```
/* x7_7.sas */
/* Purpose:  */

title1 'Example 7.7.';

data a (type=cov);
  length _type_ $ 8 _name_ $ 8;
  input pop$ _type_ @;
  if _type_='COV' or _type_='CORR' then input _name_ @;
  input x1 x2;
  cards;
pop1 COV  X1       1       1
pop1 COV  X2       1       2
pop1 MEAN          4       2
pop1 N       1000000 1000000
pop2 COV  X1       1       1
pop2 COV  X2       1       2
pop2 MEAN          1       1
pop2 N       1000000 1000000
pop3 COV  X1       1       1
pop3 COV  X2       1       2
pop3 MEAN          2       6
pop3 N       1000000 1000000
;

proc print;
title2 'List of data. NB special SAS-dataset (type=cov). Nobs set "high".';

data b;
  input x1 x2;
cards;
0 0
1 0
2 0
3 0
4 0
0 1
0 2
0 3
0 4
;

proc print data=b;
title2 'List of testdata.';

proc discrim data=a testdata=b testlist pool=yes
    bcov pcov tcov wcov bcorr pcorr tcorr wcorr simple;
  class pop;
  var x1 x2;
title2 'Linear discriminant analysis - equal prior probabilities.';

proc discrim data=a testdata=b testlist pool=yes;
  class pop;
  priors 'pop1'=0.1 'pop2'=0.2 'pop3'=0.7;
  var x1 x2;
title2 'Linear discriminant analysis - unequal prior probabilities.';

run;
```

Example 7.7. 1

List of data. NB special SAS-dataset (type=cov). Nobs set "high".

Obs	_type_	_name_	pop	x1	x2
1	COV	X1	pop1	1	1
2	COV	X2	pop1	1	2
3	MEAN		pop1	4	2
4	N		pop1	1000000	1000000
5	COV	X1	pop2	1	1
6	COV	X2	pop2	1	2
7	MEAN		pop2	1	1
8	N		pop2	1000000	1000000
9	COV	X1	pop3	1	1
10	COV	X2	pop3	1	2
11	MEAN		pop3	2	6
12	N		pop3	1000000	1000000

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x2	0.999999333	6.666666889

Example 7.7. 6
Linear discriminant analysis – equal prior probabilities.

The DISCRIM Procedure
Within-Class Correlation Coefficients / Pr > |r|

pop = pop1

Variable	x1	x2
x1	1.00000	0.70711 <.0001
x2	0.70711 <.0001	1.00000

---

pop = pop2

Variable	x1	x2
x1	1.00000	0.70711 <.0001
x2	0.70711 <.0001	1.00000

---

pop = pop3

Variable	x1	x2
x1	1.00000	0.70711 <.0001
x2	0.70711 <.0001	1.00000

Example 7.7. 7
Linear discriminant analysis - equal prior probabilities.

The DISCRIM Procedure

Pooled Within-Class Correlation Coefficients / Pr > |r|

Variable	x1	x2
x1	1.00000	0.70711 <.0001
x2	0.70711 <.0001	1.00000

Between-Class Correlation Coefficients / Pr > |r|

Variable	x1	x2
x1	1.00000	0.00000 1.0000
x2	0.00000 1.0000	1.00000

Total-Sample Correlation Coefficients / Pr > |r|

Variable	x1	x2
x1	1.00000	0.24227 <.0001
x2	0.24227 <.0001	1.00000

Example 7.7. 8
Linear discriminant analysis - equal prior probabilities.

The DISCRIM Procedure
Simple Statistics

Total-Sample

Variable	N	Sum	Mean	Variance	Standard Deviation
x1	3E6	7000000	2.33333	2.55556	1.5986
x2	3E6	9000000	3.00000	6.66667	2.5820

---

pop = pop1

Variable	N	Sum	Mean	Variance	Standard Deviation
x1	1E6	4000000	4.00000	1.00000	1.0000
x2	1E6	2000000	2.00000	2.00000	1.4142

---

pop = pop2

Variable	N	Sum	Mean	Variance	Standard Deviation
x1	1E6	1000000	1.00000	1.00000	1.0000
x2	1E6	1000000	1.00000	2.00000	1.4142

---

pop = pop3

Variable	N	Sum	Mean	Variance	Standard Deviation
x1	1E6	2000000	2.00000	1.00000	1.0000
x2	1E6	6000000	6.00000	2.00000	1.4142

Pooled Covariance Matrix Information

Covariance Matrix Rank	Natural Log of the Determinant of the Covariance Matrix
2	0

Example 7.7. 9
Linear discriminant analysis - equal prior probabilities.

The DISCRIM Procedure

Pairwise Generalized Squared Distances Between Groups

$$D^2(i|j) = (\bar{X}_i - \bar{X}_j)' \, COV^{-1} \, (\bar{X}_i - \bar{X}_j)$$

Generalized Squared Distance to pop

From pop	pop1	pop2	pop3
pop1	0	13.00000	40.00000
pop2	13.00000	0	17.00000
pop3	40.00000	17.00000	0

Linear Discriminant Function

$$Constant = -.5 \, \bar{X}_j' \, COV^{-1} \, \bar{X}_j \qquad Coefficient\ Vector = COV^{-1} \, \bar{X}_j$$

Linear Discriminant Function for pop

Variable	pop1	pop2	pop3
Constant	-10.00000	-0.50000	-10.00000
x1	6.00000	1.00000	-2.00000
x2	-2.00000	0	4.00000

Example 7.7. 10
Linear discriminant analysis - equal prior probabilities.

The DISCRIM Procedure
Classification Results for Test Data: WORK.B
Classification Results using Linear Discriminant Function

Generalized Squared Distance Function

$$D_j^2(X) = (X-\bar{X}_j)' \, COV^{-1} \, (X-\bar{X}_j)$$

Posterior Probability of Membership in Each pop

$$Pr(j|X) = \exp(-.5 \, D_j^2(X)) \, / \, SUM_k \exp(-.5 \, D_k^2(X))$$

Posterior Probability of Membership in pop

Obs	Classified into pop	pop1	pop2	pop3
1	pop2	0.0001	0.9999	0.0001
2	pop2	0.0110	0.9890	0.0000
3	pop1	0.6225	0.3775	0.0000
4	pop1	0.9959	0.0041	0.0000
5	pop1	1.0000	0.0000	0.0000
6	pop2	0.0000	0.9959	0.0041
7	pop2	0.0000	0.8176	0.1824
8	pop3	0.0000	0.0759	0.9241
9	pop3	0.0000	0.0015	0.9985

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Priors	0.33333	0.33333	0.33333

Linear discriminant analysis – unequal prior probabilities.

The DISCRIM Procedure

Observations	3000000	DF Total	2999999
Variables	2	DF Within Classes	2999997
Classes	3	DF Between Classes	2

Class Level Information

pop	Variable Name	Frequency	Weight	Proportion	Prior Probability
pop1	pop1	1000000	1000000	0.333333	0.100000
pop2	pop2	1000000	1000000	0.333333	0.200000
pop3	pop3	1000000	1000000	0.333333	0.700000

Pooled Covariance Matrix Information

Covariance Matrix Rank	Natural Log of the Determinant of the Covariance Matrix
2	0

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x1	6.00000	1.00000	-2.00000
x2	-2.00000	0	4.00000

Example 7.7. 14

Linear discriminant analysis - unequal prior probabilities.

The DISCRIM Procedure
Classification Results for Test Data: WORK.B
Classification Results using Linear Discriminant Function

Generalized Squared Distance Function

$$D_j^2(X) = (X-\bar{X}_j)' \, COV^{-1} \, (X-\bar{X}_j)$$

Posterior Probability of Membership in Each pop

$$Pr(j|X) = \exp(-.5\, D_j^2(X)) \,/\, SUM_k \exp(-.5\, D_k^2(X))$$

Posterior Probability of Membership in pop

Obs	Classified into pop	pop1	pop2	pop3
1	pop2	0.0000	0.9997	0.0003
2	pop2	0.0055	0.9945	0.0000
3	pop2	0.4519	0.5481	0.0000
4	pop1	0.9919	0.0081	0.0000
5	pop1	0.9999	0.0001	0.0000
6	pop2	0.0000	0.9859	0.0141
7	pop2	0.0000	0.5615	0.4385
8	pop3	0.0000	0.0229	0.9771
9	pop3	0.0000	0.0004	0.9996

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Priors	0.1	0.2	0.7