```
/* x5_7.sas */
/* Purpose:  */

title1 'Example 5.7.';

data a;
  do e=1 to 4;
    do d=1 to 2;
      do c=1 to 4;
        do b=1 to 2;
          do a=1 to 2;
            input x @;
            output;
          end;
        end;
      end;
    end;
  end;
  cards;
3810 6340 4080 6660 3004 5470 2833 6220 4210 7640 3910 6510 3810
                                                  8100 4570 8310
6230 9050 6370 8650 5720 8390 5620 7300 6190 9720 6860 9960 8530
                                                  10210 7450 10650
4350 6450 3800 6920 3380 6330 3308 6280 3310 6040 3302 6670 4380
                                                  8010 4860 8100
6250 9090 6700 9000 6060 8770 5820 9180 5660 8540 5690 8720 7130
                                                  10170 7350 9430
3690 6510 3590 6510 2758 5740 2945 6950 3445 6690 3273 5980 4160
                                                  7370 4670 7380
6630 8680 6360 8440 5630 8200 5170 8310 6180 8060 5700 8210 7380
                                                  9560 7610 9400
4390 7470 4700 7210 3810 6830 3320 6020 2684 6010 3080 5630 5270
                                                  8010 4640 7540
7170 9370 7460 8800 6280 9240 6170 8680 5790 7980 5180 7960 7350
                                                  10510 7590 10420
;

proc print data=a;
title2 'List of data.';

proc anova data=a;
  class a b c d e;
  model x=a|b|c|d|e;
title2 'Sums of squares and degrees of freedom from crossed model.';

proc glm data=a;
  class a b c d e;
  model x=a
          b(e*c) a*b(e*c)
          c a*c
          d a*d b*d(e*c) a*b*d(e*c) c*d a*c*d
          e(c) a*e(c) d*e(c) a*d*e(c);
  random
          b(e*c) a*b(e*c)
          c a*c
                b*d(e*c) a*b*d(e*c) c*d a
          e(c) a*e(c) d*e(c) a*d*e(c);
/* random .... /test; cannot be performed due to Var(Error)=. */
title2 'Model as specified.';

proc glm data=a;
  class a b c d e;
```



```
  model x=a
          c a*c
          d a*d
          e(c);
  random
          c a*c
          e(c) /test;
title2 'Resulting model.';

run;
```

Example 5.7.
List of data. 1

Obs	e	d	c	b	a	x
1	1	1	1	1	1	3810
2	1	1	1	1	2	6340
3	1	1	1	2	1	4080
4	1	1	1	2	2	6660
5	1	1	2	1	1	3004
6	1	1	2	1	2	5470
7	1	1	2	2	1	2833
8	1	1	2	2	2	6220
9	1	1	3	1	1	4210
10	1	1	3	1	2	7640
11	1	1	3	2	1	3910
12	1	1	3	2	2	6510
13	1	1	4	1	1	3810
14	1	1	4	1	2	8100
15	1	1	4	2	1	4570
16	1	1	4	2	2	8310
17	1	2	1	1	1	6230
18	1	2	1	1	2	9050
19	1	2	1	2	1	6370
20	1	2	1	2	2	8650
21	1	2	2	1	1	5720
22	1	2	2	1	2	8390
23	1	2	2	2	1	5620
24	1	2	2	2	2	7300
25	1	2	3	1	1	6190
26	1	2	3	1	2	9720
27	1	2	3	2	1	6860
28	1	2	3	2	2	9960
29	1	2	4	1	1	8530
30	1	2	4	1	2	10210
31	1	2	4	2	1	7450
32	1	2	4	2	2	10650
33	2	1	1	1	1	4350
34	2	1	1	1	2	6450
35	2	1	1	2	1	3800
36	2	1	1	2	2	6920
37	2	1	2	1	1	3380
38	2	1	2	1	2	6330
39	2	1	2	2	1	3308
40	2	1	2	2	2	6280
41	2	1	3	1	1	3310
42	2	1	3	1	2	6040
43	2	1	3	2	1	3302
44	2	1	3	2	2	6670
45	2	1	4	1	1	4380
46	2	1	4	1	2	8010
47	2	1	4	2	1	4860
48	2	1	4	2	2	8100
49	2	2	1	1	1	6250
50	2	2	1	1	2	9090
51	2	2	1	2	1	6700
52	2	2	1	2	2	9000
53	2	2	2	1	1	6060
54	2	2	2	1	2	8770
55	2	2	2	2	1	5820
56	2	2	2	2	2	9180
57	2	2	3	1	1	5660

58	2	2	3	1	2	8540
59	2	2	3	2	1	5690
60	2	2	3	2	2	8720

Example 5.7. 2
List of data.

Obs	e	d	c	b	a	x
61	2	2	4	1	1	7130
62	2	2	4	1	2	10170
63	2	2	4	2	1	7350
64	2	2	4	2	2	9430
65	3	1	1	1	1	3690
66	3	1	1	1	2	6510
67	3	1	1	2	1	3590
68	3	1	1	2	2	6510
69	3	1	2	1	1	2758
70	3	1	2	1	2	5740
71	3	1	2	2	1	2945
72	3	1	2	2	2	6950
73	3	1	3	1	1	3445
74	3	1	3	1	2	6690
75	3	1	3	2	1	3273
76	3	1	3	2	2	5980
77	3	1	4	1	1	4160
78	3	1	4	1	2	7370
79	3	1	4	2	1	4670
80	3	1	4	2	2	7380
81	3	2	1	1	1	6630
82	3	2	1	1	2	8680
83	3	2	1	2	1	6360
84	3	2	1	2	2	8440
85	3	2	2	1	1	5630
86	3	2	2	1	2	8200
87	3	2	2	2	1	5170
88	3	2	2	2	2	8310
89	3	2	3	1	1	6180
90	3	2	3	1	2	8060
91	3	2	3	2	1	5700
92	3	2	3	2	2	8210
93	3	2	4	1	1	7380
94	3	2	4	1	2	9560
95	3	2	4	2	1	7610
96	3	2	4	2	2	9400
97	4	1	1	1	1	4390
98	4	1	1	1	2	7470
99	4	1	1	2	1	4700
100	4	1	1	2	2	7210
101	4	1	2	1	1	3810
102	4	1	2	1	2	6830
103	4	1	2	2	1	3320
104	4	1	2	2	2	6020
105	4	1	3	1	1	2684
106	4	1	3	1	2	6010
107	4	1	3	2	1	3080
108	4	1	3	2	2	5630
109	4	1	4	1	1	5270
110	4	1	4	1	2	8010
111	4	1	4	2	1	4640
112	4	1	4	2	2	7540
113	4	2	1	1	1	7170
114	4	2	1	1	2	9370
115	4	2	1	2	1	7460
116	4	2	1	2	2	8800
117	4	2	2	1	1	6280

118	4	2	2	1	2	9240
119	4	2	2	2	1	6170
120	4	2	2	2	2	8680

Example 5.7. 3
List of data.

Obs	e	d	c	b	a	x
121	4	2	3	1	1	5790
122	4	2	3	1	2	7980
123	4	2	3	2	1	5180
124	4	2	3	2	2	7960
125	4	2	4	1	1	7350
126	4	2	4	1	2	10510
127	4	2	4	2	1	7590
128	4	2	4	2	2	10420

```
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```

Example 5.7. 4
Sums of squares and degrees of freedom from crossed model.

The ANOVA Procedure

Class Level Information

Class	Levels	Values
a	2	1 2
b	2	1 2
c	4	1 2 3 4
d	2	1 2
e	4	1 2 3 4

Number of Observations Read 128
Number of Observations Used 128

Example 5.7. 5
Sums of squares and degrees of freedom from crossed model.

The ANOVA Procedure

Dependent Variable: x

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	127	516717565.9	4068642.3	.	.
Error	0	0.0	.		
Corrected Total	127	516717565.9			

R-Square	Coeff Var	Root MSE	x Mean
1.000000	.	.	6524.781

Source	DF	Anova SS	Mean Square	F Value	Pr > F
a	1	247331040.5	247331040.5	.	.
b	1	80500.8	80500.8	.	.
a*b	1	27907.0	27907.0	.	.
c	3	47874600.1	15958200.0	.	.
a*c	3	1010358.3	336786.1	.	.
b*c	3	61802.1	20600.7	.	.
a*b*c	3	164078.3	54692.8	.	.
d	1	191805698.0	191805698.0	.	.
a*d	1	1802151.1	1802151.1	.	.
b*d	1	113407.0	113407.0	.	.
a*b*d	1	5125.8	5125.8	.	.
c*d	3	282293.3	94097.8	.	.
a*c*d	3	349272.6	116424.2	.	.
b*c*d	3	356120.8	118706.9	.	.
a*b*c*d	3	526729.6	175576.5	.	.
e	3	2658565.3	886188.4	.	.
a*e	3	356797.2	118932.4	.	.
b*e	3	415994.9	138665.0	.	.
a*b*e	3	236394.9	78798.3	.	.
c*e	9	11951766.7	1327974.1	.	.
a*c*e	9	1207332.3	134148.0	.	.
b*c*e	9	708652.0	78739.1	.	.
a*b*c*e	9	1066019.5	118446.6	.	.
d*e	3	251008.4	83669.5	.	.
a*d*e	3	357499.1	119166.4	.	.
b*d*e	3	174138.7	58046.2	.	.
a*b*d*e	3	142968.7	47656.2	.	.
c*d*e	9	884268.6	98252.1	.	.
a*c*d*e	9	1408735.4	156526.2	.	.
b*c*d*e	9	1827483.2	203053.7	.	.
a*b*c*d*e	9	1278855.7	142095.1	.	.

Example 5.7. 6
Model as specified.

The GLM Procedure

Class Level Information

Class	Levels	Values
a	2	1 2
b	2	1 2
c	4	1 2 3 4
d	2	1 2
e	4	1 2 3 4

Number of Observations Read	128
Number of Observations Used	128

Example 5.7. 7
Model as specified.

The GLM Procedure

Dependent Variable: x

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	127	516717565.9	4068642.3	.	.
Error	0	0.0	.		
Corrected Total	127	516717565.9			

R-Square	Coeff Var	Root MSE	x Mean
1.000000	.	.	6524.781

Source	DF	Type I SS	Mean Square	F Value	Pr > F
a	1	247331040.5	247331040.5	.	.
b(c*e)	31	63751881.9	2056512.3	.	.
a*b(c*e)	31	4068887.5	131254.4	.	.
c	0	0.0	.	.	.
a*c	0	0.0	.	.	.
d	1	191805698.0	191805698.0	.	.
a*d	1	1802151.1	1802151.1	.	.
b*d(c*e)	31	3888720.0	125442.6	.	.
a*b*d(c*e)	31	4069186.9	131264.1	.	.
c*d	0	0.0	.	.	.
a*c*d	0	0.0	.	.	.
e(c)	0	0.0	.	.	.
a*e(c)	0	0.0	.	.	.
d*e(c)	0	0.0	.	.	.
a*d*e(c)	0	0.0	.	.	.

Source	DF	Type III SS	Mean Square	F Value	Pr > F
a	1	247331040.5	247331040.5	.	.
b(c*e)	16	1266949.8	79184.4	.	.
a*b(c*e)	16	1494399.7	93400.0	.	.
c	3	47874600.1	15958200.0	.	.
a*c	3	1010358.3	336786.1	.	.
d	1	191805698.0	191805698.0	.	.
a*d	1	1802151.1	1802151.1	.	.
b*d(c*e)	16	2471149.7	154446.9	.	.
a*b*d(c*e)	16	1953679.8	122105.0	.	.
c*d	3	282293.2	94097.7	.	.
a*c*d	3	349272.6	116424.2	.	.
e(c)	12	14610332.0	1217527.7	.	.
a*e(c)	12	1564129.5	130344.1	.	.
d*e(c)	12	1135277.0	94606.4	.	.
a*d*e(c)	12	1766234.5	147186.2	.	.

```
                              Example 5.7.                                  8
                           Model as specified.

                            The GLM Procedure

Source                    Type III Expected Mean Square

a                         Var(Error) + 2 Var(a*d*e(c)) + 4 Var(a*e(c)) +
                          Var(a*b*d(c*e)) + 16 Var(a*c) + 2 Var(a*b(c*e)) + 64
                          Var(a) + Q(a*d,a*c*d)

b(c*e)                    Var(Error) + Var(a*b*d(c*e)) + 2 Var(b*d(c*e)) + 2
                          Var(a*b(c*e)) + 4 Var(b(c*e))

a*b(c*e)                  Var(Error) + Var(a*b*d(c*e)) + 2 Var(a*b(c*e))

c                         Var(Error) + 2 Var(a*d*e(c)) + 4 Var(d*e(c)) + 4
                          Var(a*e(c)) + 8 Var(e(c)) + 16 Var(c*d) + Var(a*b*d(c*e))
                          + 2 Var(b*d(c*e)) + 16 Var(a*c) + 32 Var(c) + 2
                          Var(a*b(c*e)) + 4 Var(b(c*e)) + Q(a*c*d)

a*c                       Var(Error) + 2 Var(a*d*e(c)) + 4 Var(a*e(c)) +
                          Var(a*b*d(c*e)) + 16 Var(a*c) + 2 Var(a*b(c*e)) + Q(a*c*d)

d                         Var(Error) + 2 Var(a*d*e(c)) + 4 Var(d*e(c)) + 16 Var(c*d)
                          + Var(a*b*d(c*e)) + 2 Var(b*d(c*e)) + Q(d,a*d,a*c*d)

a*d                       Var(Error) + 2 Var(a*d*e(c)) + Var(a*b*d(c*e)) +
                          Q(a*d,a*c*d)

b*d(c*e)                  Var(Error) + Var(a*b*d(c*e)) + 2 Var(b*d(c*e))

a*b*d(c*e)                Var(Error) + Var(a*b*d(c*e))

c*d                       Var(Error) + 2 Var(a*d*e(c)) + 4 Var(d*e(c)) + 16 Var(c*d)
                          + Var(a*b*d(c*e)) + 2 Var(b*d(c*e)) + Q(a*c*d)

a*c*d                     Var(Error) + 2 Var(a*d*e(c)) + Var(a*b*d(c*e)) + Q(a*c*d)

e(c)                      Var(Error) + 2 Var(a*d*e(c)) + 4 Var(d*e(c)) + 4
                          Var(a*e(c)) + 8 Var(e(c)) + Var(a*b*d(c*e)) + 2
                          Var(b*d(c*e)) + 2 Var(a*b(c*e)) + 4 Var(b(c*e))

a*e(c)                    Var(Error) + 2 Var(a*d*e(c)) + 4 Var(a*e(c)) +
                          Var(a*b*d(c*e)) + 2 Var(a*b(c*e))

d*e(c)                    Var(Error) + 2 Var(a*d*e(c)) + 4 Var(d*e(c)) +
                          Var(a*b*d(c*e)) + 2 Var(b*d(c*e))

a*d*e(c)                  Var(Error) + 2 Var(a*d*e(c)) + Var(a*b*d(c*e))
```



```
                              Example 5.7.                                  9
                            Resulting model.

                            The GLM Procedure

                         Class Level Information

                     Class         Levels    Values

                     a                  2    1 2

                     b                  2    1 2

                     c                  4    1 2 3 4

                     d                  2    1 2

                     e                  4    1 2 3 4


                 Number of Observations Read         128
                 Number of Observations Used         128
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                              Example 5.7.                                 10
                            Resulting model.

                            The GLM Procedure

Dependent Variable: x

                                        Sum of
 Source                      DF        Squares    Mean Square   F Value   Pr > F

 Model                       21    504434180.0     24020675.2    207.29   <.0001

 Error                      106     12283385.9       115881.0

 Corrected Total            127    516717565.9


              R-Square     Coeff Var      Root MSE        x Mean

              0.976228      5.217232      340.4130      6524.781


 Source                      DF      Type I SS    Mean Square   F Value   Pr > F

 a                            1    247331040.5    247331040.5   2134.35   <.0001
 c                            3     47874600.1     15958200.0    137.71   <.0001
 a*c                          3      1010358.3       336786.1      2.91   0.0381
 d                            1    191805698.0    191805698.0   1655.20   <.0001
 a*d                          1      1802151.1      1802151.1     15.55   0.0001
 e(c)                        12     14610332.0      1217527.7     10.51   <.0001


 Source                      DF    Type III SS    Mean Square   F Value   Pr > F

 a                            1    247331040.5    247331040.5   2134.35   <.0001
 c                            3     47874600.1     15958200.0    137.71   <.0001
 a*c                          3      1010358.3       336786.1      2.91   0.0381
 d                            1    191805698.0    191805698.0   1655.20   <.0001
```

```
a*d                             1       1802151.1        1802151.1      15.55    0.0001
e(c)                           12      14610332.0        1217527.7      10.51    <.0001
```



```
                                  Example 5.7.                                      11
                                Resulting model.

                               The GLM Procedure

   Source                       Type III Expected Mean Square

   a                            Var(Error) + 16 Var(a*c) + Q(a,a*d)

   c                            Var(Error) + 8 Var(e(c)) + 16 Var(a*c) + 32 Var(c)

   a*c                          Var(Error) + 16 Var(a*c)

   d                            Var(Error) + Q(d,a*d)

   a*d                          Var(Error) + Q(a*d)

   e(c)                         Var(Error) + 8 Var(e(c))

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                                  Example 5.7.                                      12
                                Resulting model.

                               The GLM Procedure
            Tests of Hypotheses for Mixed Model Analysis of Variance

Dependent Variable: x

     Source                       DF   Type III SS   Mean Square  F Value  Pr > F

  *  a                             1     247331041     247331041   734.39  0.0001

     Error: MS(a*c)                3       1010358        336786
  * This test assumes one or more other fixed effects are zero.


 Source                        DF    Type III SS    Mean Square   F Value   Pr > F

 c                              3       47874600       15958200     11.09   0.0007

 Error                     12.814       18432630        1438433
 Error: MS(a*c) + MS(e(c)) - MS(Error)


     Source                       DF   Type III SS   Mean Square  F Value  Pr > F

     a*c                           3       1010358        336786     2.91  0.0381
  *  d                             1     191805698     191805698  1655.20  <.0001
     a*d                           1       1802151       1802151    15.55  0.0001
     e(c)                         12      14610332       1217528    10.51  <.0001

     Error: MS(Error)            106      12283386        115881
  * This test assumes one or more other fixed effects are zero.
```