Figure 4: SPOT HRV MAD1, absolute values, three bands in 1987 and in 1989 (left), three bands in 1987 and two bands in 1989 (right)

This is of course a somewhat constructed example but in historical change detection studies comparisons between data from for instance Landsat MSS (four bands) and Landsat TM (six or seven bands) or SPOT HRV (three bands) may be relevant.

4.1.4 Geometric Illustration of Canonical Variates

To hopefully give a better feel for what canonical variates are and to illustrate geometrically the solution to the real, symmetric, generalized (RSG) eigenproblem involved in finding them (cf. Appendix A), we have generated two sets of data both consisting of two variables. The data consist of every 50'th row and every 50'th column of the image data analyzed above. The first set of variables are bands 1 and 2 from the 1987 data and the second set of variables are bands 2 and 3 from the 1989 data. The 1987 (1989) data are estimated from the 1989 (1987) data by regression.

The two top plots in Figure 5 show scatterplots and ellipses corresponding to $\chi^2_{0.95}(2) = 5.991$ contours for the 1987 and 1989 data. These contour ellipses are (top-left; see Appendix A for a description of the mathematics illustrated here) $a^T\hat{\Sigma}_{11}^{-1}a = 5.991$ (for the data) and $a^T(\hat{\Sigma}_{12}\hat{\Sigma}_{22}^{-1}\hat{\Sigma}_{21})^{-1}a = 5.991$ (for the regressions), and (top-right) $b^T\hat{\Sigma}_{22}^{-1}b = 5.991$ (for the data) and $b^T(\hat{\Sigma}_{21}\hat{\Sigma}_{11}^{-1}\hat{\Sigma}_{12})^{-1}b = 5.991$ (for the regressions). The open circles symbolize observations and the crosses symbolize regressions made from the opposite set of variables.

The two bottom plots show the solution to the eigenproblem. The ellipses shown are contours for $a^T\mathsf{D}a = 1$, $a^T\mathsf{N}a = 1$, $a^T\mathsf{N}a = \lambda_1$, $a^T\mathsf{N}a = \lambda_2$, where N means the matrix in the numerator of the Rayleigh coefficient identifying the canonical correlation problem and D means the matrix in the denominator. In this case $\lambda_1 = 0.2730$