The 7 Seas

A Domain of Waterways, Vessels and Harbours

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Abstract

We present a domain description of "The 7 Seas" ! By "The 7 Seas" we mean the navigable segments of oceans, the canals between oceans, and the navigable segments of rivers, canals and lakes "reachable" from oceans. To this we "add" ship, boats and vessels, as well as the harbours they sail between, and hence the land masses where these harbours are located. We focus on these and their relations.

Contents

1 Introduction

In this model we shall treat waterways, not as fluids, but as solids! That is, we may considers waterways as parts, and hence, by transcendental deductions, as possibly having behaviours. Similarly we shall consider many composite endurants, not as elements of structures, but as parts, while not considering their internal qualities, that is, not considering their possible behaviours.

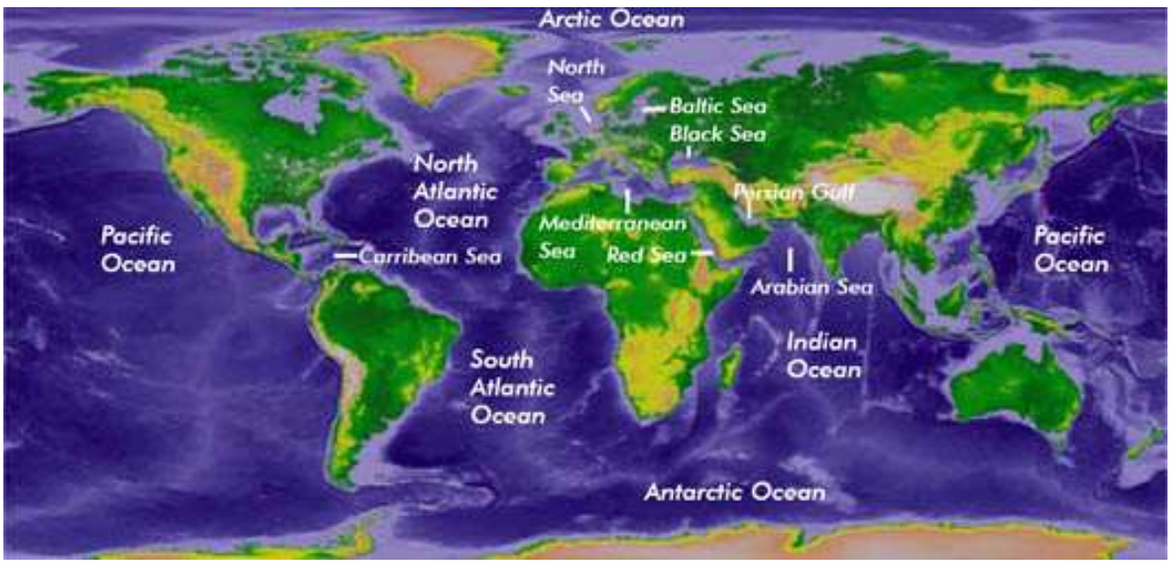
2 Endurants

2.1 External Qualities

2.1.1 Informal Introduction

- Waterways include seas, rivers and navigable "k"anals.
- One can take the view that there are the following eight seas: the *Arctic Ocean*, the *North Atlantic Ocean*, the *South Atlantic Ocean*, the *Indian Ocean*, the *North Pacific Ocean*, the *South Pacific Ocean*, the *Southern* (or *Antarctic*) *Ocean*, and the *Kaspian Sea*. Another view "collapses" the north and south into one, leaving just 6 oceans and seas. Yet a third view is that there are just 2 oceans and seas: The *Kaspian Sea* and the others – since they are all "tightly" connected! The *Kaspian Sea* cannot be reached by ship or boat from the ocean[s]! *The Mediterranean* and

The Black Seas are both considered segments of *The Atlantic Ocean*. *The Arab Sea* is considered a segment of *The Indian Ocean*. Etcetera.



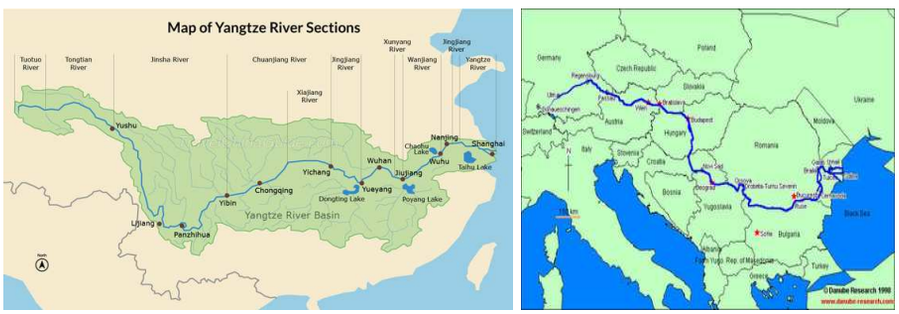
A World Map of Oceans and Seas

The Mediterranean and Arab Seas

The Black Sea and the Kaspian Ocean

- By navigable rivers, "k"anals and status mean such rivers, "k"anals and straits that are connected to the seas and can be navigated by boats and ships. Such areas of rivers and "k"anals that are not navigable by ocean-going boats and ships are area-wise elements of "their" continents. Notice that we "lump" "k"anals and straits:

The Mississippi and the Amazon Rivers



The Yang Tse and the Danube Rivers

The Panama and Suez Canals

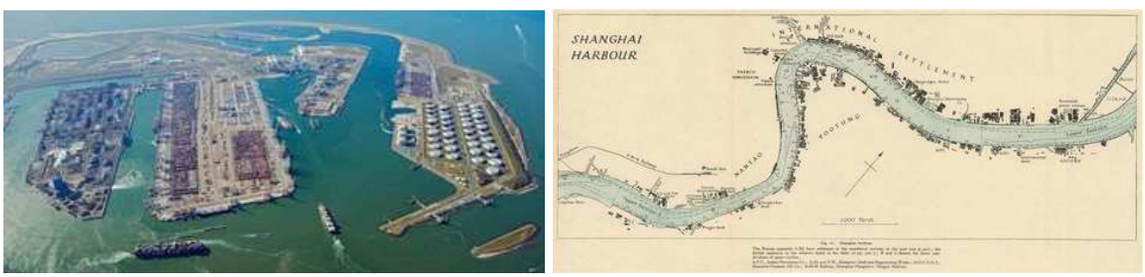
The Gibraltar and Malacca Straits

- By continents we loosely mean some connected land area.

The left map counts *Central America*, *The Caribbean* and *Middle East* as continents!

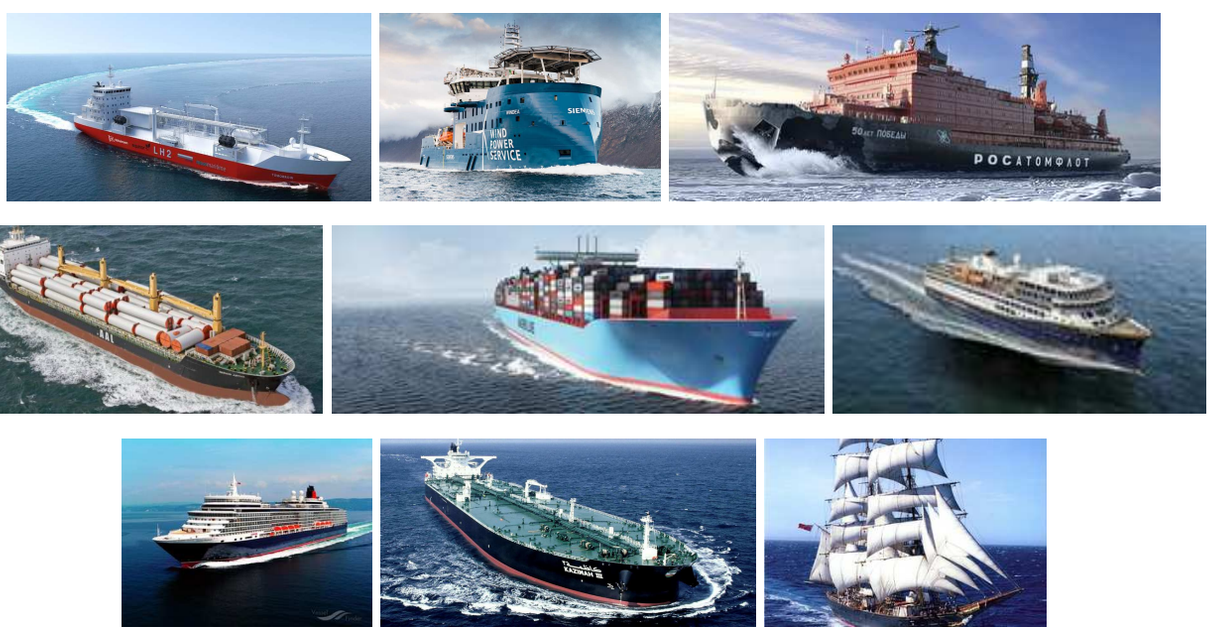
- By harbours we mean places at the edge of continents, seas, rivers, "k"anals and straits where vessels can berth, unload and load cargo and/or passengers.

Singapore and Los Angeles Harbours



Rotterdam and Shanghai Harbours

- By vessels we mean ocean-going ships and boats. Without loss of generality we omit consideration of such vessels as floats, barges, etc.



Miscellaneous Vessels

2.1.2 Formal Introduction

2.1.2.1 Parts and Fluids

1. "The 7 Seas" is a structure composite of the waterways, the continents, the harbours and the vessels.
2. The waterways aggregate consists of an structure composite of a fluids: seas, rivers and "k"anal/straits aggregates.
3. The seas aggregate is a set of seas.
4. The rivers aggregate is a set of [atomic] rivers.
5. The "k"anal/straits aggregate is a set of [atomic] "k"anals and straits.
6. The continents aggregate is a set of [atomic] continents.
7. The harbour aggregate is a set of [atomic] harbours.
8. The Vessel aggregate is a set of [atomic] vessels.

type
1. 7Seas, WA, CA, HA, VA
2. SA, RA, KA
3. Ss = S-**set**
4. Rs = R-**set**
5. Ks = K-**set**
6. Cs = C-**set**
7. Hs = H-**set**
8. Vs = V-**set**

value
1. obs_WA: 7Seas→WA, obs_CA: 7Seas→CA, obs_HA: 7Seas→HA, obsVA: 7Seas→VA
2. obs_SA: WA → SA, obs_RA: WA → RA, obs_KA: WA → KA
3. obs_Ss: SA → Ss
4. obs_Rs: RA → Rs
5. obs_Ks: KA → Ks
6. obs_Cs: CA → Cs
7. obs_Hs: HA → Hs
8. obs_Vs: VA → Vs

2.1.2.2 The 7 Seas State

9. By "The 7 Seas state" we mean the collection of all atomic "The 7 Seas" endurants – a collection which is the distributed union of all continents, rivers, canals, continents, harbours and vessels.

value
1. $7seas$:7Seas
3. ss:Ss = obs_Ss(obs_SA(obs_WA($7seas$)))
4. rs:Rs = obs_Rs(obs_RA(obs_WA($7seas$)))
5. ks:Ks = obs_Ks(obs_KA(obs_WA($7seas$)))
6. cs:Cs = obs_Cs(obs_CA($7seas$))

7. hs:Hs = obs_Hs(obs_HA($7seas$))
8. vs:Vs = obs_Vs(obs_VA($7seas$))
9. 7σ:(S|R|K|C|H|V)-**set** = $ss \cup rs \cup ks \cup cs \cup hs \cup vs$

Please not the *type font* names for the state values.

2.2 Internal Qualities

2.2.1 Unique Identifiers

2.2.1.1 Observers

10.

type
10. SI, RI, KI, CI, HI, VI

value
10. uid_S: S→SI, uid_R: R→RI, uid_K: K→KI, uid_C: C→CI, uid_H: H→HI, uid_V: V→VI

2.2.1.2 All Unique Identifiers

11. We can calculate the sets of all sea, river, canal, continent, harbor and vessel identifiers,
12. as well as the set of all atomic part and fluid identifiers of the 7 Seas domain.

value
11. sis:SI-**set** = {uid_S(s)|s:S•s ∈ ss}
11. ris:RI-**set** = {uid_R(r)|r:R•r ∈ rs}
11. kis:KI-**set** = {uid_K(k)|k:K•k ∈ ks}
11. cis:CI-**set** = {uid_C(c)|c:C•c ∈ cs}
11. his:HI-**set** = {uid_H(h)|h:H•h ∈ hs}
11. vis:VI-**set** = {uid_V(v)|v:V•v ∈ vs}
12. $7is$:(SI|RI|KI|CI|HI|VI)-**set** = $sis \cup ris \cup kis \cup cis \cup his \cup vis$

2.2.1.3 Axiom

13. All atomic parts and separate fluids have unique identifiers.

axiom
13. **card** 7σ = **card** ais

2.2.1.4 Extraction of Atomic Elements

14. From a sea identifier we can, via the domain state ss, obtain the seal.
15. From a river identifier we can, via the domain state rs, obtain the river.
16. From a canal identifier we can, via the domain state ks, obtain the canal.

17. From a continent identifier we can, via the domain state cs, obtain the continent.
18. From a harbour identifier we can, via the domain state hs, obtain the harbour.
19. From a vessel identifier we can, via the domain state vs, obtain the vessel.

value
14. xtr_S: SI→S; xtr_S(si) ≡ **let** s:S • s ∈ ss ∧ uid_S(s) = si **in** s **end**
15. xtr_R: RI→R; xtr_R(ri) ≡ **let** r:R • r ∈ rs ∧ uid_R(r) = ri **in** r **end**
16. xtr_K: KI→K; xtr_K(ki) ≡ **let** k:K • k ∈ ks ∧ uid_K(k) = ki **in** k **end**
17. xtr_C: CI→C; xtr_C(ci) ≡ **let** c:C • c ∈ cs ∧ uid_C(c) = ci **in** c **end**
18. xtr_H: HI→H; xtr_H(hi) ≡ **let** h:H • h ∈ hs ∧ uid_H(h) = hi **in** h **end**
19. xtr_V: VI→V; xtr_V(vi) ≡ **let** v:V • v ∈ vs ∧ uid_V(v) = vi **in** v **end**

2.2.2 Mereology

2.2.2.1 Types, Observers and Axioms

2.2.2.1.1 Seas

20. The mereology of a sea is a triplet of the sets of unique identifiers of
    - the vessels that may sail on it,
    - the continents that borders it and
    - the harbours that confront it.

type
20. MS = VI-**set** × CI-**set** × HI-**set**
value
20. mereo_S: S → MS
axiom
20. ∀ s:S: s ∈ ss ⇒ **let** (vis,cis,his) = mereo_S(s) **in** vis ⊆ vis ∧ cis ⊆ cis ∧ his ⊆ his **end**

2.2.2.1.2 Rivers

21. The mereology of a river is the triplet of
    - the non-empty set of unique identifiers of the continents it is embedded in,
    - the [one] unique identifier of the sea (or ocean) it is connected to, and
    - the set of unique identifiers of the vessels that may sail on that river.

type
21. MR = CI-**set** × SI × VI-**set**
value
21. mereo_R: R → MR
axiom
21. ∀ r:R: r ∈ rs ⇒ **let** (cis,si,vis) = mereo_R(r) **in** {} ≠ cis ⊆ cis ∧ si ∈ sis ∧ vis ⊆ vis **end**

2.2.2.1.3 Canals and Straits

22. The mereology of a canal or a strait is the triplet of
    - a set of one or two unique identifiers of the seas that the canal or strait connects,
    - the set of unique identifiers of the harbours it offers,
    - the set of unique identifiers of the vessels that may sail through the canal or strait.

type
22. MK = SI-**set** × HI_set × VI-**set**
value
22. mereo_K: K → MK
axiom
22. ∀ r:K: k ∈ ks ⇒ **let** (sis,cis,vis) =
mereo_K(k) **in** 1 ≤**card** sis ≤ 2 ∧ sis ⊆ *sis* ∧ his ∈ *his* ∧ vis ⊆ *vis* **end**

2.2.2.1.4 Continents

23. The mereology of a continent is the triplet of
    - the set of unique identifiers of the [other[1]] continents that the continent borders with,
    - the set of unique identifiers of the harbours on that continent, and
    - the set of unique identifiers of the rivers flowing through that continent.

type
23. MC = CI-**set** × HI_set × RI-**set**
value
23. mereo_C: C → MC
axiom
23. ∀ c:C: c ∈ *cs* ⇒ **let** (cis,his,ris) = mereo_C(c) **in** cis ⊆ *cis* ∧ his ⊆ *his* ∧ ris ⊆ *ris* **end**

2.2.2.1.5 Harbours

24. The mereology of a harbour is the triplet of
    - the unique identifier of the continent to which the harbour belongs, and
    - the set of unique identifiers of the vessels that may berth at that harbour.

type
24. MH = CI × VI-**set**
value
24. mereo_H: H → MH
axiom
24. ∀ h:H • h ∈ *hs* ⇒ **let** (ci,vis) = mereo_H(j) **in** ci ∈ *cis* ∧ vis ∈ *vis* **end**

[1] The **axiom** (22) does not model "the other" clause!

2.2.2.1.6 Vessels

25. The mereology of a vessel is the pair of
    - the set of unique identifiers of the seas on which the vessel may sail, and
    - the set of unique identifiers of the harbours at which the vessel may berth,

type
25. MV = SI-**set** × HI-**set**
value
25. mereo_V: V → MV
axiom
25. ∀ v:V • v ∈ *vis* ⇒ **let** (sis,his) = mereo_V(v) **in** sis ⊆ *sis* ∧ his ⊆ *his* **end**

2.2.2.2 A Remark Please note that we have not [yet] had a need to describe the sea and land 𝔸ℝ𝔼𝔸s of seas and continents.

2.2.2.3 A Domain Axiom The axioms of Sect. 2.2.2.1 pertains to the individual atomic elements of the domain, not to their occurrence in the context of the aggregates to which they are elements.

26. The mereology of a sea of a domain states the unique identifiers of the vessels that may sail on it, so we must, vice-versa, expect that the mereology of the identified vessels likewise identify that sea as one on which it may sail.

axiom
26. ∀ s:S • s ∈ *ss* ⇒
26. **let** (vis,cis,his) = mereo_S(s) **in**
26. ∀ vi:VI • vi ∈ vis ⇒
26. **let** v:V • v = xtr_V(vi) **in**
26. **let** (sis,his) = mereo_V(v) **in**
26. uid_S(s) ∈ sis **end end end**

We leave it to the reader to narrate and formalise similar "cross-mereology" axioms for [all other] relevant "pairs" of different sort atomic elements of the domain.

2.2.3 Attributes

Seas, rivers, canals, continents and harbours have spatial attributes of kind 𝕊𝕌ℝ𝔽𝔸ℂ𝔼, 𝕃𝕀ℕ𝔼 and ℙ𝕆𝕀ℕ𝕋. We refer to [1, Sect. 3.4].

2.2.3.1 Seas

27. We ascribe names to seas.
28. Seas spread over contiguous surface (𝕊𝕌ℝ𝔽𝔸ℂ𝔼).
29. Seas have borders/edges (𝕃𝕀ℕ𝔼).
30.
31.

32.

33.

type
27. SeaName
28. SeaSurface = 𝕊𝕌ℝ𝔽𝔸ℂ𝔼
29. SeaBorder = 𝕃𝕀ℕ𝔼
30.
31.
32.

value
27. attr_SeaName: S → SeaName
28. attr_SeaSurface: S → SeaSurface
29. attr_SeaBorder: S → SeaBorder
30. attr_: →
31. attr_: →
32. attr_: →

2.2.3.2 Rivers

34.

35.

36.

37.

38.

39.

40.

type
27.
28.
29.
30.
31.
32.

value
27. attr_: →
28. attr_: →
29. attr_: →
30. attr_: →
31. attr_: →
32. attr_: →

2.2.3.3 Canals and Straits

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42.

43.

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45.

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47.

type
41.
42.
43.
44.
45.
46.
value
41. attr_: →
42. attr_: →
43. attr_: →
44. attr_: →
45. attr_: →
46. attr_: →

2.2.3.4 Continents

48.

49.

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53.

54.

type
48.
49.
50.
51.
52.

53.
value
48. attr_: →
49. attr_: →
50. attr_: →
51. attr_: →
52. attr_: →
53. attr_: →

2.2.3.5 Harbours

55.

56.

57.

58.

59.

60.

61.

type
55.
56.
57.
58.
59.
60.
value
55. attr_: →
56. attr_: →
57. attr_: →
58. attr_: →
59. attr_: →
60. attr_: →

2.2.3.6 Vessels

62. Vessels have names.
63. Vessels have kind: passenger, ordinary freight, crude oil, container, ...
64. Vessels, at any one "point" in time has a position.
65. Vessels, when sailing, follow a route.
66. Vessel positions are well-formed if they are on the current route.

67. Vessels have a speed

68. and a velocity.

69. A vessel is **on course** if its position (at some time) is on that vessel's route.

type
62. VesselName
63. VesselKind = ...
64. VesselPos = TIME × POSITION
65. VesselRoute = BezierCurve
67. VesselSpeed
67. VesselVelocity

value
62. attr_VesselName: V → VesselName
63. attr_VesselKind: V → VesselKind
64. attr_VesselPos: V → VesselPos
65. attr_VesselRoute: V → VesselRoute
67. attr_VesselSpeed: V → Speed
68. attr_VesselVelocity: V → Velocity
69. Vessel_on_course: V → **Bool**
69. Vessel_on_course(v) ≡ **let** (vp,_) = attr_VesselPos(v) **in** Position_on_curve(vp,attr_VesselRoute(v)) **end**
69. Position_on_curve: POSITION × Bezier → **Bool**

3 Perdurants

3.1 Channels

3.2 Behaviours

3.3 Signatures

3.4 Definitions

3.5 System

4 Conclusion

5 Bibliography

5.1 Bibliographical Notes

References

[1] Dines Bjørner. *Domain Science & Engineering – A Foundation for Software Development*. EATCS Monographs in Theoretical Computer Science. Springer, 2021.

A Indexes

A.1 Sorts and Types

A.2 Predicates and Functions

A.3 Values

A.4 Axioms