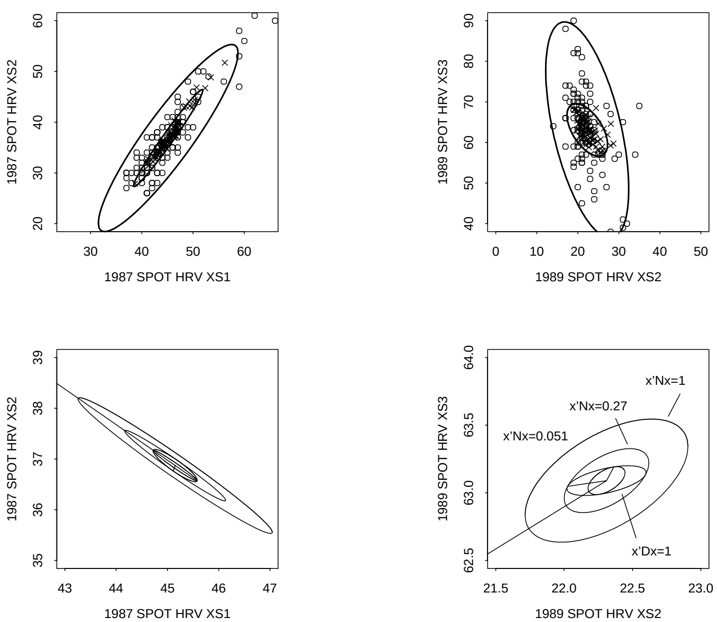

Figure 3.10: Canonical variates geometrically

$b^T(\hat\Sigma_{21}\hat\Sigma_{11}^{-1}\hat\Sigma_{12})^{-1}b = 5.991$ (for the regressions). The open circles symbolize observations and the crosses symbolize regressions made from the opposite set of variables.

The two bottom plots show the solution to the eigenproblem. The ellipses shown are contours for (bottom-left) $a^T\hat\Sigma_{11}a = 1$, $a^T\hat\Sigma_{12}\hat\Sigma_{22}^{-1}\hat\Sigma_{21}a = 1$, $a^T\hat\Sigma_{12}\hat\Sigma_{22}^{-1}\hat\Sigma_{21}a = \rho_1^2$ and $a^T\hat\Sigma_{12}\hat\Sigma_{22}^{-1}\hat\Sigma_{21}a = \rho_2^2$, and (bottom-right) $b^T\hat\Sigma_{22}b = 1$, $b^T\hat\Sigma_{21}\hat\Sigma_{11}^{-1}\hat\Sigma_{12}b = 1$, $b^T\hat\Sigma_{21}\hat\Sigma_{11}^{-1}\hat\Sigma_{12}b = \rho_1^2$ and $b^T\hat\Sigma_{21}\hat\Sigma_{11}^{-1}\hat\Sigma_{12}b = \rho_2^2$. In the bottom-right plot the contour lines are identified; here N means the matrix in the numerator of the Rayleigh coefficient identifying the canonical correlation problem and D means the matrix in the denominator. In this case $\rho_1^2 = 0.2730$ and $\rho_2^2 = 0.05147$ corresponding to canonical correlations 0.5199 and 0.2269.

In the two bottom plots the eigenvectors to the canonical correlation problem are vectors with end points in the center of the ellipses and the points where the ellipses have a common tangent (indicated with short lines). The square root of the eigenvalues (the canonical correlations) ρ_i are the ratios of the the lengths of the major (or minor) axes in the ellipses corresponding to $a^T\hat\Sigma_{12}\hat\Sigma_{22}^{-1}\hat\Sigma_{21}a = 1$ and $a^T\hat\Sigma_{12}\hat\Sigma_{22}^{-1}\hat\Sigma_{21}a = \rho_i^2$ (bottom-left). The same is true for $b^T\hat\Sigma_{21}\hat\Sigma_{11}^{-1}\hat\Sigma_{12}b = 1$ and $b^T\hat\Sigma_{21}\hat\Sigma_{11}^{-1}\hat\Sigma_{12}b = \rho_i^2$ (bottom-right). The major axes of $a^T\hat\Sigma_{12}\hat\Sigma_{22}^{-1}\hat\Sigma_{21}a$ and $b^T\hat\Sigma_{21}\hat\Sigma_{11}^{-1}\hat\Sigma_{12}b$ (the matrices in the numerators of the Rayleigh coefficient) are indicated with long lines.

3.3.2 Landsat TM Data in Forestry (MUSECC)

The applicability of multiset canonical correlations analysis to multivariate and truly multitemporal change detection studies is demonstrated in a case study using Landsat-5 Thematic Mapper (TM) data covering a small forested area approximately 20 kilometers north of Umeå in northern Sweden (data from the Swedish Space Corporation). The data consist of six times six spectral bands with 512×512 20 meter pixels rectified to the Swedish national grid from the summers 1984–89. The acquisition dates are 1 August 1984, 26 June 1985, 6 June 1986, 12 August 1987, 27 June 1988 and 21 June 1989. Results from such analyses are linear combinations that transform the original bands into new variables that show decreasing similarity over six points in time. The minimum similarity variables are measures of change in all bands simultaneously. This analysis of correlations between variables where observations are considered as repetitions is termed R-mode analysis. In this case, in R-mode analysis we consider Landsat TM bands 1, 2, 3, 4, 5 and 7 for each of the years 1984–1989 as one set of variables. In Q-mode analysis of correlations between observations where variables are considered as repetitions we consider TM bands 1 for all years 1984–1989 as one set of variables, TM bands 2 for all years 1984–1989 as another set of variables, etc. For a sketch of R- and Q-mode analysis set-up see Figures 3.11 and 3.12. In both figures the sets of variables indicated on the top are transformed into new variables on the bottom.