Automated 3D Rigid Registration of Open 2D Manifolds

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***Abstract*— We present a robust method for 3D rigid registration of 2D open manifolds based on distance maps using an initialization step, a Gibbs sampler for optimization supplemented by an Iterated Conditional Modes (ICM) optimizer. The algorithm is fully automatic based on very simple information of the initial orientation of the given objects.**

I. Introduction

In the industry the use of CAD modeling and 3D scanning of objects become more and more widespread. Thus huge amounts of data are becoming available and this calls for automated data analysis. One major challenge is the registration of 2D surfaces embedded in a 3D Euclidian space. Most solutions to this problem has for some time been primarily based on annotations and ICP-like algorithms [2], [5], [10], [13]. However, a recent advance in registration and building of 2D and 3D shape models has inspired several new approaches to the registration process. In this paper we put emphasis on registering scans of ear canal impressions. The hearing aid industry has a need for automatic algorithms, since the amount of data stored and ready for analysis is increasing with thousands of ear impressions scanned each day, all open surfaces. The method presented here have also successfully been applied to other data sets.

II. Previous work

A common solution to the rigid registration problem is a landmark based least squares solution [7]. This solution demands expert annotation, and for larger data sets this is undesirable. The foremost non-landmark based solution used today is the iterative closest point (ICP) [2] algorithm which is fast and reliable. The ICP-algorithm exists in a wide variety of forms, but has one flaw. It needs a good initialization which is a part of the algorithm presented in this paper.

Given the right cost function, a initialization can be formulated and the registration problem can be viewed as an optimization problem. There exist a huge variety of optimization algorithms, one is Markov Random Fields and the Potts Model[9], others are gradient descent, Metropolis Hastings [6] etc. In order to use one of these methods we need to define a cost function on which we can optimize. Tsai et. al [12] uses the difference of level sets [8] to generate a cost function but this poses a problem since level sets are not defined on the boundary of open surfaces so this is not the right choice for our data.

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III. The Cost Function

Inspired by the level set method and the level set function shape model in [12] we define a cost function in Euclidian space on which we can make initialization and optimize on. We propose to use the difference of distance maps which can be generated using various algorithms (e.g. fast marching [11] or implicit surfaces [3])or other functions. Integrating the difference of the two distance maps over the entire space results in either infinity or zero (for a perfect match). We propose to sample the distance map at a finite number of locations in a box containing the zero levels sets of the two surfaces to be registered. A cost function based on this can then be formulated as follows.

Let p_i be a point in a Euclidian space $p_i \in Q$ and let S be a surface embedded in Q so that $S \subset Q$. Let p_s be a point on the surface S hence $p_s \in S$. We then define the distance between p_i and S, $f_{\text{dist}}(p_i, S)$ as,

$$f_{\text{dist}}(p_i, S) = min(||p_i - p_s||^2) \ \forall p_s \in S. \tag{1}$$

The distance between two shapes S_1 and S_2 is then defined by the following. Given N points $P = p_1 \dots p_N$ the cost function $f_{\text{cost}}(S_1, S_2)$ is

$$f_{\text{cost}}(S_1, S_2) = \frac{1}{N}\sum_{i=1}^{N} ||f_{\text{dist}}(p_i, S_1) - f_{\text{dist}}(p_i, S_2)||^2. \tag{2}$$

In practice the N points are organized in a cubic grid with the same center as one of the shapes. We emphasize that other distances i.e. $|p_i - p_s|$ and $\sqrt{|(p_i - p_s)|}$ can be used in eq.1 and other norms in eq.2. However, since the functions presented in eq.1 and 2 yield the desired result of matching the concha within a single individual we focus on the use of these.

IV. Rigid Registration

Rigid registration is basically applying a series of translations and rotations in order to minimize some cost function. Let T be the set of all translations and rotations, then applying a transformation $t \in T$ to a shape S_1, we obtain $t(S_1)$.
For a transformation $t \in T'$, where $T' \in T$ is a set of admissible transformations, we define the Gibbs probability function

$$P(t) = \frac{\exp(-f_{\text{cost}}(t(S_1), S_2))}{\sum_{t' \in T'} exp(-f_{\text{cost}}(t'(S_1), S_2))} c, \tag{3}$$

where c is a constant controlling the peakedness of the distribution. By randomly sampling from this distribution we have a Gibbs sampler for translation and rotation to help avoiding ending up in a local minima. To finish the optimization we use Iterated Conditional Modes [1] where we simply select the transformation with the highest probability. To increase speed we use scale space wrt. the translation and rotation in a coarse to fine manner.

A. *Open surfaces*

Since all our scanned impressions are open surfaces we need to handle edges. If this fact is ignored the edges would get to much power in the registration. In order to deal with this, all distances $f_{\text{dist}}(p_i, S)$, where p_s (see eq. 1) is on the edge of S, are discarded. In this way the edges are not given any power in the matching and we end up with pure surface matching on both open and closed surfaces.

B. *Initialization*

In most cases prior knowledge of the orientation of the surfaces in question exist. As with the ICP algorithm a reasonably good initialization must be made to avoid local minima. Often when you scan items there is some consistency in the way that the object is placed in the scanner, and in the case of ear impression scanning we know one axis of orientation. Furthermore, the way the objects are scanned causes translation to be virtually non-existing. As initialization, we simply rotate the object around this axis of orientation in steps of $\Delta\theta$ degrees and calculate the cost for each rotation. We then select the rotation with the lowest cost as our starting guess. In practice $\Delta\theta$ is selected to be $20° - 25°$. Since the implementation is rather fast, <2min for rigid registration, we consider it feasible to try rotations around all three axis without extending the calculation time by more than a few minutes.

V. RESULTS

A benefit of this algorithm is that we have some sort of accuracy parameter to tune i.e. the steps of rotation and translation. To actually evaluate the results, a visual inspection of the resulting registration is made as well as a quantitative analysis by comparing the average point to point distance and the cost function both calculated for our method and ICP. We also briefly compare our method and ICP with a landmark based. We know that ICP fails randomly with un-initialized data. To avoid this problem we have used the initialization method described in the previous section to initialize both methods. The easy initialization built-in in our method gives our method an advantage which we prove by making raw matches and compare the success rate.

A. *Data*

The method has been tested on a data set consisting of 30 ear impressions from 15 people. Two impressions have been made from each individual, one with closed mouth and one with open mouth. A typical ear impression is 30-35 mm. deep and the same at the widest point. The impressions are scanned with at state-of-the-art laser scanner, resulting in more than 10000 points for each. The intention is to align the two impressions from each individual in order to se the difference between the two impressions. Furthermore, a registration between the impressions across the population is desirable.

B. *Registration results and performance*

The registration of the two different impressions from a single individual is presented in fig. 1. The results of ICP and our methods looks almost the same even though there is a small difference of up to 0.1-0.2 mm. in some places. It is difficult to determine the best method based on visual inspection. However, it seems that both ICP and our method perform marginally better than the landmark based wrt. consistency. Performance wise the ICP implementation used in this paper is in VTK and executes 1-2 minutes and our method uses 8x8x8 sample points and executes equally fast including initialization which takes less than 10 seconds on a 1.7GHz Pentium M.

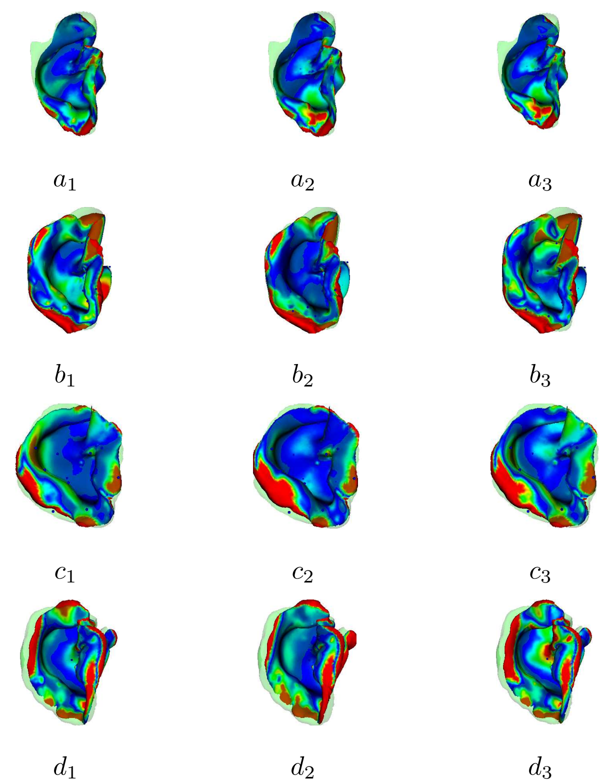

Fig. 1. Two different impressions from a single individual registered. Each row represents a individual and each column a method. Column 1 is the ICP results, column 2 is the method described in this paper and column 3 is the annotation based method. The images are distance difference images: Blue is 0.0 mm. and red is 0.5 mm. and above distance between the two impressions.

Figure 2 depicts the inter-subject registration. All three methods are here able to align the shapes in a satisfying manner. In our opinion all 3 methods perform this task equally well, however, the registration has been performed with different norms and hence yield different results. To quantify the performance of our method vs. ICP we have calculated the mean point to point distance and the norm in eq. 2 for our method and ICP and performed a T-test for mean for both experimens. The results seen in fig. 3 and fig. 4 shows that both methods perform very well with respect to the different norms respectively and the hypothesis of same mean can be rejected with >99% confidence in 3 cases and is only barely accepted in the last case. As fig. 4 depicts it is impossible to tell which method is best and the choice of the best method depends on the analysis of the data and the desired alignment. But within the same individual it seems our method is more desirable than ICP if we take both norms into consideration.

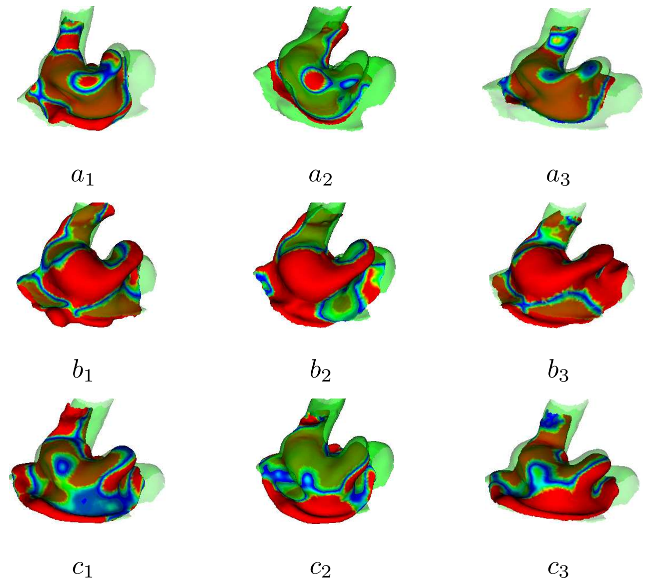

Fig. 2. One ear impression has been selected as a reference and then each individual has been matched to this reference. Each row represents a individual and each column a method. Column 1 is ICP, column 2 is the method described in this paper, column 3 is annotation based. This figure has the same color coding as fig. 1.

$Score/Norm$	f_{cost}	$meandist$
Our method (Mean)	2.4620	0.4833
ICP (Mean)	6.3048	0.4576
Our method (Std)	1.6085	0.0964
ICP (Std)	3.2447	0.0753
T-Test p-value for means	0.0000	0.0532

Fig. 3. The mean and standard deviation of the two different distance measures taken over 15 matching individuals to themselves. Closed mouth impression to open mouth impression

$Score/Norm$	f_{cost}	$meandist$
Our method (Mean)	35.9285	1.6153
ICP (Mean)	65.1232	1.4071
Our method (Std)	13.8721	0.3622
ICP (Std)	31.3325	0.3748
T-Test p-value for means	$0.3990 \cdot 10^{-6}$	$0.8850 \cdot 10^{-6}$

Fig. 4. The mean and standard deviation of the two different distance measures taken over 29 matching between individuals

C. Raw data processing

A study on raw un-initialized data has been made to investigate the benefit of our methods initialization compared to ICP without initialization. One ear has been selected as the reference and all others have been matched to this one. Figure 5 show a successful registration and a failed registration. As the results show (fig. 6), our method outperforms ICP which only gives the right result as a function of the initial alignment. It seems that the ICP performs with a success rate of approximately 45% (fig. 6) which is useless for fully automatic registration. Our method shows a success rate of 100% (fig. 6) which is ideal in an automated environment. We mention here that ICP performs equally well with the initialization i.e. 100%

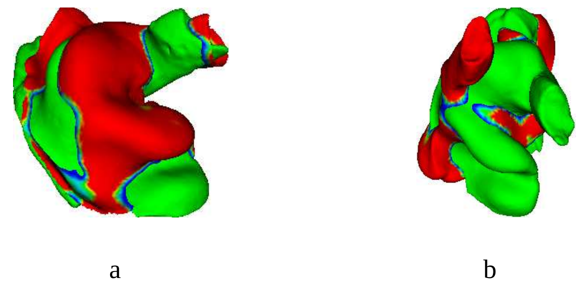

Fig. 5. **a)** Successful registration. **b** Failed Registration.

Matching results		
Method	ICP	Our Method
Success	$\frac{13}{29}$	$\frac{29}{29}$

Fig. 6. The success rate of ICP and our method on the same un-initialized data.

D. Other Norms

In order to investigate the concept of substituting eq. 1 and eq. 2 with other norms we have conducted the same experiment as in fig. 3. We match the impression of the ear with the mouth closed to the impression with the mouth open using eq. 4 instead of eq. 1 and eq. 5 instead of eq. 2. The results in fig. 7 show that the algorithm performs just as well as ICP minimizing the average point to point distance within a 95% confidence interval and much better with it's own distance norm.

$$f_{dist}(p_i, S) = min(\sqrt{|p_i - p_s|})\ \forall p_s \in S. \tag{4}$$

$$f_{cost} = \frac{1}{N}\sum_{i=1}^{N} |f_{dist}(p_i, S_1) - f_{dist}(p_i, S_2)|. \tag{5}$$

$Score/Norm$	f_{cost}	$meandist$
Our (sqrt) method (Mean)	0.0721	0.4460
ICP (Mean)	0.0948	0.4576
Our (sqrt) method (Std)	0.0182	0.0815
ICP (Std)	0.0229	0.0753
T-Test p-value for means	0.0000	0.1623

Fig. 7. The mean and standard deviation of the two different distance measures taken over 15 matching individuals to themselves. Closed mouth impression to open mouth impression using eq.4 and eq.5

VI. Conclusion

We have seen that the method presented in this paper performs just as well as the ICP (fig. 3, 4 and 7) when both have been initialized. However, our method has a built-in initialization routine making our method much more robust than the standard ICP (without initialization). The ICP can use our method as an initialization routine yielding similar results. However, this would be doing double the work. It is obvious that our method is well suited for fully automatic rigid registration (fig.6) of ear impressions. Further more the concept of a global cost function (eq. 2) is very useful since a measure now exist for 3D surfaces on which other optimization routines besides the one presented here can be applied to. Furthermore the cost function can easily be modified to use different norms.

VII. Implementation issues

The method has been implemented in C++ using GEL as a framework and VTK (www.vtk.org) has been used for importing the *.stl files and providing the ICP. Since our data is very dense sampled > 0.05 mm. between each sampling point we have made an implementation shortcut by actually matching point to point and not point to surface. This enables us to use a KD-tree [4] for looking up nearest point and gives a considerable increase in performance. Furthermore by pre-calculating the distance to each point in the grid a further increase in performance can be achieved. GEL can be found at www.imm.dtu.dk/GEL

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