text{hub}_{hui} : \\
245a \quad h_{ui} : \text{H}_{UI} \times (vuis,luis,) : \text{H}_{Mere} \times H \Omega \\
245b \quad \rightarrow (H \Sigma \times H_{\text{Traffic}}) \\
245c \quad \rightarrow \text{in,out} \{ \text{h}_{ui} \text{ch}[h_{ui},l_{ui}] \mid l_{ui} : \text{L}_{UI} \land l_{ui} \in luis \} \\
245d \quad \{ \text{bar}_r \text{ch}[h_{ui},v_{ui}] \mid v_{ui} : \text{V}_{UI} \land v_{ui} \in vuis \} \text{ Unit} \\
245a \quad \text{pre} : vuis = v_{ui}s \land luis = l_{ui}s
\]

#### A.4.2.1.2 Link Behaviour Signature

246 $\text{link}_{lui}$:

- a there is the usual “triplet” of arguments: unique identifier, mereology and static attributes;
- b then there are the programmable attributes;
- c and finally there are the input/output channel references: first those allowing communication between hub and link behaviours,
- d and then those allowing communication between link and vehicle (bus and automobile) behaviours.
A.4 Perdurants

value

246 \( \text{link}_{ui}^{lu} : \text{Lui} \times (\text{vuis}, \text{huis}, \_): \text{Lmer} \times \Omega \)

246b \( \rightarrow (\text{L} \times \text{Ltraffic}) \)

246c \( \rightarrow \text{in.out} | h_{lu} = h_{ui}, \text{ui} \in \text{huis} \)

246d \( \{ \text{ba}_{ui} : \text{ui} : \text{vuis}, \text{ui} \in \text{vuis} \} \text{ Unit} \)

246a \( \text{pre: vuis} = \text{vuis} \wedge \text{huis} = \text{huis} \)

A.4.2.1.3 Bus Company Behaviour Signature

247 \( \text{bus}_\text{company}_{bcui} : \)

\( \begin{align*} 
\text{a} & \: \text{there is here just a “doublet” of arguments: unique identifier and mereology;} \\
\text{b} & \: \text{then there is the one programmable attribute;} \\
\text{c} & \: \text{and finally there are the input/output channel references allowing communication between the bus company and buses.} 
\end{align*} \)

value

247 \( \text{bus}_\text{company}_{bcui} : \)

247a \( \text{bc}_{ui} : \text{BCui} \times (\_ , \text{buis}) : \text{BCmer} \)

247b \( \rightarrow \text{BusTimTbl} \)

247c \( \text{in.out} (\text{bc}_{ui}, \text{bui}) | \text{bui} : \text{Bui} \wedge \text{bui} \in \text{buis} \) \text{ Unit} 

247a \( \text{pre: buis} = \text{buis} \wedge \text{huis} = \text{huis} \)

A.4.2.1.4 Bus Behaviour Signature

248 \( \text{bus}_{bui} : \)

\( \begin{align*} 
\text{a} & \: \text{there is here just a “doublet” of arguments: unique identifier and mereology;} \\
\text{b} & \: \text{then there are the programmable attributes;} \\
\text{c} & \: \text{and finally there are the input/output channel references: first the input/output allowing communication between the bus company and buses,} \\
\text{d} & \: \text{and the input/output allowing communication between the bus and the hub and link behaviours.} 
\end{align*} \)

value

248 \( \text{bus}_{bui} : \)

248a \( \text{bui} : \text{Bui} \times (\text{bc}_{ui}, \_ , \text{ruis}) : \text{BJmer} \)

248b \( \rightarrow (\text{LN} \times \text{BTT} \times \text{BPOS}) \)

248c \( \rightarrow \text{out bc}_{ui} , \text{bui} | \text{bui} : \text{Bui} \wedge \text{bui} \in \text{buis} \)

248d \( \{ \text{ba}_{ui} : h_{ui} = h_{ui}, \text{ui} : \text{Hui} \wedge \text{ui} \in \text{vuis} \} \text{ Unit} \)

248a \( \text{pre: ruis} = \text{ruis} \wedge \text{bui} \in \text{bui} \)

A.4.2.1.5 Automobile Behaviour Signature

249 \( \text{automobile}_{aui} : \)

\( \begin{align*} 
\text{a} & \: \text{there is the usual “triplet” of arguments: unique identifier, mereology and static attributes;} 
\end{align*} \)
b then there is the one programmable attribute;
c and finally there are the input/output channel references allowing communication between the automobile and the hub and link behaviours.

A.4.2.2 Behaviour Definitions

We only illustrate automobile, hub and link behaviours.

A.4.2.2.1 Automobile Behaviour at a Hub

We define the behaviours in a different order than the treatment of their signatures. We “split” definition of the automobile behaviour into the behaviour of automobiles when positioned at a hub, and into the behaviour automobiles when positioned at on a link. In both cases the behaviours include the “idling” of the automobile, i.e., its “not moving”, standing still.

We abstract automobile behaviour at a Hub (hui).

The vehicle remains at that hub, “idling”,

informing the hub behaviour,

or, internally non-deterministically,

a moves onto a link, Vì, whose “next” hub, identified by th_u, is obtained from the mereology of the link identified by tl_u;

b informs the hub it is leaving and the link it is entering of its initial link position,

c whereupon the vehicle resumes the vehicle behaviour positioned at the very beginning (0) of that link,

or, again internally non-deterministically,

the vehicle “disappears — off the radar”!

The TUV Lectures, Vienna, Austria, October–November 2022

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A.4.2.2.2 Automobile Behaviour On a Link

We abstract automobile behaviour on a Link.

a Internally non-deterministically, either
i the automobile remains, “idling”, i.e., not moving, on the link,
ii however, first informing the link of its position,
b or
i if the automobile’s position on the link has not yet reached the hub, then
1 then the automobile moves an arbitrary small, positive Real-valued increment
   along the link
2 informing the hub of this,
3 while resuming being an automobile at the new position, or
ii else,
1 while obtaining a “next link” from the mereology of the hub (where that next
   link could very well be the same as the link the vehicle is about to leave),
2 the vehicle informs both the link and the imminent hub that it is now at that hub,
   identified by th_ui,
3 whereupon the vehicle resumes the vehicle behaviour positioned at that hub;

c or
d the vehicle “disappears — off the radar!”!

A.4.2.2.3 Hub Behaviour

The hub behaviour

a non-deterministically, externally offers
b to accept timed vehicle positions —
c which will be at the hub, from some vehicle, $v_{ui}$.
d The timed vehicle hub position is appended to the front of that vehicle’s entry in the
hub’s traffic table;
e whereupon the hub proceeds as a hub behaviour with the updated hub traffic table.
f The hub behaviour offers to accept from any vehicle.
g A post condition expresses what is really a proof obligation: that the hub traffic, $ht'$
satisfies the axiom of the endurant hub traffic attribute Item 92 Pg. 74.

value
257 hub$_{hi}$($h_{ui},((luis,vuis)),h\omega)(h\sigma,ht) \equiv$
257a \[
\begin{align*}
\text{let } m &= \text{bar_ch}[h_{ui},v_{ui}] ? \text{ in } \\
\text{assert: } & m = (\text{atHub}(h_{ui},h\omega)) \\
\text{let } ht' &= ht + [h_{ui} \mapsto \langle m \rangle 'ht(h_{ui})] \text{ in } \\
\text{hub$_{hi}$($h_{ui},((luis,vuis)),(h\omega))}(h\sigma,ht') \\
\text{let } & v_{ui} : V_{UI} \text{ in } v_{ui} \in \text{dom } ht' \Rightarrow \text{time_ordered}(ht'(v_{ui}))
\end{align*}
\]

A.4.2.2.4 Link Behaviour

258 The link behaviour non-deterministically, externally offers
259 to accept timed vehicle positions —
260 which will be on the link, from some vehicle, $v_{ui}$.
261 The timed vehicle link position is appended to the front of that vehicle’s entry in the link’s
traffic table;
262 whereupon the link proceeds as a link behaviour with the updated link traffic table.
263 The link behaviour offers to accept from any vehicle.
264 A post condition expresses what is really a proof obligation: that the link traffic, $lt'$
satisfies the axiom of the endurant link traffic attribute Item 96 Pg. 75.

value
258 link$_{li}$($l_{ui},((luis,vuis)),l\omega)(l\sigma,lt) \equiv$
258a \[
\begin{align*}
\text{let } m &= \text{bar_ch}[l_{ui},v_{ui}] ? \text{ in } \\
\text{assert: } & m = (\text{onLink}(l_{ui},l\omega)) \\
\text{let } lt' &= lt + [l_{ui} \mapsto \langle m \rangle 'lt(l_{ui})] \text{ in } \\
\text{link$_{li}$($l_{ui},((luis,vuis)),h\omega)(l\sigma,lt') } \\
\text{let } & v_{ui} : V_{UI} \text{ in } v_{ui} \in \text{dom } lt' \Rightarrow \text{time_ordered}(lt'(v_{ui}))
\end{align*}
\]

A.5 System Initialisation

A.5.1 Initial States

value
$hs:\text{H-set} \equiv \equiv \text{obs_SH(obs_SH(obs_RN(rts)))}$
$ls:\text{L-set} \equiv \equiv \text{obs_SL(obs_SL(obs_RN(rts)))}$
$bcs:\text{B-set} \equiv \equiv \text{obs_BCS(obs_BCS(obs_FV(obs_RN(rts))))}$
A.5 System Initialisation

\[ bs : B \text{-set} \equiv \bigcup \{ \text{obs}_{BS}(bc) | bc : BC \in \text{bcs} \} \]

\[ as : A \text{-set} \equiv \text{obs}_{BGs}(\text{obs}_{SBG}(\text{obs}_{FV}(\text{obs}_{RN}(\text{rts})))) \]

A.5.2 Initialisation

We are reaching the end of this domain modeling example. Behind us there are narratives and formalisations. Based on these we now express the signature and the body of the definition of a “system build and execute” function.

265 The system to be initialised is

- a the parallel compositions (\( \parallel \)) of
- b the distributed parallel composition (\( \parallel \{ \ldots \} \)) of all hub behaviours, and
- c the distributed parallel composition (\( \parallel \{ \ldots \} \)) of all link behaviours, and
- d the distributed parallel composition (\( \parallel \{ \ldots \} \)) of all bus company behaviours, and
- e the distributed parallel composition (\( \parallel \{ \ldots \} \)) of all bus behaviours, and
- f the distributed parallel composition (\( \parallel \{ \ldots \} \)) of all automobile behaviours.

\[ \text{initial system}() \equiv \bigcup \{ \text{hub} : (\text{hui}, \text{me}, \text{h}) (\text{htrf}, \text{h}^\omega) | \text{h} : H, \text{hui} : HUI \text{me} : HMetL, \text{htrf} : \text{Traffic}_{Ht} \text{h}^\omega : \text{\Omega}, \text{h} : \text{\Sigma}, \text{h}^\Sigma : \text{\Sigma} \} \]

\[ \bigcup \{ \text{link} : (\text{lui}, \text{me}, \text{l}) (\text{ltrf}, \text{l}^\omega) | \text{l} : L, \text{lui} : LUI \text{me} : LMet, \text{ltrf} : \text{Traffic}_{Lt} \text{l}^\omega : \text{\Omega}, \text{l} : \text{\Sigma}, \text{l}^\Sigma : \text{\Sigma} \} \]

\[ \bigcup \{ \text{bus company} : (\text{bcui}, \text{me}) (\text{btt}) | \text{bc} : BC \text{me} : \text{BC}_{\text{me}}, \text{btt} : \text{BusTimTbl} \} \]

\[ \bigcup \{ \text{bus} : (\text{bui}, \text{me}, \text{rn}) (\text{apos}) | \text{b} : B \text{me} : \text{BMet}, \text{apos} : \text{APos} \} \]

\[ \bigcup \{ \text{automobile} : (\text{aui}, \text{me}, \text{rn}) (\text{apos}) | \text{a} : A \text{me} : \text{AMet}, \text{apos} : \text{APos} \} \]
Appendix B
An RSL (Raise Specification Language) Primer

We present an RSL Primer. Indented text, in slanted font, such as this, presents informal material and examples. Non-indented text, in roman font, presents narrative and formal explanation of RSL constructs.

This RSL Primer omits treatment of a number of language constructs, notably the RSL module concepts of schemes, classes and objects. Although we do cover the imperative language construct of [declaration of] variables and, hence, assignment, we shall omit treatment of structured imperative constructs like for ..., do s while b, while b do s loops.

Section B.13 on page 159 introduces additional language constructs, thereby motivation the + in the RSL+ name.

B.1 Types and Values

Types are, in general, set-like structures of things, i.e., values, having common characteristics.

A bunch of zero, one or more apples (type apples) may thus form a [sub]set of type Belle de Boskoop apples. A bunch of zero, one or more pears (type pears) may thus form a [sub]set of type Concorde pears. A union of zero, one or more of these apples and pears then form a [sub]set of entities of type fruits.

B.1.1 Sort and Type Expressions

Sort and type expressions are expressions whose values are types, that is, possibly infinite set-like structures of values (of “that” type).

B.1.1.1 Atomic Types: Identifier Expressions and Type Values

Atomic types have (atomic) values. That is, values which we consider to have no proper constituent (sub-)values, i.e., cannot, to us, be meaningfully “taken apart”.

The letter I shall designate begin of informal text.

The ■ symbol shall designate end-of-informal text.

We shall not, in this primer, go into details as to the mathematics of types.
RSL has a number of [so-called] built-in atomic types. They are expressed in terms of literal identifiers. These are the **Booleans**, integers, **Natural numbers**, **Reals**, **Characters**, and **Texts**. **Texts** are free-form texts and are more general than just texts of RSL-11ke formulas. **RSL-Text**'s will be introduced in Sect. B.13 on page 159.

We shall not need the base types **Characters**, nor the general type **Texts** for domain modelling in this primer. They will be listed below, but not mentioned further.

The base types are:

```
type
  [1] Bool
  [2] Int
  [3] Nat
  [4] Real
  [5] Char
```

1. The Boolean type of truth values **false** and **true**.
2. The integer type on integers ..., \(-2\), \(-1\), 0, 1, 2, ...
3. The natural number type of positive integer values 0, 1, 2, ...
4. The real number type of real values, i.e., values whose numerals can be written as an integer, followed by a period ("."), followed by a natural number (the fraction).
5. The character type of character values "a", "bbb", ...
6. The text type of character string values "aa", "aaa", ..., "abc", ...

### B.1.1.2 Composite Types: Expressions and Type Values

Composite types have composite values. That is, values which we consider to have proper constituent (sub-)values, i.e., can, to us, be meaningfully “taken apart”.

From these one can form type expressions: finite sets, infinite sets, Cartesian products, lists, maps, etc.

Let A, B and C be any type names or type expressions, then these are the composite types, hence, type expressions:

```
[8] A-infset
[9] A × B × ... × C
[10] A'
[12] A → B
[14] A | B | ... | C
[15] mk_id(sel_a:A,...,sel_b:B)
[16] sel_a:A ... sel_b:B
```

The following are generic type expressions:

7. The set type of finite cardinality set values.
8. The set type of infinite and finite cardinality set values.
9. The Cartesian type of Cartesian values.
10. The list type of finite length list values.
11. The list type of infinite and finite length list values.
B.1 Types and Values

12 The map type of finite definition set map values.
13 The function type of total function values.
14 The function type of partial function values.
15 The postulated disjoint union of types $A$, $B$, $C$.

16 The record type of $\text{mk}_i\text{id}$-named record values $\text{mk}_i\text{id}(av, \ldots, bv)$, where $av, \ldots, bv$ are values of respective types. The distinct identifiers $sel_a$, etc., designate selector functions.

17 The record type of unnamed record values $(av, \ldots, bv)$, where $av, \ldots, bv$ are values of respective types. The distinct identifiers $sel_a$, etc., designate selector functions.

B.1.2 Type Definitions

B.1.2.1 Sorts — Abstract Types

Types can be (abstract) sorts in which case their structure is not specified:

\[
\text{type } A, B, \ldots, C
\]

B.1.2.2 Concrete Types

Types can be concrete in which case the structure of the type is specified by type expressions:

\[
\text{type } A = \text{Type}\_\text{expr}
\]

RSL Example: Sets. Narrative: $H$ stand for the domain type of street intersections – we shall call then hubs, and let $L$ stand for the domain type of segments of streets between immediately neighboring hubs – we shall call then links. Then $Hs$ and $Ls$ are to designate the types of finite sets of zero, one or more hubs, respectively links. Formalisation:

\[
\text{type } H, L, Hs = \text{H-set}, Ls = \text{L-set}
\]

RSL Example: Cartesians. Narrative: Let $RN$ stand for the domain type of road nets consisting of hub aggregates, $HA$, and link aggregates, $LA$. Hub and link aggregates can be observed from road nets, and hub sets and link sets can be observed from hub, respectively link aggregates. Formalisation:

\[
\text{type } RN = HA \times LA, Hs, Ls
\]

\[
\text{value } obs_{HA}: RN \rightarrow HA, obs_{LA}: RN \rightarrow LA, obs_{Hs}: HA \rightarrow Hs, obs_{Ls}: LA \rightarrow Ls
\]

Observer functions, $obs_{\ldots}$, are not further defined – beyond their signatures. They will (subsequently) be defined through axioms over their results.

Some schematic type definitions are:

\[
[18] \text{Type}_\text{name} = \text{Type}_\text{expr} / * \text{without } s \text{or subtypes } */
\]

\[
[19] \text{Type}_\text{name} = \text{Type}_\text{expr}_1 \mid \text{Type}_\text{expr}_2 \mid \ldots \mid \text{Type}_\text{expr}_n
\]

\[
[20] \text{Type}_\text{name} ==
\]

\[
\text{mk}_i\text{id}_1(s_{a1}:\text{Type}_\text{name}_a1, \ldots, s_{ai}:\text{Type}_\text{name}_ai) \mid \\
\ldots \mid
\text{mk}_i\text{id}_n(s_{z1}:\text{Type}_\text{name}_z1, \ldots, s_{zk}:\text{Type}_\text{name}_zk)
\]

\[
[21] \text{Type}_\text{name} :: sel_a:Type\_name\_a \ldots sel_z:Type\_name\_z
\]

\[
[22] \text{Type}_\text{name} = || v:Type\_name \cdot P(v) ||
\]
where a form of [19–20] is provided by combining the types:

\[
\begin{align*}
\text{Type name} & = A \mid B \mid \ldots \mid Z \\
A & = \text{mk}_{\text{id}1}(s_{a1}:A_1, s_{a2}:A_2, \ldots, s_{ai}:A_i) \\
B & = \text{mk}_{\text{id}2}(s_{b1}:B_1, s_{b2}:B_2, \ldots, s_{bj}:B_j) \\
\ldots \\
Z & = \text{mk}_{\text{id}n}(s_{z1}:Z_1, s_{z2}:Z_2, \ldots, s_{zk}:Z_k)
\end{align*}
\]

Of these we shall almost exclusively make use of [23–27].

**Disjoint Types. Narrative:** A pipeline consists of a finite set of zero, one or more interconnected pipe units. Pipe units are either wells, or are pumps, or are valves, or are joins, or are forks, or are sinks.

**Formalisation:**

\[
\text{type PL} = \text{P-set}, \quad \text{P} == \text{WU} \mid \text{PU} \mid \text{VU} \mid \text{JO} \mid \text{FO} \mid \text{SU}, \\
\text{WU}::\text{mk}_{\text{WU}}(s_{wu}:\text{Wu}), \quad \text{PU}::\text{mk}_{\text{PU}}(s_{pu}:\text{Pu}), \\
\text{VU}::\text{mk}_{\text{VU}}(s_{vu}:\text{Vu}), \quad \text{JO}::\text{mk}_{\text{JO}}(s_{ju}:\text{Ju}), \\
\text{FO}::\text{mk}_{\text{FO}}(s_{fu}:\text{Fu}), \quad \text{SU}::\text{mk}_{\text{SU}}(s_{su}:\text{Su})
\]

where we leave types Wu, Pu, VU, Ju, Fu and Su further undefined •

Types A, B, ..., Z are disjoint, i.e., shares no values, provided all \(\text{mk}_{\text{id}k}\) are distinct and due to the use of the disjoint record type constructor ==.

**axiom**

\[
\forall a_1:A_1, a_2:A_2, \ldots, a_i:A_i \cdot \begin{align*}
& s_{a1}(\text{mk}_{\text{id}1}(a_1, a_2, \ldots, a_i)) = a_1 \wedge s_{a2}(\text{mk}_{\text{id}1}(a_1, a_2, \ldots, a_i)) = a_2 \wedge \\
& \ldots \wedge s_{ai}(\text{mk}_{\text{id}1}(a_1, a_2, \ldots, a_i)) = a_i \\
& \forall a:A \cdot \text{let} \text{mk}_{\text{id}1}(a_1', a_2', \ldots, a_i') = a \text{ in} \\
& a_1' = s_{a1}(a) \wedge a_2' = s_{a2}(a) \wedge \ldots \wedge a_i' = s_{ai}(a) \text{ end}
\end{align*}
\]

**Note:** Values of type A, where that type is defined by \(A::B \times C \times D\), can be expressed \(A(b, c, d)\) for \(b:B, c:D, d:D\).

**B.1.2.3 Subtypes**

In RSL, each type represents a set of values. Such a set can be delimited by means of predicates. The set of values \(b\) which have type \(B\) and which satisfy the predicate \(\mathcal{P}\), constitute the subtype \(A:\)

\[
\text{type} \quad A = \{ b:B \mid \mathcal{P}(b) \}
\]

**Subtype. Narrative:** The subtype of even natural numbers.

**Formalisation:**

\[
\text{type ENat} = \{ \text{en} \mid \text{en}:\text{Nat} \cdot \text{is_even_number}(en) \}
\]

**B.2 The Propositional and Predicate Calculi**

**B.2.1 Propositions**

In logic, a proposition is the meaning of a declarative sentence. [A declarative sentence is a type of sentence that makes a statement]

---

\[13\] Although interconnected we shall not model pipelines as lists.
## 2.1 Propositional Expressions

Propositional expressions, informally speaking, are quantifier-free expressions having truth (or chaos) values. ∀, ∃ and ∃! are quantifiers, see below.

Below, we will first treat propositional expressions all of whose identifiers denote truth values. As we progress, in sections on arithmetic, sets, list, maps, etc., we shall extend the range of propositional expressions.

Let identifiers (or propositional expressions) a, b, ..., c designate Boolean values (true or false). Then:

- false, true
- a, b, ..., c
- a ∧ b, a ∨ b, a ⇒ b, a = b, a ≠ b

are propositional expressions having Boolean values. ¬, ∧, ∨, ⇒ are Boolean connectives (i.e., operators). They can be read as: not, and, or, if then (or implies), equal, not equal and always.

### 2.1.2 Propositional Calculus

Propositional calculus is a branch of logic. It is also called propositional logic, statement logic, sentential calculus, sentential logic, or sometimes zeroth-order logic. It deals with propositions (which can be true or false) and relations between propositions, including the construction of arguments based on them. Compound propositions are formed by connecting propositions by logical connectives. Propositions that contain no logical connectives are called atomic propositions.

A simple two-value Boolean logic can be defined as follows:

<table>
<thead>
<tr>
<th>type</th>
<th>Bool</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>true, false</td>
</tr>
</tbody>
</table>

- ¬: Bool → Bool
- ∧, ∨, ⇒: Bool × Bool → Bool

**axiom**

- ∀ b, b': Bool
  - ¬b ≡ if b then false else true end
  - b ∧ b' ≡ if b then b' else false end
  - b ∨ b' ≡ if b then true else b' end
  - b ⇒ b' ≡ if b then b' else true end
  - b = b' ≡ if (b ∧ b') ∨ (¬b ∧ ¬b') then true else false end
  - (b ≠ b') = ~ (b = b')
  - (b ⊢ b') ≡ (b = b')

We shall, however, make use of a three-value Boolean logic. The model-theory explanation of the meaning of propositional expressions is now given in terms of the truth tables for the logic connectives:

<table>
<thead>
<tr>
<th>¬</th>
<th>true</th>
<th>false</th>
<th>chaos</th>
</tr>
</thead>
<tbody>
<tr>
<td>true</td>
<td>true</td>
<td>false</td>
<td>chaos</td>
</tr>
<tr>
<td>false</td>
<td>false</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>chaos</td>
<td>chaos</td>
<td>chaos</td>
<td>chaos</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>∧</th>
<th>true</th>
<th>false</th>
<th>chaos</th>
</tr>
</thead>
<tbody>
<tr>
<td>true</td>
<td>true</td>
<td>true</td>
<td>true</td>
</tr>
<tr>
<td>false</td>
<td>true</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>chaos</td>
<td>chaos</td>
<td>chaos</td>
<td>chaos</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>⇒</th>
<th>true</th>
<th>false</th>
<th>chaos</th>
</tr>
</thead>
<tbody>
<tr>
<td>true</td>
<td>true</td>
<td>false</td>
<td>false</td>
</tr>
<tr>
<td>false</td>
<td>true</td>
<td>true</td>
<td>true</td>
</tr>
<tr>
<td>chaos</td>
<td>chaos</td>
<td>chaos</td>
<td>chaos</td>
</tr>
</tbody>
</table>
The two-value logic defined earlier ‘transpires’ from the true, false columns and rows of the above truth tables.

**B.2.2 Predicates**

Predicates are mathematical assertions that contains variables, sometimes referred to as predicate variables, and may be true or false depending on those variables’ value or values\(^\text{14}\)

**B.2.2.1 Predicate Expressions**

Let \(x, y, ..., z\) (or term expressions) designate non-Boolean values, and let \(P(x)\), \(Q(y)\) and \(R(z)\) be propositional or predicate expressions, then:

\[
\begin{align*}
[28] & \forall x : X \cdot P(x) \\
[29] & \exists y : Y \cdot Q(y) \\
[30] & \exists ! z : Z \cdot R(z)
\end{align*}
\]

are quantified, i.e., predicate expressions. \(\forall, \exists\) and \(\exists !\) are the quantifiers.

**B.2.2.2 Predicate Calculus**

They are “read” as:

- [28] For all \(x\) (values in type \(X\)) the predicate \(P(x)\) holds – if that is not the case the expression yields truth value false.
- [29] There exists (at least) one \(y\) (value in type \(Y\)) such that the predicate \(Q(y)\) holds – if that is not the case the expression yields truth value false.
- [30] There exists a unique \(z\) (value in type \(Z\)) such that the predicate \(R(z)\) holds – if that is not the case the expression yields truth value false.

[28–30] The predicates \(P(x)\), \(Q(y)\) or \(R(z)\) may yield chaos in which case the whole expression yields chaos.

**B.3 Arithmetic**

<table>
<thead>
<tr>
<th>type</th>
<th>Nat, Int, Real</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td></td>
</tr>
<tr>
<td>+, −, ∗:</td>
<td>Nat × Nat → Nat</td>
</tr>
<tr>
<td>/:</td>
<td>Nat × Nat → Nat</td>
</tr>
<tr>
<td>&lt;, ≤, =, ≠, ≥, &gt;: (Nat</td>
<td>Int</td>
</tr>
</tbody>
</table>

\(^{14}\)https://calcworkshop.com/logic/predicate-logic/, and: predicate logic, first-order logic or quantified logic is a formal language in which propositions are expressed in terms of predicates, variables and quantifiers. It is different from propositional logic which lacks quantifiers https://brilliant.org/wiki/predicate-logic/.
Let the below a’s denote values of type A, then the below designate simple set enumerations:

\[
\{\emptyset, \{a\}, \{e_1,e_2,\ldots,e_n\}, \ldots\} \in A\text{-set}
\]

\[
\{\emptyset, \{a\}, \{e_1,e_2,\ldots,e_n\}, \ldots, \{e_1,e_2,\ldots\}\} \in A\text{-infset}
\]

### B.4.1.1 Set Comprehension

The expression, last line below, to the right of the \(\equiv\) expresses set comprehension. The expression “builds” the set of values satisfying the given predicate. It is abstract in the sense that it does not do so by following a concrete algorithm.

**type**

\[
A, B
\]

\[
P : A \rightarrow \text{Bool}
\]

\[
Q : A \rightarrow B
\]

**value**

\[
\text{comprehend} : A\text{-infset} \times P \times Q \rightarrow B\text{-infset}
\]

\[
\text{comprehend}(s, P, Q) \equiv \{ Q(a) \mid a : A \cdot a \in s \land P(a) \}
\]

### B.4.2 Cartesian Expressions

#### B.4.2.1 Cartesian Enumerations

Let \(e\) range over values of Cartesian types involving \(A, B, \ldots, C\), then the below expressions are simple Cartesian enumerations:

**type**

\[
A, B, \ldots, C
\]

\[
A \times B \times \ldots \times C
\]

**value**

\[
(e_1, e_2, \ldots, e_n)
\]

### B.4.3 List Expressions

#### B.4.3.1 List Enumerations

Let \(a\) range over values of type \(A\), then the below expressions are simple list enumerations:

\[
\langle \emptyset, \langle e\rangle, \ldots, \langle e_1, e_2, \ldots, e_n\rangle, \ldots\rangle \in A^\ast
\]

\[
\langle \emptyset, \langle e\rangle, \ldots, \langle e_1, e_2, \ldots, e_n\rangle, \ldots, \langle e_1, e_2, \ldots, e_n, \ldots\rangle, \ldots\rangle \in A^\omega
\]

\[
\langle a_1 \ldots a_n \rangle
\]
The last line above assumes \( a_i \) and \( a_j \) to be integer-valued expressions. It then expresses the set of integers from the value of \( e_i \) to and including the value of \( e_j \). If the latter is smaller than the former, then the list is empty.

### B.4.3.2 List Comprehension

The last line below expresses list comprehension.

```plaintext
type A, B, P = A → Bool, Q = A → B
value
comprehend: A\omega × P × Q → B\omega
comprehend(l,P,Q) ≡
  ⟨ Q(l(i)) | i in (1..len l) • P(l(i))⟩
```

### B.4.4 Map Expressions

#### B.4.4.1 Map Enumerations

Let (possibly indexed) \( u \) and \( v \) range over values of type \( T_1 \) and \( T_2 \), respectively, then the below expressions are simple map enumerations:

```plaintext
type T1, T2
M = T1 → m T2
value
u,u1,u2,...,un:T1, v,v1,v2,...,vn:T2
[ ], [u↦v], ..., [u1↦v1,u2↦v2,...,un↦vn] ∀ ∈ M
```
B.4 Set Expressions

B.4.5 Set Operations

B.4.5.1 Set Operator Signatures

value
18 $\in$: $A \times A$-infset $\rightarrow$ Bool
19 $\notin$: $A \times A$-infset $\rightarrow$ Bool
20 $\cup$: $A$-infset $\times$ $A$-infset $\rightarrow$ $A$-infset
21 $\cup$: $(A$-infset$)$-infset $\rightarrow$ $A$-infset
22 $\cap$: $A$-infset $\times$ $A$-infset $\rightarrow$ $A$-infset
23 $\cap$: $(A$-infset$)$-infset $\rightarrow$ $A$-infset
24 $\setminus$: $A$-infset $\times$ $A$-infset $\rightarrow$ $A$-infset
25 $\subseteq$: $A$-infset $\times$ $A$-infset $\rightarrow$ Bool
26 $\subset$: $A$-infset $\times$ $A$-infset $\rightarrow$ Bool
27 $\neq$: $A$-infset $\times$ $A$-infset $\rightarrow$ Bool
28 $\neq$: $A$-infset $\times$ $A$-infset $\rightarrow$ Bool
29 card: $A$-infset $\rightarrow$ Nat

B.4.5.2 Set Examples

examples
18 a $\in$ {a,b,c}
19 a $\notin$ {}, a $\notin$ {b,c}
20 {a,b,c} $\cup$ {a,b,d,e} = {a,b,c,d,e}
21 $\cup$$\{a\}$$\cup$$\{a,b\}$$\cup$$\{a,d\}$ = {a,b,d}
22 {a,b,c} $\cap$ {c,d,e} = {c}
23 $\cap$$\{a\}$$\cap$$\{a,b\}$$\cap$$\{a,d\}$ = {a}
24 {a,b,c} $\setminus$ {c,d} = {a,b}
25 {a,b} $\subset$ {a,b,c}
26 {a,b,c} $\subseteq$ {a,b,c}
27 {a,b,c} = {a,b,c}
28 {a,b,c} $\neq$ {a,b}

B.4.5.3 Informal Explication

18 $\in$: The membership operator expresses that an element is a member of a set.
19 $\notin$: The nonmembership operator expresses that an element is not a member of a set.
20 $\cup$: The infix union operator. When applied to two sets, the operator gives the set whose members are in either or both of the two operand sets.
21 $\cup$: The distributed prefix union operator. When applied to a set of sets, the operator gives the set whose members are in some of the operand sets.
22 $\cap$: The infix intersection operator. When applied to two sets, the operator gives the set whose members are in both of the two operand sets.
23 $\cap$: The prefix distributed intersection operator. When applied to a set of sets, the operator gives the set whose members are in some of the operand sets.
24 \(\setminus\): The set complement (or set subtraction) operator. When applied to two sets, the operator gives the set whose members are those of the left operand set which are not in the right operand set.

25 \(\subseteq\): The proper subset operator expresses that all members of the left operand set are also in the right operand set.

26 \(\subset\): The proper subset operator expresses that all members of the left operand set are also in the right operand set, and that the two sets are not identical.

27 \(\equiv\): The equal operator expresses that the two operand sets are identical.

28 \(\neq\): The nonequal operator expresses that the two operand sets are not identical.

29 \(\text{card}\): The cardinality operator gives the number of elements in a finite set.

### B.4.5.4 Set Operator Definitions

The operations can be defined as follows (\(\equiv\) is the definition symbol):

**value**

\[
\begin{align*}
\mathsf{s}' \cup \mathsf{s}'' & \equiv \{ a \mid a : A \land a \in \mathsf{s}' \lor a \in \mathsf{s}'' \} \\
\mathsf{s}' \cap \mathsf{s}'' & \equiv \{ a \mid a : A \land a \in \mathsf{s}' \land a \in \mathsf{s}'' \} \\
\mathsf{s}' \setminus \mathsf{s}'' & \equiv \{ a \mid a : A \land a \in \mathsf{s}' \land a \notin \mathsf{s}'' \} \\
\mathsf{s}' \subseteq \mathsf{s}'' & \equiv \forall a : A : a \in \mathsf{s}' \Rightarrow a \in \mathsf{s}'' \\
\mathsf{s}' \subset \mathsf{s}'' & \equiv \mathsf{s}' \subseteq \mathsf{s}'' \land \exists a : A : a \in \mathsf{s}' \land a \notin \mathsf{s}' \\
\mathsf{s}' = \mathsf{s}'' & \equiv \mathsf{s}' \subseteq \mathsf{s}'' \land \mathsf{s}'' \subseteq \mathsf{s}' \\
\mathsf{s}' \neq \mathsf{s}'' & \equiv \mathsf{s}' \cap \mathsf{s}'' \neq \emptyset \\
\text{card} \; \mathsf{s} & \equiv \\
& \text{if } \mathsf{s} = \emptyset \text{ then 0 else} \\
& \text{let } a : A : a \in \mathsf{s} \text{ in 1 + card } (\mathsf{s} \setminus \{a\}) \end{align*}
\]

### B.4.6 Cartesian Operations

**type**

\[
\begin{align*}
A, B, C & \rightarrow (va, vb, vc):G0 \\
A \times B \times C & \rightarrow (va, vb, vc):G1 \\
(A \times B \times C) & \rightarrow (va3, vb3, vc3):G2 \\
A \times (B \times C) & \rightarrow \text{decomposition expressions} \\
\mathsf{g0} & = A \times B \times C \\
\mathsf{g1} & = (A \times B \times C) \\
\mathsf{g2} & = (A \times B) \times C \\
\mathsf{g3} & = A \times (B \times C)
\end{align*}
\]

**value**

\[
\begin{align*}
va : A, vb : B, vc : C, vd : D & \rightarrow (va, vb, vc):G0,
\end{align*}
\]
B.4.7 List Operations

B.4.7.1 List Operator Signatures

value
- \text{hd}: A^\omega \to A
- \text{tl}: A^\omega \to A^\omega
- \text{len}: A^\omega \to \text{Nat}
- \text{inds}: A^\omega \to \text{Nat-infset}
- \text{elems}: A^\omega \to A^{\text{infset}}
- \langle . \rangle: A^\omega \times \text{Nat} \to A
- \text{=} : A^\omega \times A^\omega \to \text{Bool}
- \neq : A^\omega \times A^\omega \to \text{Bool}

B.4.7.2 List Operation Examples

examples
- \text{hd}(a_1,a_2,\ldots,a_m)=a_1
- \text{tl}(a_1,a_2,\ldots,a_m)=(a_2,\ldots,a_m)
- \text{len}(a_1,a_2,\ldots,a_m)=m
- \text{inds}(a_1,a_2,\ldots,a_m)\{1,2,\ldots,m\}
- \text{elems}(a_1,a_2,\ldots,a_m)\{a_1,a_2,\ldots,a_m\}
- \langle a_1,a_2,\ldots,a_m\rangle(i)=a_i
- \langle a,b,c\rangle\langle a,b,d\rangle = \langle a,b,c,a,b,d\rangle
- \langle a,b,c\rangle = (a,b,c)
- \langle a,b,c\rangle \neq (a,b,d)

B.4.7.3 Informal Explication

- \text{hd}: Head gives the first element in a nonempty list.
- \text{tl}: Tail gives the remaining list of a nonempty list when Head is removed.
- \text{len}: Length gives the number of elements in a finite list.
- \text{inds}: Indices give the set of indices from 1 to the length of a nonempty list. For empty lists, this set is the empty set as well.
- \text{elems}: Elements gives the possibly infinite set of all distinct elements in a list.
- \langle . \rangle: Indexing with a natural number, \( i \) larger than 0, into a list \( \ell \) having a number of elements larger than or equal to \( i \), gives the \( i \)th element of the list.
- \text{\}: Concatenates two operand lists into one. The elements of the left operand list are followed by the elements of the right. The order with respect to each list is maintained.
- \text{=} : The equal operator expresses that the two operand lists are identical.
- \text{\neq} : The nonequal operator expresses that the two operand lists are not identical.

The operations can also be defined as follows:
B.4.7.4 List Operator Definitions

value

\( \text{is\_finite\_list}: A^\omega \rightarrow \text{Bool} \)

len \( q \) ≡

\begin{align*}
\text{case } \text{is\_finite\_list}(q) \text{ of} \\
\text{true} & \rightarrow \text{if } q = \langle \rangle \text{ then } 0 \text{ else } 1 + \text{len} \text{tl } q \text{ end}, \\
\text{false} & \rightarrow \text{chaos end}
\end{align*}

inds \( q \) ≡

\begin{align*}
\text{case } \text{is\_finite\_list}(q) \text{ of} \\
\text{true} & \rightarrow \{ i \mid i : \text{Nat} \cdot 1 \leq i \leq \text{len } q \}, \\
\text{false} & \rightarrow \{ i \mid i : \text{Nat} \cdot i \neq 0 \} \text{ end}
\end{align*}

elems \( q \) ≡ \{ q(i) \mid i : \text{Nat} \cdot i \in \text{inds } q \}

\( q(i) \) ≡

\begin{align*}
\text{if } i = 1 \\
\text{then} \\
\text{if } q(\langle \rangle) \\
\text{then } \text{let } a : A, q' : Q \cdot q = (a) \hat{q} \text{ in } a \text{ end} \\
\text{else } \text{chaos end} \\
\text{else } q(i-1) \text{ end}
\end{align*}

\( f \hat{q} \hat{i} q \equiv \langle \text{if } 1 \leq i \leq \text{len } f q \text{ then } f q(i) \text{ else } i q(i - \text{len } f q) \text{ end} \\
\mid i : \text{Nat} \cdot \text{if } \text{len } i q \neq \text{chaos} \text{ then } i \leq \text{len } f q + \text{len end} \rangle \)

\( \text{pre } \text{is\_finite\_list}(f q) \)

\( i q' = i q'' \equiv \text{inds } i q' = \text{inds } i q'' \land \forall i : \text{Nat} \cdot i \in \text{inds } i q' \Rightarrow i q'(i) = i q''(i) \)

\( i q' \neq i q'' \equiv \sim (i q' = i q'') \)

B.5 Map Operations

B.5.1 Map Operator Signatures and Map Operation Examples

value

\( m(a) : M \rightarrow A \rightarrow B, m(a) = b \)

\( \text{dom} : M \rightarrow A \rightarrow \text{infset} \) \hspace{1em} [domain of map]

\text{dom} [a1\rightarrow b1,a2\rightarrow b2,\ldots,an\rightarrow bn] = \{a1,a2,\ldots,an\}

\( \text{rng} : M \rightarrow B \rightarrow \text{infset} \) \hspace{1em} [range of map]

\text{rng} [a1\rightarrow b1,a2\rightarrow b2,\ldots,an\rightarrow bn] = \{b1,b2,\ldots,bn\}

\( \vdash : M \times M \rightarrow M \) \hspace{1em} [override extension]
### B.5 Map Operations

\[ [a \mapsto b, a' \mapsto b', a'' \mapsto b''] \uplus [a' \mapsto b'', a'' \mapsto b'] = [a \mapsto b, a' \mapsto b'', a'' \mapsto b'] \]

\( \cup : M \times M \rightarrow M \) [merge \( \cup \)]
\[ [a \mapsto b, a' \mapsto b', a'' \mapsto b''] \cup [a'' \mapsto b''] = [a \mapsto b, a' \mapsto b', a'' \mapsto b'', a''' \mapsto b'''] \]

\( \setminus : M \times A-\text{infset} \rightarrow M \) [restriction by]
\[ [a \mapsto b, a' \mapsto b', a'' \mapsto b''] \setminus \{a\} = [a' \mapsto b', a'' \mapsto b''] \]

\( / : M \times A-\text{infset} \rightarrow M \) [restriction to]
\[ [a \mapsto b, a' \mapsto b', a'' \mapsto b''] / \{a, a''\} = [a' \mapsto b', a'' \mapsto b''] \]

\( =, \neq : M \times M \rightarrow \text{Bool} \)
\[ (A \mapsto B) \times (B \mapsto C) \rightarrow (A \mapsto C) \] [composition]
\[ [a \mapsto b, a' \mapsto b', a'' \mapsto b', b' \mapsto c', b'' \mapsto c''] = [a \mapsto c, a' \mapsto c'] \]

#### B.5.1.1 Map Operation Explication

- **\( m(a) \)**: Application gives the element that \( a \) maps to in the map \( m \).
- **\( \text{dom} \)**: Domain/Definition Set gives the set of values which maps to in a map.
- **\( \text{rng} \)**: Range/Image Set gives the set of values which are mapped to in a map.
- **\( \hat{} \)**: Override/Extend. When applied to two operand maps, it gives the map which is like an override of the left operand map by all or some “pairings” of the right operand map.
- **\( \cup \)**: Merge. When applied to two operand maps, it gives a merge of these maps.
- **\( \setminus \)**: Restriction. When applied to two operand maps, it gives the map which is a restriction of the left operand map to the elements that are not in the right operand set.
- **\( / \)**: Restriction. When applied to two operand maps, it gives the map which is a restriction of the left operand map to the elements of the right operand set.
- **\( = \)**: The equal operator expresses that the two operand maps are identical.
- **\( \neq \)**: The nonequal operator expresses that the two operand maps are not identical.
- **\( \circ \)**: Composition. When applied to two operand maps, it gives the map from definition set elements of the left operand map, \( m_1 \), to the range elements of the right operand map, \( m_2 \), such that if \( a \) is in the definition set of \( m_1 \) and maps into \( b \), and if \( b \) is in the definition set of \( m_2 \) and maps into \( c \), then \( a \), in the composition, maps into \( c \).

#### B.5.1.2 Map Operation Redefinitions

The map operations can also be defined as follows:

\[ \text{value} \]
\[ \text{rng} m \equiv \{ m(a) \mid a : A \land a \in \text{dom} m \} \]
\[ m_1 \uplus m_2 \equiv [\ a \mapsto b \mid a : A, b : B \]
\[ a \in \text{dom} m_1 \land \text{dom} m_2 \land b = m_1(a) \lor a \in \text{dom} m_2 \land b = m_2(a) \] \[

\]
\[
m \setminus s \equiv \{ a \mapsto m(a) \mid a : A \in \text{dom } m \setminus s \} \\
m / s \equiv \{ a \mapsto m(a) \mid a : A \in \text{dom } m \cap s \}
\]

\[
m_1 = m_2 \equiv \\
\text{dom } m_1 = \text{dom } m_2 \land \forall a : A \in \text{dom } m_1 \Rightarrow m_1(a) = m_2(a) \\
m_1 \neq m_2 \equiv \neg (m_1 = m_2)
\]

\[
m^n \equiv \\
\{ a \mapsto c \mid a : A, c : C \in \text{dom } m \land c = n(m(a)) \}
\]

\[
\text{pre rng } m \subseteq \text{dom } n
\]

B.6 \(\lambda\)-Calculus + Functions

B.6.1 The \(\lambda\)-Calculus Syntax

**type /\ */ A BNF Syntax: */

\[
\langle L \rangle ::= \langle V \rangle \mid \langle F \rangle \mid \langle A \rangle \mid (\langle A \rangle)
\]

\[
\langle V \rangle ::= /\ */ \text{variables, i.e. identifiers }*
\]

\[
\langle F \rangle ::= \lambda \langle V \rangle \cdot \langle L \rangle
\]

\[
\langle A \rangle ::= (\langle L \rangle)\langle L \rangle
\]

**value /\ */ Examples */

\[
\langle L \rangle: e, f, a, ...
\]

\[
\langle V \rangle: x, ...
\]

\[
\langle F \rangle: \lambda x \cdot e, ...
\]

\[
\langle A \rangle: f a, (f a), f(a), (f)(a), ...
\]

B.6.2 Free and Bound Variables

Let \(x, y\) be variable names and \(e, f\) be \(\lambda\)-expressions.

- \(\langle V \rangle\): Variable \(x\) is free in \(x\).
- \(\langle F \rangle\): \(x\) is free in \(\lambda y \cdot e\) if \(x \neq y\) and \(x\) is free in \(e\).
- \(\langle A \rangle\): \(x\) is free in \(f(c)\) if it is free in either \(f\) or \(e\) (i.e., also in both).

B.6.3 Substitution

In RSL, the following rules for substitution apply:

- \(\text{subst}[\langle N \rangle/x]x \equiv N\);
- \(\text{subst}[\langle N \rangle/x]a \equiv a\),
  
  for all variables \(a \neq x\);
- \(\text{subst}[\langle N \rangle/x](P Q) \equiv (\text{subst}[\langle N \rangle/x]P) \text{ subst}[\langle N \rangle/x]Q)\);
- \(\text{subst}[\langle N \rangle/x](\lambda x \cdot P) \equiv \lambda y \cdot \text{subst}[\langle N \rangle/x](P)\);
- \(\text{subst}[\langle N \rangle/x](\lambda y \cdot P) \equiv \lambda y \cdot \text{subst}[\langle N \rangle/x](P)\),
B.6 \(\lambda\)-Calculus + Functions

if \(x \neq y\) and \(y\) is not free in \(N\) or \(x\) is not free in \(P\);

- subst\([N/x]\(\lambda y\cdot P\)) \(\equiv\) \(\lambda z\cdot\text{subst}(\text{subst}(\text{subst}(z/y)P))\),

  if \(y \neq x\) and \(y\) is free in \(N\) and \(x\) is free in \(P\)
  (where \(z\) is not free in \((N P)\)).

B.6.4 \(\alpha\)-Renaming and \(\beta\)-Reduction

- \(\alpha\)-renaming: \(\lambda x\cdot M\)
  If \(x\), \(y\) are distinct variables then replacing \(x\) by \(y\) in \(\lambda x\cdot M\) results in \(\lambda y\cdot\text{subst}(y/x|M)\).
  We can rename the formal parameter of a \(\lambda\)-function expression provided that no free
  variables of its body \(M\) thereby become bound.

- \(\beta\)-reduction: \((\lambda x\cdot M)(N)\)
  All free occurrences of \(x\) in \(M\) are replaced by the expression \(N\) provided that no free
  variables of \(N\) thereby become bound in the result. \((\lambda x\cdot M)(N) \equiv\) subst\(([N/x]M)\)

B.6.5 Function Signatures

For sorts we may want to postulate some functions:

- type
  A, B, C
- value
  \(\text{obs}_B\): A \(\rightarrow\) B,
  \(\text{obs}_C\): A \(\rightarrow\) C,
  \(\text{gen}_A\): B\(\times\)C \(\rightarrow\) A

B.6.6 Function Definitions

Functions can be defined explicitly:

- value
  \(f\): Arguments \(\rightarrow\) Result
  \(f(\text{args}) \equiv \text{DValueExpr}\)

  \(g\): Arguments \(\rightarrow\) Result
  \(g(\text{args}) \equiv \text{ValueAndStateChangeClause}\)
  \(\text{pre P}(\text{args})\)

Or functions can be defined implicitly:

- value
  \(f\): Arguments \(\rightarrow\) Result
  \(f(\text{args})\) as result
  \(\text{post P1}(\text{args},\text{result})\)
An RSL (Raise Specification Language) Primer

The symbol \( \sim \rightarrow \) indicates that the function is partial and thus not defined for all arguments. Partial functions should be assisted by preconditions stating the criteria for arguments to be meaningful to the function.

### B.7 Other Applicative Expressions

#### B.7.1 Simple Let Expressions

Simple (i.e., nonrecursive) let expressions:

\[
\text{let } a = E_d \text{ in } E_b(a) \text{ end}
\]

is an “expanded” form of:

\[
(\lambda a.E_b(a))(E_d)
\]

#### B.7.2 Recursive Let Expressions

Recursive let expressions are written as:

\[
\text{let } f = \lambda a:A \cdot E(f) \text{ in } B(f,a) \text{ end}
\]

is “the same” as:

\[
\text{let } f = YF \text{ in } B(f,a) \text{ end}
\]

where:

\[
F \equiv \lambda g.\lambda a.(E(g)) \text{ and } YF = F(YF)
\]

#### B.7.3 Predicative Let Expressions

Predicative let expressions:

\[
\text{let } a:A \cdot \mathcal{P}(a) \text{ in } B(a) \text{ end}
\]

express the selection of a value \( a \) of type \( A \) which satisfies a predicate \( \mathcal{P}(a) \) for evaluation in the body \( B(a) \).
B.7 Other Applicative Expressions

B.7.4 Pattern and “Wild Card” let Expressions

Patterns and wild cards can be used:

```
let {a} ∪ s = set in ... end
let {a, _} ∪ s = set in ... end

let (a, b, ..., c) = cart in ... end
let (a, _, b) = cart in ... end

let ⟨a⟩^ℓ = list in ... end
let ⟨a, _, b⟩^ℓ = list in ... end

let [a↦→ b] ∪ m = map in ... end
let [a↦→ b, _] ∪ m = map in ... end
```

B.7.4.1 Conditionals

Various kinds of conditional expressions are offered by RSL:

```
if b expr then c expr else a expr end

if b expr then c expr end ≡ /* same as: */
if b expr then c expr else skip end

if b expr_1 then c expr_1
elsif b expr_2 then c expr_2
elsif b expr_3 then c expr_3
... elsif b expr_n then c expr_n end

case expr of
  choice_pattern_1 → expr_1,
  choice_pattern_2 → expr_2,
  ...
  choice_pattern_n_or_wild_card → expr_n
end
```

B.7.5 Operator/Operand Expressions

```
⟨Expr⟩ ::= ⟨Prefix_Op⟩ ⟨Expr⟩
| ⟨Expr⟩ ⟨Infix_Op⟩ ⟨Expr⟩
| ⟨Expr⟩ ⟨Suffix_Op⟩
| ...

⟨Prefix_Op⟩ ::= − | ~ | ∪ | ∩ | card | len | inds | elems | hd | tl | dom | rng
```
An RSL (Raise Specification Language) Primer

⟨Infix Op⟩ ::= 
= | ≠ | + | − | ↑ | ↓ | ≤ | ≥ | > | ∧ | ∨ | ⇒ | < | ≤ | ≥ | > | ∈ | nelement |
⟨Suffix Op⟩ ::= !

B.8 Imperative Constructs

B.8.1 Statements and State Changes

Often, following the RAISE method, software development starts with highly abstract-applicative constructs which, through stages of refinements, are turned into concrete and imperative constructs. Imperative constructs are thus inevitable in RSL.

Unit value
stmt: Unit → Unit
stmt()

- Statements accept no arguments.
- Statement execution changes the state (of declared variables).
- Unit → Unit designates a function from states to states.
- Statements, stmt, denote state-to-state changing functions.
- Writing () as “only” arguments to a function “means” that () is an argument of type Unit.

B.8.2 Variables and Assignment

0. variable v:Type := expression
1. v := expr

B.8.3 Statement Sequences and skip

Sequencing is expressed using the ‘;’ operator. skip is the empty statement having no value or side-effect.

2. skip
3. stm_1;stm_2;...;stm_n

B.8.4 Imperative Conditionals

4. if expr then stm_c else stm_a end
5. case e of: p_1→S_1(p_1),...,p_n→S_n(p_n) end
B.10 Process Constructs

B.10.5 Iterative Conditionals

6. while expr do stmt end
7. do stmt until expr end

B.9 Iterative Sequencing

8. for e in list\_expr \cdot P(b) do S(b) end

B.10 Process Constructs

B.10.1 Process Channels

As for channels we deviate from common RSL [66] in that we directly declare channels – and not via common RSL objects etc. Let A and B stand for two types of (channel) messages and i:K\_Idx for channel array indexes, then:

channel c:A
channel { k[i]:B \cdot i:Idx }
channel { k[j],...,k]:B \cdot i:Idx,j:Jdx,...,k:Kdx }

declare a channel, c, and a set (an array) of channels, k[i], capable of communicating values of the designated types (A and B).

B.10.2 Process Composition

Let P and Q stand for names of process functions, i.e., of functions which express willingness to engage in input and/or output events, thereby communicating over declared channels. Let P() and Q stand for process expressions, then:

P || Q Parallel composition
P [\_] Q Nondeterministic external choice (either/or)
P [\_\_] Q Nondeterministic internal choice (either/or)
P -\_\_ Q Interlock parallel composition

express the parallel (||) of two processes, or the nondeterministic choice between two processes: either external ([\_]) or internal ([\_\_]). The interlock (-\_\_) composition expresses that the two processes are forced to communicate only with one another, until one of them terminates.

B.10.3 Input/Output Events

Let c, k[i] and e designate channels of type A and B, then:
expresses the willingness of a process to engage in an event that “reads” an input, respectively “writes” an output.

### B.10.4 Process Definitions

The below signatures are just examples. They emphasise that process functions must somehow express, in their signature, via which channels they wish to engage in input and output events.

{\[
\text{value}
\]

\[
P: \text{Unit} \rightarrow \text{in c out k[i]}
\]

\[
\text{Unit}
\]

\[
Q: \text{i:Kdx} \rightarrow \text{out c in k[i] Unit}
\]

\[
P() \equiv ... \text{c? ... k[i]! e ...}
\]

\[
Q(i) \equiv ... \text{k[i]? ... c! e ...}
\]

The process function definitions (i.e., their bodies) express possible events.

### B.11 RSL Module Specifications

We shall not include coverage nor use of the RSL module concepts of schemes, classes and objects.

### B.12 Simple RSL Specifications

Often, we do not want to encapsulate small specifications in schemas, classes, and objects, as is often done in RSL. An RSL specification is simply a sequence of one or more types, values (including functions), variables, channels and axioms:

- type
  - ...
- variable
  - ...
- channel
  - ...
- value
  - ...
- axiom
  - ...
TO BE WRITTEN
Appendix C
Indexes

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C.2 Concepts

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MORE TO COME
C.7 Analysis Predicate Prompts

There are 17 analysis predicates.

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p : \{\{p_j\} \cup \{p_j\}, 157} \\
p : \{|i|p_j\}, 157} \\
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