```
/* x8_3.sas */
/* Purpose:  */

title1 'Example 8.3';

data a;
input x1 x2 x3 x4 x5 x6 x7;
cards;
 3.76    3.66    0.54     5.275     9.768    13.741    4.782
 8.59    4.99    1.34    10.022     7.500    10.162    2.130
 6.22    6.14    4.52     9.842     2.175     2.732    1.089
 7.57    7.28    7.07    12.662     1.791     2.101    0.822
 9.03    7.08    2.59    11.762     4.539     6.217    1.276
 5.51    3.98    1.30     6.924     5.326     7.304    2.403
 3.27    0.62    0.44     3.357     7.629     8.838    8.389
 8.74    7.00    3.31    11.675     3.529     4.757    1.119
 9.64    9.49    1.03    13.567    13.133    18.519    2.354
 9.73    1.33    1.00     9.871     9.871    11.064    3.704
 8.59    2.98    1.17     9.170     7.851     9.909    2.616
 7.12    5.49    3.68     9.716     2.642     3.430    1.189
 4.69    3.01    2.17     5.983     2.760     3.554    2.013
 5.51    1.34    1.27     5.808     4.566     5.382    3.427
 1.66    1.61    1.57     2.799     1.783     2.087    3.716
 5.90    5.76    1.55     8.388     5.395     7.497    1.973
 9.84    9.27    1.51    13.604     9.017    12.668    1.745
 8.39    4.92    2.54    10.053     3.956     5.237    1.432
 4.94    4.38    1.03     6.678     6.494     9.059    2.807
 7.23    2.30    1.77     7.790     4.393     5.374    2.274
 9.46    7.31    1.04    11.999    11.579    16.182    2.415
 9.55    5.35    4.25    11.742     2.766     3.509    1.054
 4.94    4.52    4.50     8.067     1.793     2.103    1.292
 8.21    3.08    2.42     9.097     3.753     4.657    1.719
 9.41    6.44    5.11    12.495     2.446     3.103    0.914
;
proc print data=a;
title2 'List of data.';

proc factor data=a eigenvectors mineigen=0;
  var x1-x7;
title2 'Principal factor analysis - all factors.';

proc factor data=a eigenvectors nfactors=2 residuals
    preplot nplot=2 plot rotate=varimax;
  var x1-x7;
title2 'Principal factor analysis - 2 factors + varimax rotation.';

run;
```

Example 8.3 1
List of data.

Obs	x1	x2	x3	x4	x5	x6	x7
1	3.76	3.66	0.54	5.275	9.768	13.741	4.782
2	8.59	4.99	1.34	10.022	7.500	10.162	2.130
3	6.22	6.14	4.52	9.842	2.175	2.732	1.089
4	7.57	7.28	7.07	12.662	1.791	2.101	0.822
5	9.03	7.08	2.59	11.762	4.539	6.217	1.276
6	5.51	3.98	1.30	6.924	5.326	7.304	2.403
7	3.27	0.62	0.44	3.357	7.629	8.838	8.389
8	8.74	7.00	3.31	11.675	3.529	4.757	1.119
9	9.64	9.49	1.03	13.567	13.133	18.519	2.354
10	9.73	1.33	1.00	9.871	9.871	11.064	3.704
11	8.59	2.98	1.17	9.170	7.851	9.909	2.616
12	7.12	5.49	3.68	9.716	2.642	3.430	1.189
13	4.69	3.01	2.17	5.983	2.760	3.554	2.013
14	5.51	1.34	1.27	5.808	4.566	5.382	3.427
15	1.66	1.61	1.57	2.799	1.783	2.087	3.716
16	5.90	5.76	1.55	8.388	5.395	7.497	1.973
17	9.84	9.27	1.51	13.604	9.017	12.668	1.745
18	8.39	4.92	2.54	10.053	3.956	5.237	1.432
19	4.94	4.38	1.03	6.678	6.494	9.059	2.807
20	7.23	2.30	1.77	7.790	4.393	5.374	2.274
21	9.46	7.31	1.04	11.999	11.579	16.182	2.415
22	9.55	5.35	4.25	11.742	2.766	3.509	1.054
23	4.94	4.52	4.50	8.067	1.793	2.103	1.292
24	8.21	3.08	2.42	9.097	3.753	4.657	1.719
25	9.41	6.44	5.11	12.495	2.446	3.103	0.914

Example 8.3 2
Principal factor analysis - all factors.

The FACTOR Procedure
Initial Factor Method: Principal Components

Prior Communality Estimates: ONE

Eigenvalues of the Correlation Matrix: Total = 7 Average = 1

	Eigenvalue	Difference	Proportion	Cumulative
1	3.39464225	0.58916459	0.4849	0.4849
2	2.80547765	2.36816685	0.4008	0.8857
3	0.43731080	0.15936357	0.0625	0.9482
4	0.27794723	0.19699468	0.0397	0.9879
5	0.08095255	0.07754549	0.0116	0.9995
6	0.00340706	0.00314459	0.0005	1.0000
7	0.00026247		0.0000	1.0000

7 factors will be retained by the NFACTOR criterion.

Eigenvectors

	1	2	3	4
x1	0.40529	0.29290	-0.66736	0.08884
x2	0.43158	0.22244	0.69799	-0.03378
x3	0.38544	-0.35588	0.14769	0.62755
x4	0.49389	0.23227	-0.11864	0.21029
x5	-0.12771	0.57510	0.02944	0.11085
x6	-0.09680	0.58000	0.17430	-0.00615
x7	-0.48094	0.13030	0.01759	0.73525

Eigenvectors

	5	6	7
x1	-0.22666	0.40981	0.27816
x2	-0.43657	0.14430	0.25399
x3	0.51215	0.18753	0.10809
x4	-0.10543	-0.58781	-0.53590
x5	0.38899	-0.42321	0.55621
x6	0.35494	0.50035	-0.49747
x7	-0.45532	0.03316	-0.04894

Factor Pattern

	Factor1	Factor2	Factor3	Factor4
x1	0.74673	0.49059	-0.44132	0.04684
x2	0.79517	0.37258	0.46158	-0.01781
x3	0.71016	-0.59609	0.09767	0.33085
x4	0.90997	0.38904	-0.07846	0.11087
x5	-0.23530	0.96328	0.01947	0.05844
x6	-0.17836	0.97148	0.11527	-0.00324
x7	-0.88611	0.21824	0.01164	0.38763

Example 8.3 3
Principal factor analysis - all factors.

The FACTOR Procedure
Initial Factor Method: Principal Components

Factor Pattern

	Factor5	Factor6	Factor7
x1	-0.06449	0.02392	0.00451
x2	-0.12421	0.00842	0.00411
x3	0.14572	0.01095	0.00175
x4	-0.03000	-0.03431	-0.00868
x5	0.11068	-0.02470	0.00901
x6	0.10099	0.02921	-0.00806
x7	-0.12955	0.00194	-0.00079

Variance Explained by Each Factor

Factor1	Factor2	Factor3	Factor4
3.3946422	2.8054777	0.4373108	0.2779472

Factor5	Factor6	Factor7
0.0809525	0.0034071	0.0002625

Final Communality Estimates: Total = 7.000000

x1	x2	x3	x4
1.0000000	1.0000000	1.0000000	1.0000000

x5	x6	x7
1.0000000	1.0000000	1.0000000

```
                               Example 8.3                                     4
           Principal factor analysis - 2 factors + varimax rotation.

                            The FACTOR Procedure
                  Initial Factor Method: Principal Components

                      Prior Communality Estimates: ONE



          Eigenvalues of the Correlation Matrix: Total = 7  Average = 1

                  Eigenvalue    Difference    Proportion    Cumulative

             1    3.39464225    0.58916459        0.4849        0.4849
             2    2.80547765    2.36816685        0.4008        0.8857
             3    0.43731080    0.15936357        0.0625        0.9482
             4    0.27794723    0.19699468        0.0397        0.9879
             5    0.08095255    0.07754549        0.0116        0.9995
             6    0.00340706    0.00314459        0.0005        1.0000
             7    0.00026247                      0.0000        1.0000

              2 factors will be retained by the NFACTOR criterion.



                               Eigenvectors

                                     1               2

                       x1         0.40529         0.29290
                       x2         0.43158         0.22244
                       x3         0.38544        -0.35588
                       x4         0.49389         0.23227
                       x5        -0.12771         0.57510
                       x6        -0.09680         0.58000
                       x7        -0.48094         0.13030


                              Factor Pattern

                               Factor1         Factor2

                       x1         0.74673         0.49059
                       x2         0.79517         0.37258
                       x3         0.71016        -0.59609
                       x4         0.90997         0.38904
                       x5        -0.23530         0.96328
                       x6        -0.17836         0.97148
                       x7        -0.88611         0.21824


                        Variance Explained by Each Factor

                               Factor1         Factor2

                             3.3946422       2.8054777


                  Final Communality Estimates: Total = 6.200120

                  x1              x2              x3              x4
```



```
           0.79829189      0.77111321      0.85964214      0.97940083
```

Example 8.3 5
Principal factor analysis - 2 factors + varimax rotation.

The FACTOR Procedure
Initial Factor Method: Principal Components

x5	x6	x7
0.98326505	0.97558652	0.83282025

Residual Correlations With Uniqueness on the Diagonal

	x1	x2	x3	x4
x1	0.20171	-0.19631	-0.03673	0.04089
x2	-0.19631	0.22889	0.02119	-0.03479
x3	-0.03673	0.02119	0.14036	0.02426
x4	0.04089	-0.03479	0.02426	0.02060
x5	-0.01354	-0.00597	0.03711	0.00240
x6	-0.05687	0.04093	0.02521	-0.01336
x7	0.02142	0.01457	0.11053	0.04589

Residual Correlations With Uniqueness on the Diagonal

	x5	x6	x7
x1	-0.01354	-0.05687	0.02142
x2	-0.00597	0.04093	0.01457
x3	0.03711	0.02521	0.11053
x4	0.00240	-0.01336	0.04589
x5	0.01673	0.01244	0.00849
x6	0.01244	0.02441	-0.01294
x7	0.00849	-0.01294	0.16718

Root Mean Square Off-Diagonal Residuals: Overall = 0.05628060

x1	x2	x3	x4
0.08701933	0.08378398	0.05262327	0.03098581

Root Mean Square Off-Diagonal Residuals: Overall = 0.05628060

x5	x6	x7
0.01745789	0.03174285	0.05038563

Partial Correlations Controlling Factors

	x1	x2	x3	x4
x1	1.00000	-0.91361	-0.21832	0.63437
x2	-0.91361	1.00000	0.11821	-0.50661
x3	-0.21832	0.11821	1.00000	0.45110
x4	0.63437	-0.50661	0.45110	1.00000
x5	-0.23311	-0.09649	0.76569	0.12930
x6	-0.81043	0.54754	0.43061	-0.59594
x7	0.11663	0.07449	0.72153	0.78197

Example 8.3 6
Principal factor analysis - 2 factors + varimax rotation.

The FACTOR Procedure
Initial Factor Method: Principal Components

Partial Correlations Controlling Factors

	x5	x6	x7
x1	-0.23311	-0.81043	0.11663
x2	-0.09649	0.54754	0.07449
x3	0.76569	0.43061	0.72153
x4	0.12930	-0.59594	0.78197
x5	1.00000	0.61535	0.16044
x6	0.61535	1.00000	-0.20248
x7	0.16044	-0.20248	1.00000

Root Mean Square Off-Diagonal Partials: Overall = 0.51266620

x1	x2	x3	x4
0.57872367	0.48648159	0.50948347	0.55466516

Root Mean Square Off-Diagonal Partials: Overall = 0.51266620

x5	x6	x7
0.42250087	0.56521745	0.45055121

```
                        Example 8.3                                     7
           Principal factor analysis - 2 factors + varimax rotation.

                          The FACTOR Procedure
                Initial Factor Method: Principal Components

Plot of Factor Pattern for Factor1 and Factor2

                                     Factor1
                                        1

                                       .9          D

                                       .8          B
                                                      A
                      C                .7

                                       .6

                                       .5

                                       .4

                                       .3

                                       .2
                                                                        F
                                       .1                               a
                                                                        c
        -1 -.9-.8-.7-.6-.5-.4-.3-.2-.1  0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0t
                                                                        o
                                      -.1                               r
                                                                        2
                                      -.2                           F
                                                                   E
                                      -.3

                                      -.4

                                      -.5

                                      -.6

                                      -.7

                                      -.8

                                      -.9     G

                                       -1

x1=A          x2=B          x3=C          x4=D          x5=E          x6=F
x7=G
```


Example 8.3 8

Principal factor analysis - 2 factors + varimax rotation.

The FACTOR Procedure

Rotation Method: Varimax

Orthogonal Transformation Matrix

	1	2
1	0.92099	-0.38960
2	0.38960	0.92099

Rotated Factor Pattern

	Factor1	Factor2
x1	0.87886	0.16091
x2	0.87750	0.03335
x3	0.42181	-0.82566
x4	0.98964	0.00378
x5	0.15858	0.97883
x6	0.21422	0.96421
x7	-0.73107	0.54622

Variance Explained by Each Factor

Factor1	Factor2
3.3052161	2.8949038

Final Communality Estimates: Total = 6.200120

x1	x2	x3	x4
0.79829189	0.77111321	0.85964214	0.97940083

x5	x6	x7
0.98326505	0.97558652	0.83282025

```
                                  Example 8.3                                    9
           Principal factor analysis - 2 factors + varimax rotation.

                              The FACTOR Procedure
                            Rotation Method: Varimax

Plot of Factor Pattern for Factor1 and Factor2

                                     Factor1
                                        D

                                       .9B   A

                                       .8

                                       .7

                                       .6

                                       .5

               C                       .4

                                       .3

                                       .2                          F
                                                                    E  F
                                       .1                              a
                                                                       c
        -1 -.9-.8-.7-.6-.5-.4-.3-.2-.1  0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0t
                                                                       o
                                      -.1                              r
                                                                       2
                                      -.2

                                      -.3

                                      -.4

                                      -.5

                                      -.6

                                      -.7
                                                        G
                                      -.8

                                      -.9

                                       -1

x1=A          x2=B          x3=C          x4=D          x5=E          x6=F
x7=G
```