DTU Mechanical Engineering
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Robust Ship Hydrodynamics via Novel Methods

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Motivation

With fuel prices, emissions standards and environmental regulations expected to increase dramatically over the next years, the shipping industry is adopting slow steaming. As the ship speed decreases dramatically, the relative importance of the added resistance due to waves compared to the resistance of the vessel in calm water conditions increases. Therefore, accurate methods for predicting the added resistance are critical for designing safe and energy efficient ships.
To achieve this goal a parallel (GPU/CUDA) finite difference (FD) potential flow solver based on structured grids will be combined with an immersed boundary technique for the body representation.

Contribution

- Representation of the complex body geometry in the structured grid solver while maintaining it's efficiency.
- Simplified derivation of the WENO (Weighted Essentially Non-Oscillatory) finite difference scheme coefficients that is suited to numerical implementation.
- Simplified WENO smoothness indicator that is computed numerically.
- Stable discretization of the nonlinear wave propagation problem in a moving frame of reference using WENO.

Governing equations

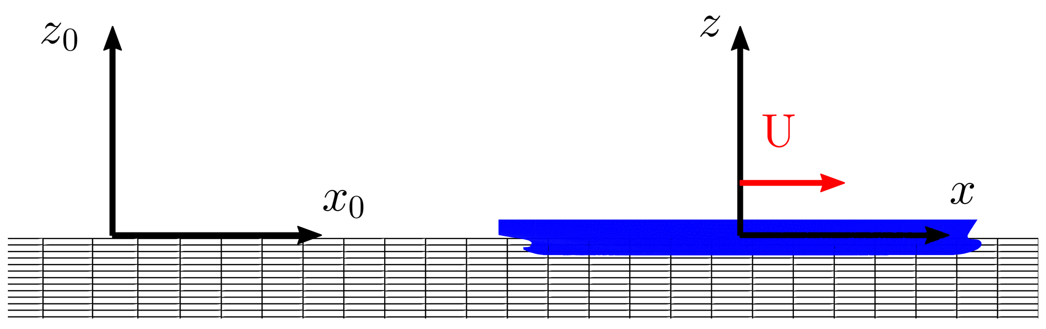


Figure 1: *Definition sketch of the seakeeping problem.*

The initial-boundary-value problem in a moving frame of reference consists of the:
Laplace equation in $\mathcal{V}$

$$\nabla^2\phi + \partial_{zz}\phi = 0$$

Kinematic Free Surface Boundary Condition (FSBC) on $z = \zeta$

$$\partial_t\zeta + \nabla\zeta\left(\nabla\tilde{\phi} - \tilde{w}\,\nabla\zeta - \mathrm{U}\right) = \tilde{w}$$

Dynamic Free Surface Boundary Condition (FSBC) on $z = \zeta$

$$\partial_t\tilde{\phi} + \nabla\tilde{\phi}\left(\frac{1}{2}\nabla\tilde{\phi} - \mathrm{U}\right) - \frac{1}{2}\tilde{w}^2\left(1 + \nabla\zeta\nabla\zeta\right) = -g\zeta$$

Bottom Boundary Condition on $z = -h$

$$\partial_z\phi + \nabla h\nabla\phi = 0$$

Body Boundary Condition (BC) on $\mathcal{S}_b$

$$\partial_n\phi = V_n$$

with $\zeta(\mathrm{x}, 0)$, & $\tilde{\phi}(\mathrm{x}, 0)$ given and

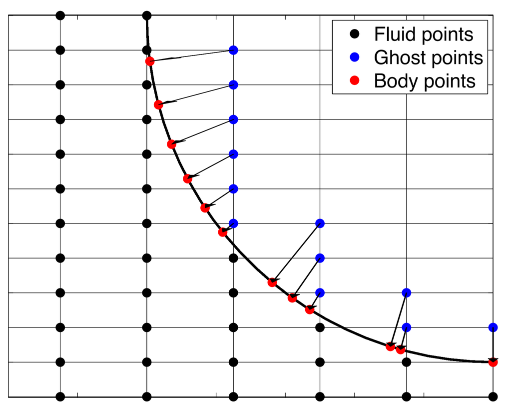
- $\nabla = (\partial_x, \partial_y)$
- $\zeta(\mathrm{x}, t)$: free surface elevation.
- $\tilde{\phi}(\mathrm{x}, t) = \phi(\mathrm{x}, \zeta, t)$: velocity potential on the free surface.
- h: water depth.
- $\mathcal{S}_b$ and V the moving ship surface and it's velocity.

As a first step the linearized problem is considered where the kinematic and dynamic FSBC evaluated on $z = 0$ reduce to:

$$\partial_t\zeta - U\nabla\zeta = \partial_z\tilde{\phi}$$
$$\partial_t\tilde{\phi} - U\nabla\tilde{\phi} = -g\zeta$$

Implementation of the body BC

The FD Laplace solver is extended to include a Weighted Least Squares (WLS) approximation of the body BC using the Immersed Boundary Method (IBM).



- Sign function distinguishes points in/out of the body.
- Ghost points: points in the body that belong to the FD stencil of a fluid point.
- Body points: projection of a ghost point on the body.
- WLS stencil for each body point: contains fluid points plus the associated ghost point.
- The WLS method is used to approximate the normal derivative of the body BC.

Solving the linearized problem

- Convective terms of the FSBC are discretized robustly using a one-point upwind-biased FD scheme.
- Time stepping: Explicit Runge-Kutta 44.
- Figure 2 shows a representative example of linear Kelvin wave patterns computed using the IBM.

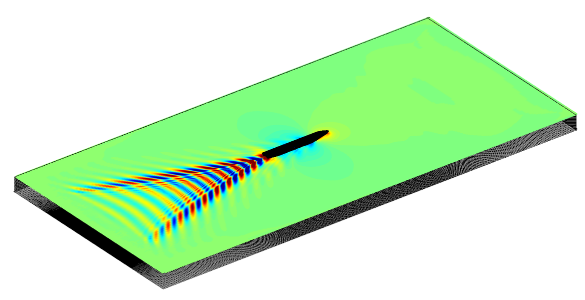


Figure 2: *Kelvin wave patterns for the KCS hull.*

Towards nonlinear wave-structure interaction

- The numerical scheme used on the FSBC has to handle all ratios of wave celerity to ship speed.
- The upwind-biased FD scheme becomes unstable when the wave speed in the direction of the ship motion is larger than the ship speed.
- Solution: the FSBC are posed in Hamilton-Jacobi form and the WENO scheme is adopted.

WENO Finite difference scheme

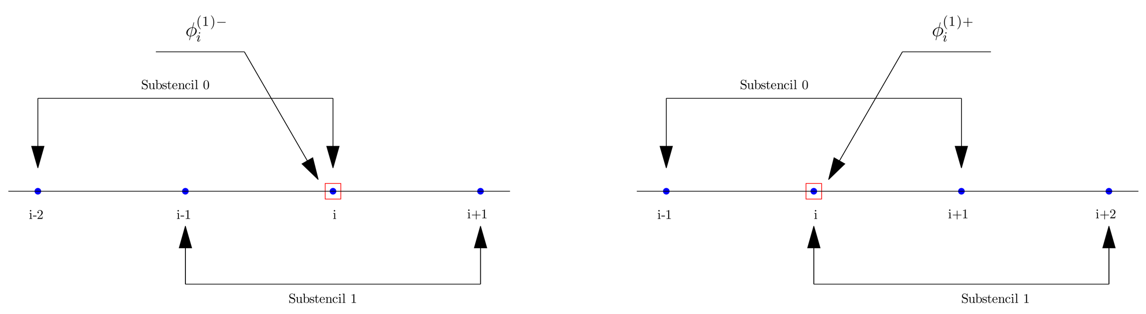
- A WENO-r scheme develops a left- and right- biased derivative approximation $\phi_{x,i}^{-/+}$.
- Each approximation is based on a weighted sum of $r-$ sub-stencil approximations (Figure 3) e.g.

$$\phi_{x,i}^{-} = \sum_{s=0}^{r-1}\omega_s\phi_{x,i}^{-,s}$$

where

$$\omega_s = \frac{a_s}{\sum_{s=0}^{r-1}a_s}, \quad a_s = \frac{d_s}{(\epsilon + \beta_s)^2}, \quad s = 0,\ldots,r-1$$

- d_s: constant linear weights. Smooth solution $\Rightarrow$ $(2r-1)^{th}$-order.
- β_s: "smoothness indicators". They become large whenever discontinuities exist in the solution.
- ω_s: nonlinear weights. Smooth solution $\Rightarrow$ $(2r-1)^{th}$-order. Discontinuity $\Rightarrow$ $(r)^{th}$-order plus stability.
- The simplified smoothness indicator is defined as $\beta_s = \sum_{l=2}^{r}\left(\frac{\partial^l\phi^s(x_i)}{\partial^l x}\,\Delta x^{l-1}\right)^2.$



(a) Left-Biased stencil. (b) Right-Biased stencil.

Figure 3: WENO stencils and sub-stencils for $r = 2$.

Linear WENO weights

We seek r coefficients which set to zero the first $r-1$ truncation error terms in the Taylor series expansion of the combined derivative approximation; and sum to one. For example, on the sub-stencils of the left biased stencil the first truncation error terms are

- $\phi_i^{(1)} = \phi_{x,i}^{-,0} - \frac{1}{3}\phi_i^{(3)}\,\Delta x^2 + \ldots$
- $\phi_i^{(1)} = \phi_{x,i}^{-,1} + \frac{1}{6}\phi_i^{(3)}\,\Delta x^2 + \ldots$

Thus we have the following linear system of equations to solve for the d_s:

$$\begin{bmatrix}-\frac{1}{3} & \frac{1}{6}\\ 1 & 1\end{bmatrix}\begin{bmatrix}d_0\\ d_1\end{bmatrix} = \begin{bmatrix}0\\ 1\end{bmatrix}$$

~~The result agrees with the values found in the literature.~~

WENO on the forward speed problem

- Express FSBC in Hamilton-Jacobi form: $\phi_t + H(\nabla\phi) = 0.$
- Discretize using the Lax-Friedrichs scheme $\hat{H} = H\left(\frac{\phi_x^- + \phi_x^+}{2}, \frac{\phi_y^- + \phi_y^+}{2}\right) - a^x\left(\frac{\phi_x^+ - \phi_x^-}{2}\right) - a^y\left(\frac{\phi_y^+ - \phi_y^-}{2}\right)$

where

- $a^x = \max|H_1(\phi_x, \phi_y)|$
- $a^y = \max|H_2(\phi_x, \phi_y)|$
- H_1 and H_2: partial derivatives of H with respect to ϕ_x and ϕ_y, respectively

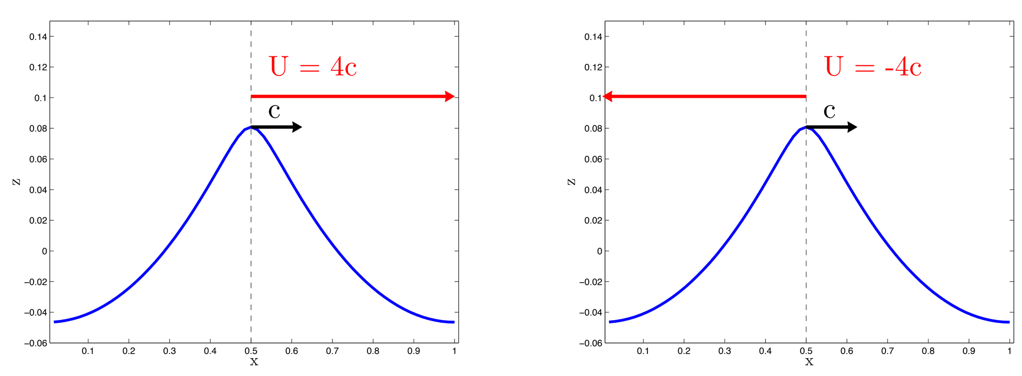
Express the FSBC in the WENO formulation as:

- $\partial_t\zeta + H_\zeta = \partial_z\tilde{\phi}$
- $\partial_t\tilde{\phi} + H_\phi = -g\zeta$

where

- $H_\zeta = \partial_x\zeta\left(\partial_x\tilde{\phi} - \partial_z\tilde{\phi}\,\partial_x\zeta - U\right)$
- $H_\phi = \partial_x\tilde{\phi}\left(\frac{1}{2}\partial_x\tilde{\phi} - U\right) - \frac{1}{2}(\partial_z\tilde{\phi})^2\left(1 + \partial_x\zeta\,\partial_x\zeta\right)$
- Right hand side terms: source terms
- $a_\zeta^x = \max|H_{1,\zeta}(\zeta_x, \tilde{\phi}_x)| = \max|\tilde{\phi}_x - 2\tilde{\phi}_z\,\zeta_x - U|$
- $a_{\tilde{\phi}}^x = \max|H_{1,\phi}(\zeta_x, \tilde{\phi}_x)| = \max|\tilde{\phi}_x - U|$

The scheme is tested on propagating steep stream function waves over a range of different ship speed (U) to wave propagation speed (c) ratios see e.g. Figure 4. WENO remains stable and upwinded in all cases.



(a) $U = 4c$. (b) $U = -4c$.

Figure 4: Stream function wave in a moving frame of reference.

Conclusion

- WENO plus IBM paves the way for a fully nonlinear wave-structure interaction solver.

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