```
/* x7_1.sas */
/* Purpose:  */

title1 'Example 7.1.';

data a (type=cov);
  length _type_ $ 8 _name_ $ 8;
  input pop$ _type_ @;
  if _type_='COV' or _type_='CORR'
    then input _name_ x1 x2;
    else input        x1 x2;
  cards;
pop1 COV  X1       1       1
pop1 COV  X2       1       2
pop1 MEAN          4       2
pop1 N       1000000 1000000
pop2 COV  X1       1       1
pop2 COV  X2       1       2
pop2 MEAN          1       1
pop2 N       1000000 1000000
;

proc print;
title2 'List of data. NB special SAS-dataset (type=cov). Nobs set "high".';

data b;
  input x1 x2;
cards;
5 6
0 0
1 0
2 0
3 0
4 0
;

proc print data=b;
title2 'List of testdata.';

proc discrim data=a testdata=b testlist pool=yes
    bcov pcov tcov wcov bcorr pcorr tcorr wcorr simple;
  class pop;
  var x1 x2;
title2 'Linear discriminant analysis - equal prior probabilities.';

proc discrim data=a testdata=b testlist pool=yes;
  class pop;
  priors 'pop1'=0.1 'pop2'=0.9;
  var x1 x2;
title2 'Linear discriminant analysis - unequal prior probabilities.';

run;
```

```
                        Example 7.1.                                  1
    List of data. NB special SAS-dataset (type=cov). Nobs set "high".

          Obs    _type_    _name_    pop          x1          x2

           1      COV        X1      pop1          1           1
           2      COV        X2      pop1          1           2
           3      MEAN               pop1          4           2
           4      N                  pop1    1000000     1000000
           5      COV        X1      pop2          1           1
           6      COV        X2      pop2          1           2
           7      MEAN               pop2          1           1
           8      N                  pop2    1000000     1000000

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                        Example 7.1.                                  2
                     List of testdata.

                       Obs    x1    x2

                        1      5     6
                        2      0     0
                        3      1     0
                        4      2     0
                        5      3     0
                        6      4     0

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                        Example 7.1.                                  3
        Linear discriminant analysis - equal prior probabilities.

                       The DISCRIM Procedure

           Observations 2000000          DF Total            1999999
           Variables          2          DF Within Classes   1999998
           Classes            2          DF Between Classes        1


                      Class Level Information

           Variable                                                    Prior
   pop     Name         Frequency        Weight    Proportion    Probability

   pop1    pop1           1000000       1000000      0.500000       0.500000
   pop2    pop2           1000000       1000000      0.500000       0.500000
```



```
                        Example 7.1.                                  4
        Linear discriminant analysis - equal prior probabilities.

                       The DISCRIM Procedure
                 Within-Class Covariance Matrices

                  pop = pop1,     DF = 999999

          Variable                x1                x2

          x1             1.000000000       1.000000000
          x2             1.000000000       2.000000000

--------------------------------------------------------------------------------


                  pop = pop2,     DF = 999999

          Variable                x1                x2

          x1             1.000000000       1.000000000
          x2             1.000000000       2.000000000

--------------------------------------------------------------------------------

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                        Example 7.1.                                  5
        Linear discriminant analysis - equal prior probabilities.

                       The DISCRIM Procedure

     Pooled Within-Class Covariance Matrix,     DF = 1999998

          Variable                x1                x2

          x1             1.000000000       1.000000000
          x2             1.000000000       2.000000000


          Between-Class Covariance Matrix,     DF = 1

          Variable                x1                x2

          x1             4.500000000       1.500000000
          x2             1.500000000       0.500000000


        Total-Sample Covariance Matrix,     DF = 1999999

          Variable                x1                x2

          x1             3.250000625       1.749999875
          x2             1.749999875       2.249999125
```

```
                         Example 7.1.                                      6
       Linear discriminant analysis - equal prior probabilities.

                       The DISCRIM Procedure
         Within-Class Correlation Coefficients  /  Pr > |r|

                           pop = pop1

              Variable            x1            x2

              x1             1.00000       0.70711
                                            <.0001

              x2             0.70711       1.00000
                              <.0001

--------------------------------------------------------------------------------


                           pop = pop2

              Variable            x1            x2

              x1             1.00000       0.70711
                                            <.0001

              x2             0.70711       1.00000
                              <.0001

--------------------------------------------------------------------------------
```



```
                         Example 7.1.                                      7
       Linear discriminant analysis - equal prior probabilities.

                       The DISCRIM Procedure

      Pooled Within-Class Correlation Coefficients  /  Pr > |r|

              Variable            x1            x2

              x1             1.00000       0.70711
                                            <.0001

              x2             0.70711       1.00000
                              <.0001


         Between-Class Correlation Coefficients  /  Pr > |r|

              Variable            x1            x2

              x1             1.00000       1.00000
                                   .             .

              x2             1.00000       1.00000
                                   .             .


         Total-Sample Correlation Coefficients  /  Pr > |r|

              Variable            x1            x2

              x1             1.00000       0.64715
                                            <.0001

              x2             0.64715       1.00000
                              <.0001
```

Example 7.1.
Linear discriminant analysis - equal prior probabilities.

The DISCRIM Procedure
Simple Statistics

Total-Sample

Variable	N	Sum	Mean	Variance	Standard Deviation
x1	2E6	5000000	2.50000	3.25000	1.8028
x2	2E6	3000000	1.50000	2.25000	1.5000

---

pop = pop1

Variable	N	Sum	Mean	Variance	Standard Deviation
x1	1E6	4000000	4.00000	1.00000	1.0000
x2	1E6	2000000	2.00000	2.00000	1.4142

---

pop = pop2

Variable	N	Sum	Mean	Variance	Standard Deviation
x1	1E6	1000000	1.00000	1.00000	1.0000
x2	1E6	1000000	1.00000	2.00000	1.4142

Pooled Covariance Matrix Information

Covariance Matrix Rank	Natural Log of the Determinant of the Covariance Matrix
2	-3.331E-16

Example 7.1.
Linear discriminant analysis - equal prior probabilities.

The DISCRIM Procedure

Pairwise Generalized Squared Distances Between Groups

$$D^2(i|j) = (\bar{X}_i - \bar{X}_j)' \, COV^{-1} \, (\bar{X}_i - \bar{X}_j)$$

Generalized Squared Distance to pop

From pop	pop1	pop2
pop1	0	13.00000
pop2	13.00000	0

Linear Discriminant Function

$$\text{Constant} = -.5 \, \bar{X}'_j \, COV^{-1} \, \bar{X}_j \qquad \text{Coefficient Vector} = COV^{-1} \, \bar{X}_j$$

Linear Discriminant Function for pop

Variable	pop1	pop2
Constant	-10.00000	-0.50000
x1	6.00000	1.00000
x2	-2.00000	-2.22E-16

```
                       Example 7.1.                                10
     Linear discriminant analysis - equal prior probabilities.

                   The DISCRIM Procedure
         Classification Results for Test Data: WORK.B
     Classification Results using Linear Discriminant Function

              Generalized Squared Distance Function

                   2         _         -1   _
                  D (X) = (X-X )' COV   (X-X )
                   j          j              j

          Posterior Probability of Membership in Each pop

                            2                     2
           Pr(j|X) = exp(-.5 D (X)) / SUM exp(-.5 D (X))
                            j         k           k


            Posterior Probability of Membership in pop

                    Classified
              Obs   into pop       pop1      pop2

                1   pop1          0.9707    0.0293
                2   pop2          0.0001    0.9999
                3   pop2          0.0110    0.9890
                4   pop1          0.6225    0.3775
                5   pop1          0.9959    0.0041
                6   pop1          1.0000    0.0000

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                       Example 7.1.                                11
     Linear discriminant analysis - equal prior probabilities.

                   The DISCRIM Procedure
         Classification Summary for Test Data: WORK.B
     Classification Summary using Linear Discriminant Function

              Generalized Squared Distance Function

                   2         _         -1   _
                  D (X) = (X-X )' COV   (X-X )
                   j          j              j

          Posterior Probability of Membership in Each pop

                            2                     2
           Pr(j|X) = exp(-.5 D (X)) / SUM exp(-.5 D (X))
                            j         k           k


        Number of Observations and Percent Classified into pop

                         pop1        pop2       Total

           Total            4           2           6
                        66.67       33.33      100.00

           Priors         0.5         0.5
```

Example 7.1. 12

Linear discriminant analysis - unequal prior probabilities.

The DISCRIM Procedure

Observations	2000000	DF Total	1999999
Variables	2	DF Within Classes	1999998
Classes	2	DF Between Classes	1

Class Level Information

pop	Variable Name	Frequency	Weight	Proportion	Prior Probability
pop1	pop1	1000000	1000000	0.500000	0.100000
pop2	pop2	1000000	1000000	0.500000	0.900000

Pooled Covariance Matrix Information

Covariance Matrix Rank	Natural Log of the Determinant of the Covariance Matrix
2	-3.331E-16

```
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```

Example 7.1. 13

Linear discriminant analysis - unequal prior probabilities.

The DISCRIM Procedure

Pairwise Generalized Squared Distances Between Groups

$$D^2(i|j) = (\bar{X}_i - \bar{X}_j)' \, COV^{-1} (\bar{X}_i - \bar{X}_j) - 2 \ln PRIOR_j$$

Generalized Squared Distance to pop

From pop	pop1	pop2
pop1	4.60517	13.21072
pop2	17.60517	0.21072

Linear Discriminant Function

$$\text{Constant} = -.5 \, \bar{X}_j' \, COV^{-1} \bar{X}_j + \ln PRIOR_j \qquad \text{Coefficient Vector} = COV^{-1} \bar{X}_j$$

Linear Discriminant Function for pop

Variable	pop1	pop2
Constant	-12.30259	-0.60536
x1	6.00000	1.00000
x2	-2.00000	-2.22E-16

```
                               Example 7.1.                                  14
         Linear discriminant analysis - unequal prior probabilities.

                          The DISCRIM Procedure
                 Classification Results for Test Data: WORK.B
          Classification Results using Linear Discriminant Function

                   Generalized Squared Distance Function

                        2         _          -1   _
                       D (X) = (X-X )' COV   (X-X )
                        j          j              j

              Posterior Probability of Membership in Each pop

                                 2                     2
               Pr(j|X) = exp(-.5 D (X)) / SUM exp(-.5 D (X))
                                 j        k            k


                 Posterior Probability of Membership in pop

                          Classified
                   Obs    into pop        pop1      pop2

                     1    pop1          0.7863    0.2137
                     2    pop2          0.0000    1.0000
                     3    pop2          0.0012    0.9988
                     4    pop2          0.1548    0.8452
                     5    pop1          0.9645    0.0355
                     6    pop1          0.9998    0.0002

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                               Example 7.1.                                  15
         Linear discriminant analysis - unequal prior probabilities.

                          The DISCRIM Procedure
                 Classification Summary for Test Data: WORK.B
          Classification Summary using Linear Discriminant Function

                   Generalized Squared Distance Function

                        2         _          -1   _
                       D (X) = (X-X )' COV   (X-X )
                        j          j              j

              Posterior Probability of Membership in Each pop

                                 2                     2
               Pr(j|X) = exp(-.5 D (X)) / SUM exp(-.5 D (X))
                                 j        k            k


            Number of Observations and Percent Classified into pop

                              pop1         pop2        Total

               Total             3            3            6
                             50.00        50.00       100.00

               Priors          0.1          0.9
```