02917: Advanced Topics in Embedded Systems

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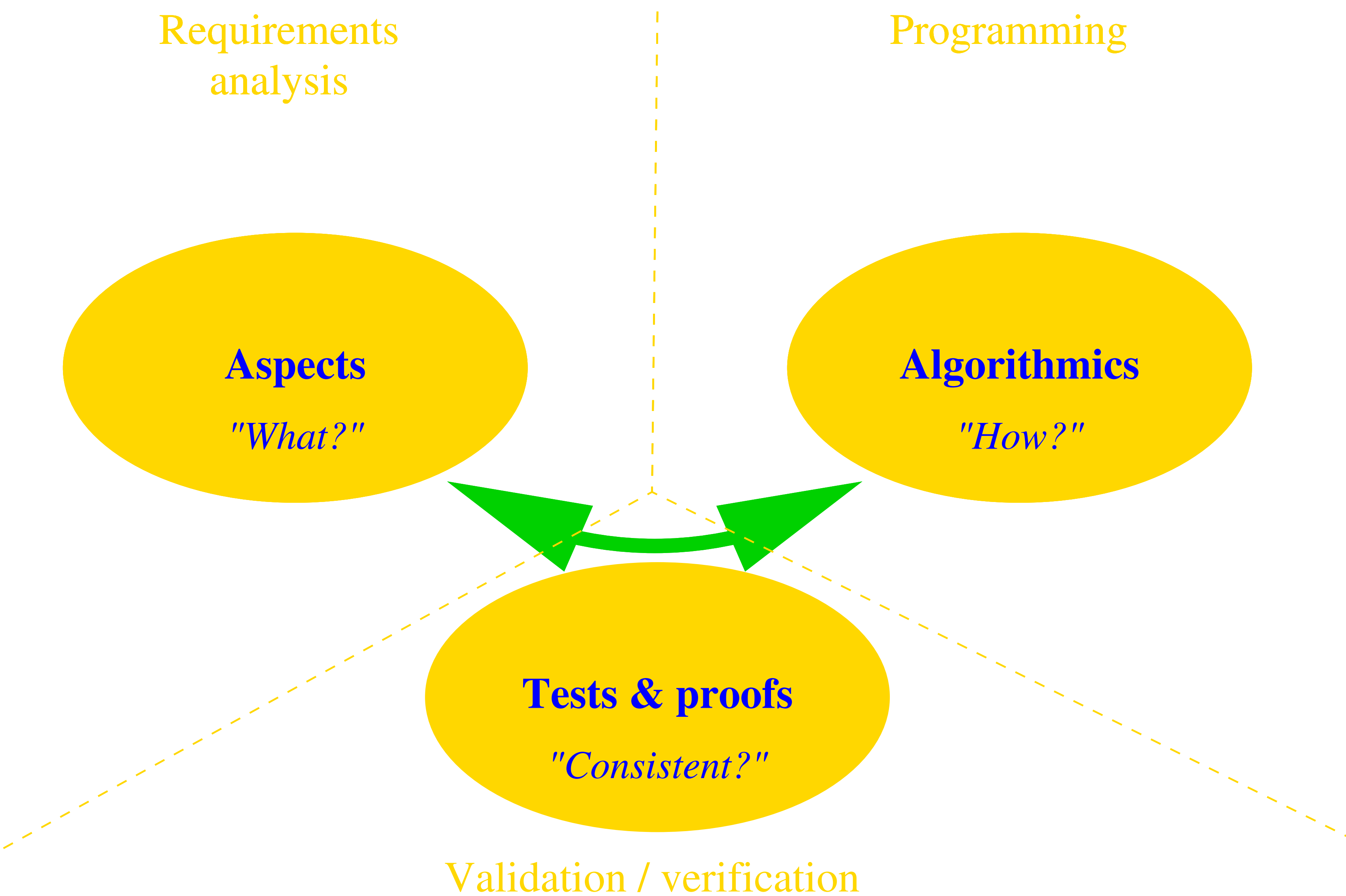
Multiple viewpoints



Formal Methods

- Formal methods are mathematically-based techniques for the specification, development and verification of software and hardware systems. [R.W. Butler, 2001]
- Motivated by the expectation that appropriate mathematical analyses can contribute to the reliability and robustness of a design. [M. Holloway, 1997]
- Alternative to less exhaustive analyses:

(Cartoon)

Embedded computer systems

Estimates for number of embedded systems in current use exceed 10^{10}.

[Rammig 2000, Motorola 2001]

Application domains

- **Consumer & household products:**
  CD players, TV sets, handheld games, electronic pets, cameras, alarm clocks, remote controls, dishwashers, microwave ovens, ...
- **Office, telecommunications, etc.:**
  Printers, network controllers, mobile phones, keyboards, CRTs and flatscreens, ...
- **Environmental control:**
  λ control, programmable heating systems, exhaust control, ...
- **Traffic systems and traffic management:**
  Cars (body, powertrain, suspension, brakes), signalling devices, balises, interlocks, autopilots, traffic information, ...
- **Medicine:**
  Measurement devices (thermometers, RR's, X-ray, sonographic imaging, EEG, ECG ...), treatment devices (perfusors, respirators, microwave radiation treatment, ...)
- **Supplies:**
  Power plants, distribution networks, ...

The roles, they are a changing...

Phase 1: Added value through add-on functionality:

- automatic climate control,
- adaptive power steering,
- keyless entry,
- navigation system.

Phase 2: Integration into/hooking onto vital safety components:

- anti-locking brake,
- electronic stability control,
- electric power steering,
- electronically variable transmission ratio of steering column.

Phase 3: Replacement of vital safety components:

- steer-by-wire,
- brake-by-wire,
- driver-less go (automatic parking, autonomous lane change, . . .).

A little mishap...

Source of problem:

- Car geometry:
  - center of gravity
  - wheelbase
- Reduced component-count axles

Solution:

- Patched with embedded control ("ESP")

How to validate the patch?

Continuity?

Around 1999, the car industry fought for a unification of European and American crash test scenarios because

> "it is hardly possible to adjust the firing delay of the airbag to both the ECE and the NCAP frontal crash."

ECE test: $56\,\frac{\mathrm{km}}{\mathrm{h}}$,

NCAP test: $64\,\frac{\mathrm{km}}{\mathrm{h}}$.

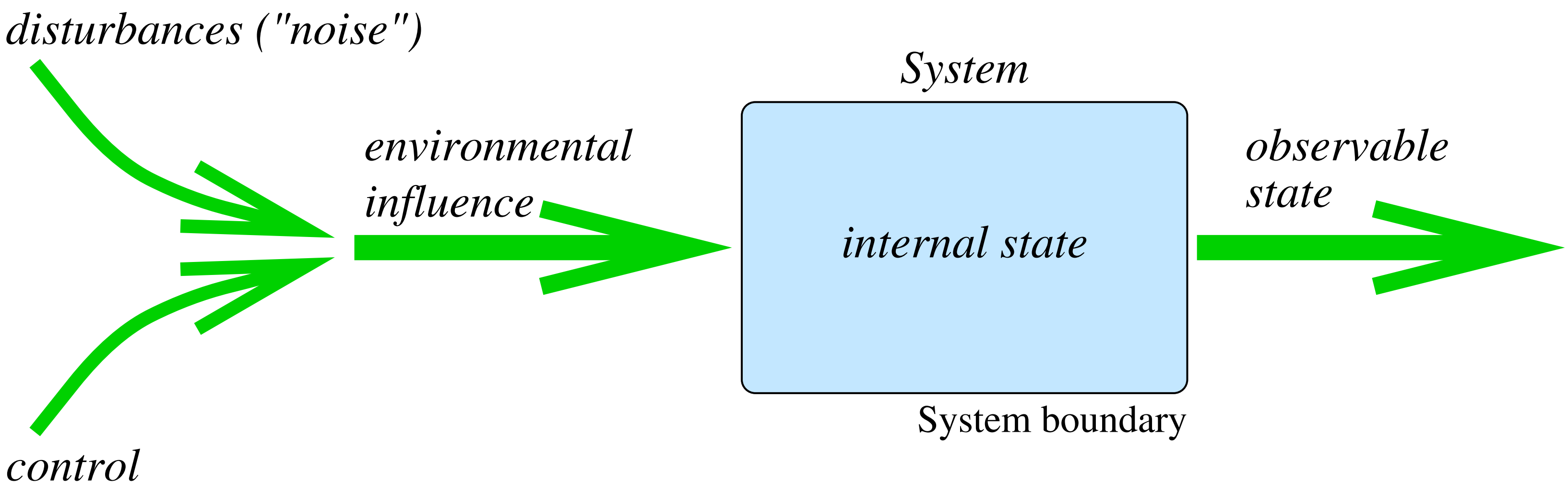
A Suggestion: Formal Methods

The term refers to a broad set of notations and tools for

1. Mathematically rigorous documentation of requirements
   - less ambiguous than prose
   - amenable to formal analysis (check for consistency, check for adherence to well-formedness criteria,...)
2. Mathematically rigorous models of designs
   - rigorous semantics, removes ambiguity of design documentation in prose, albeit only wrt. the viewpoint focused at
   - early availability of abstract, yet concise model of the system under design
   - amenable to formal analysis (check for absence of design flaws, like deadlock, wrong interfaces,...; check for side-conditions of design steps,...)
3. Check of consistency between such
   - correctness of a design relative to requirements
   - replacability of a design by another one

State-Based Models

Open (contin. time & state) dynamical system



- Time is continuous: $\mathbb{R}_{\geq 0}$,
- internal state is a bunch of real-valued (or complex-val ued) functions of time: $\vec{x}(.) : \mathit{Time} \longrightarrow \mathbb{R}^n$,
- observable state is a time-invariant function (usually projection) thereof,
- environment influence is a bunch of real-valued (or complex-valued) functions of time: $\vec{u}(.) : \mathit{Time} \longrightarrow \mathbb{R}^m$.

Some state-based models

- **Finite automata**
  - discrete state space, finitely many states
  - evolution through discrete transitions
- **Differential equations**
  - continuous state space
  - continuous flows
- **Hybrid systems**
  - continuous and discrete state components
  - both jumps ($=$ discrete transitions) and continuous flows

State-based models:

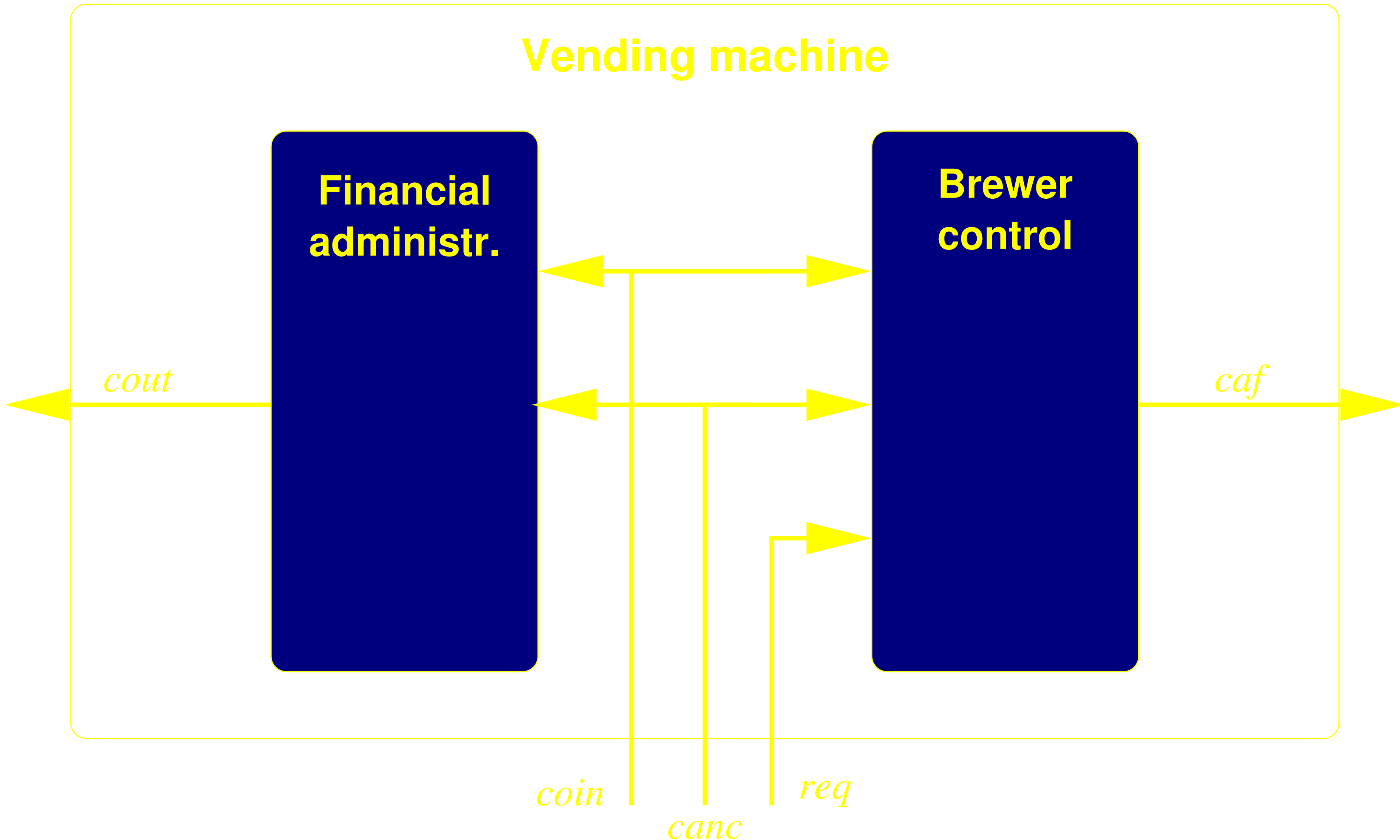
Finite Automata

Finite-state models

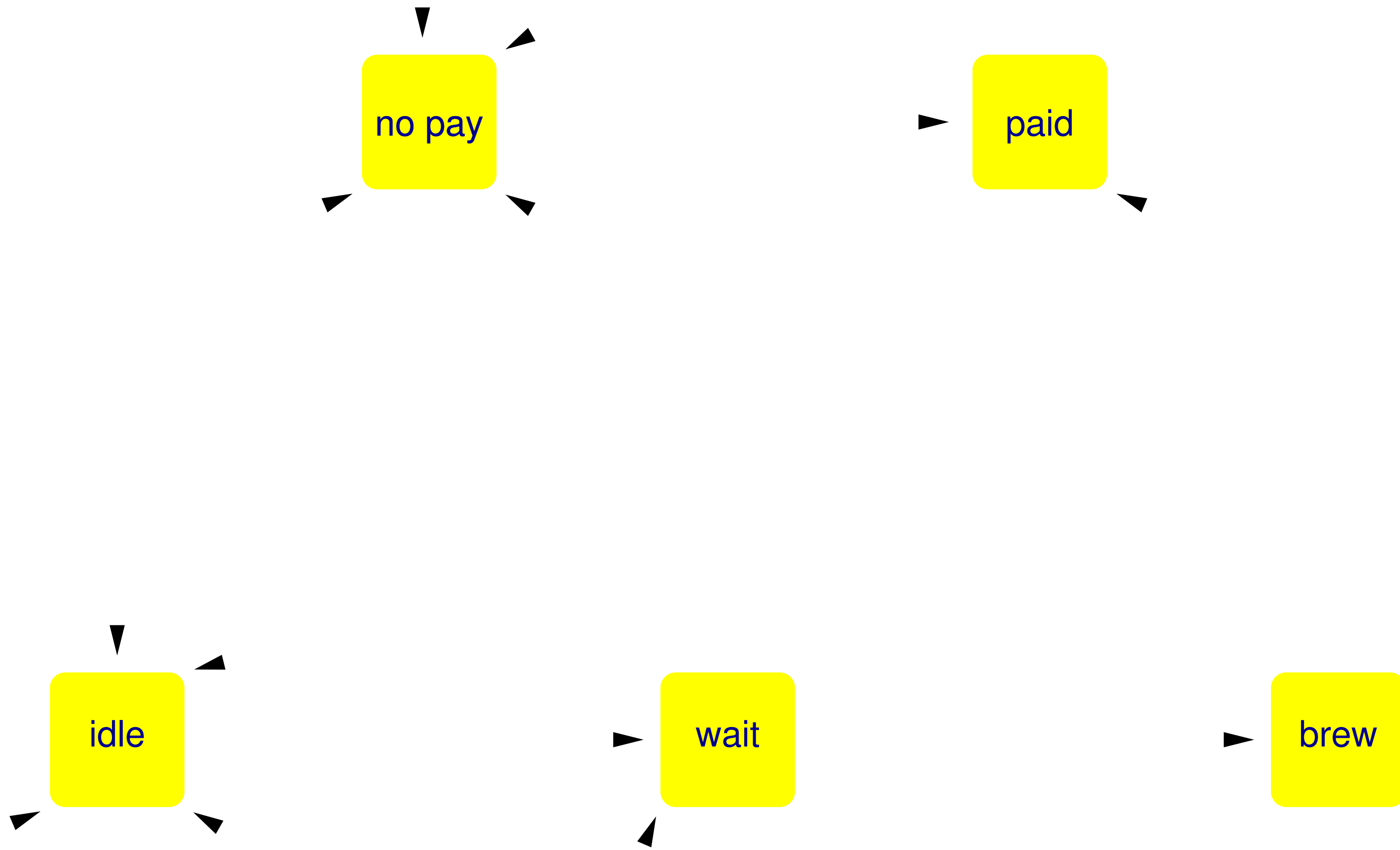
arise naturally in

- descriptions of hardware components,
  - because these are computational devices with finite (though sometimes large...) memory,
- descriptions of communication protocols,
  - because these engage into an alternation of finitely many different phases,
- descriptions of embedded computer systems,
  - because these are computers with finite memory.

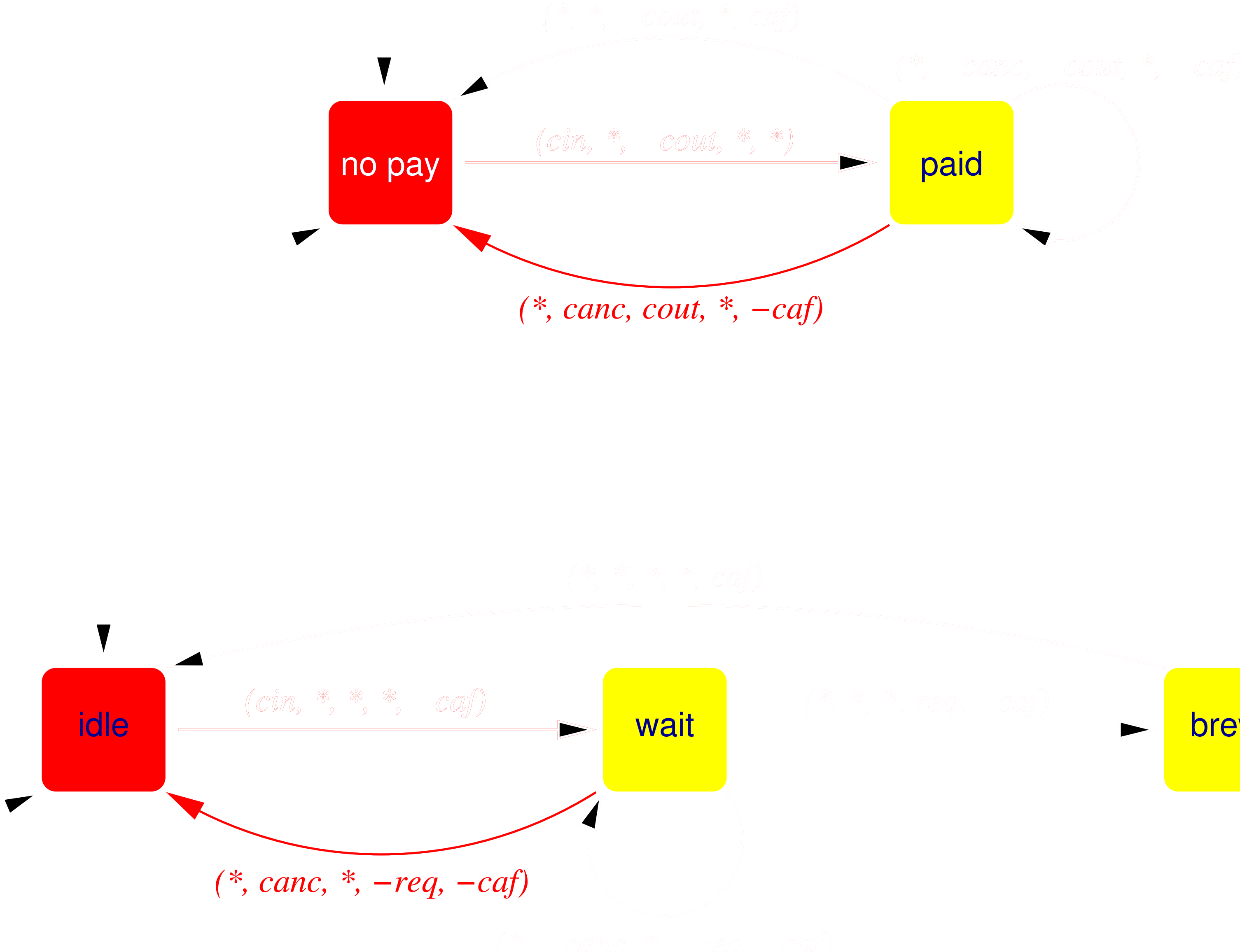
The coffee vending machine — architecture



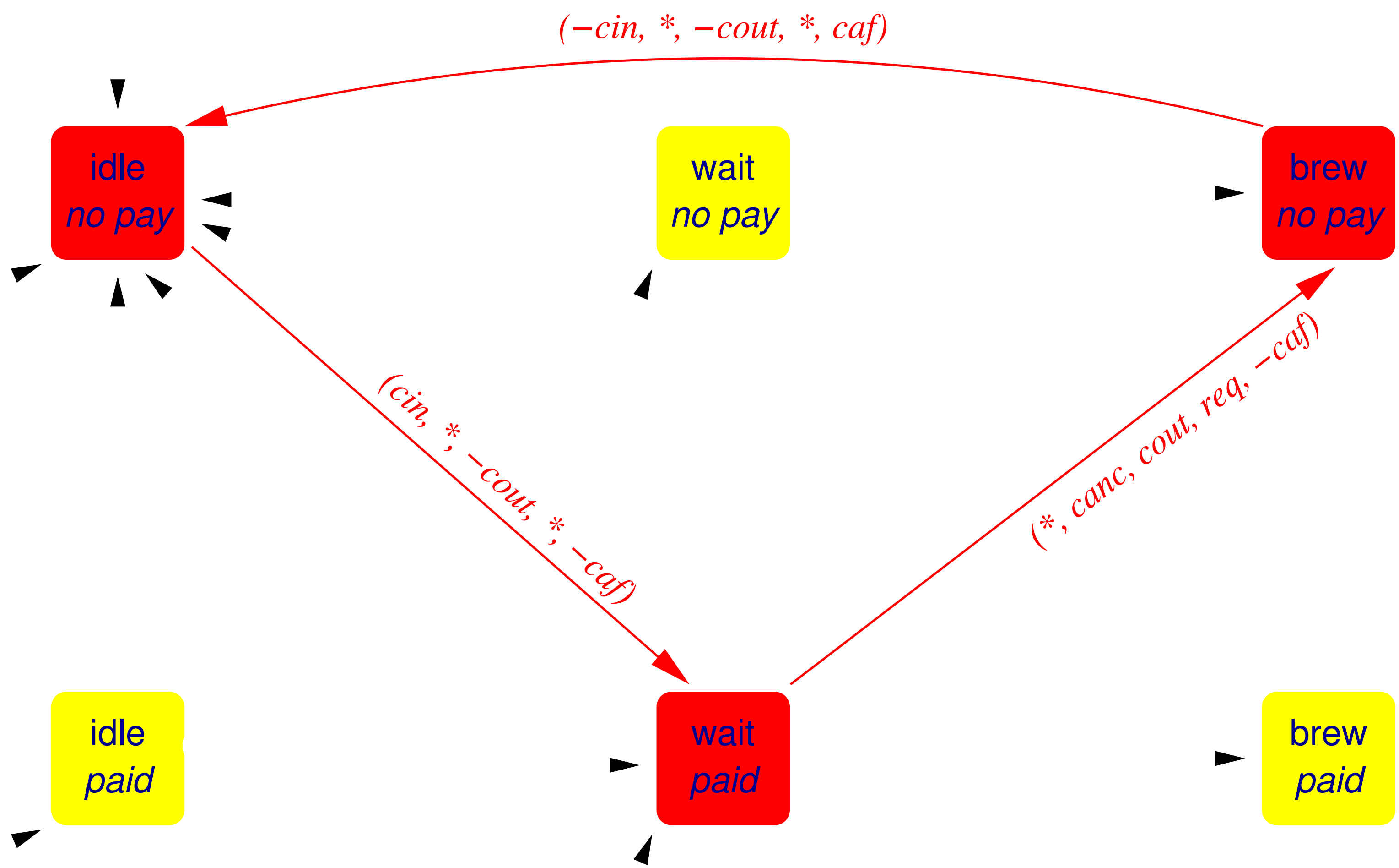
The coffee vending machine — dynamics



An example run



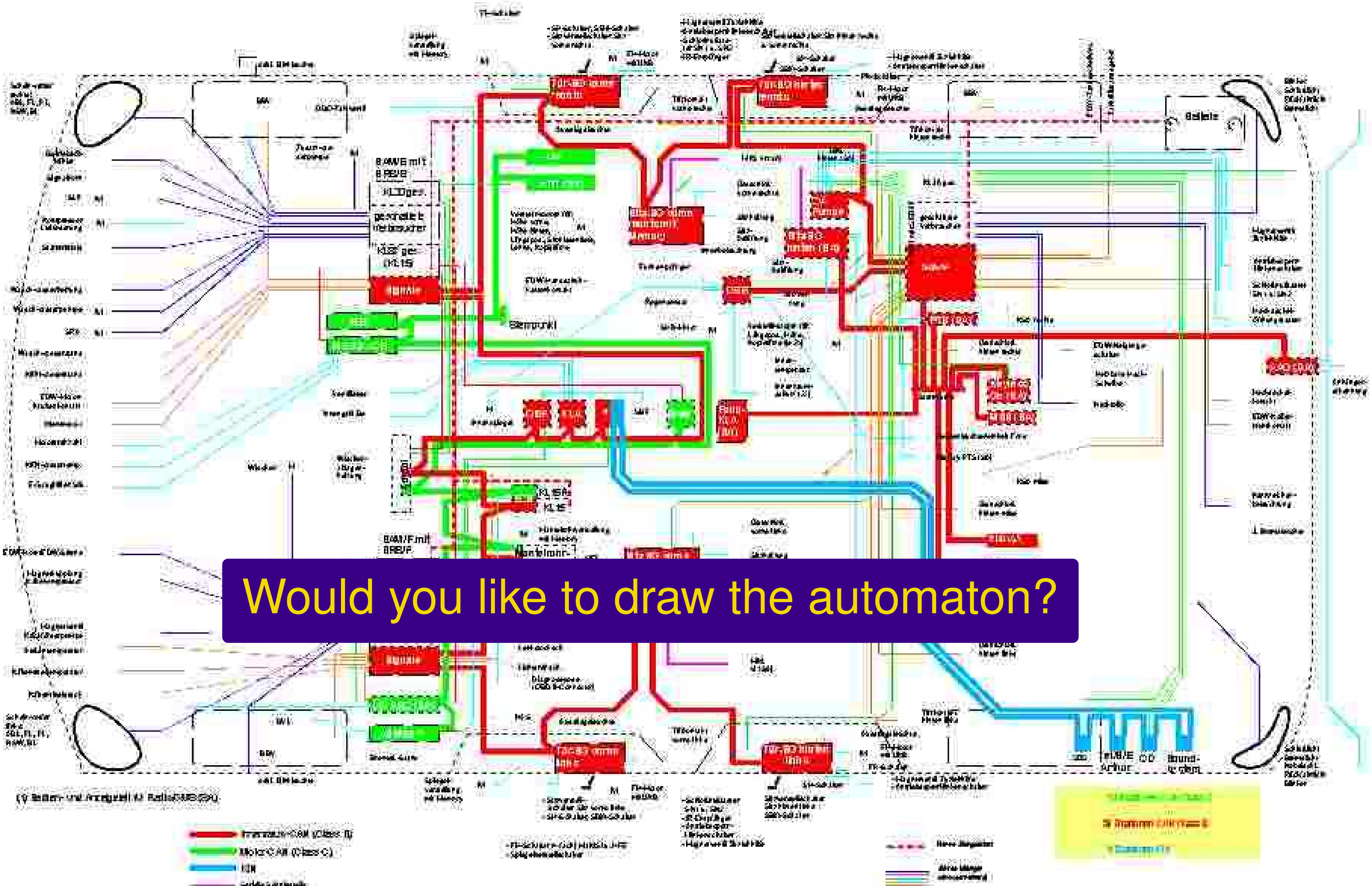
The coffee vending machine



Unreachable state

Free coffee!

Embedded systems in-the-large

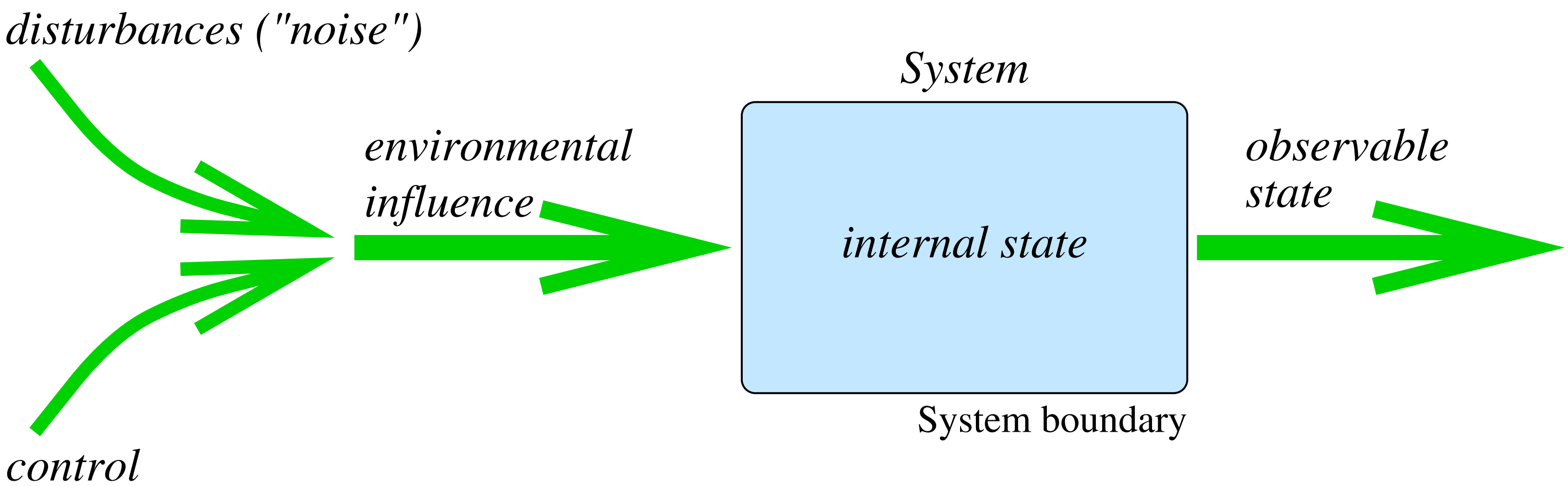


Would you like to draw the automaton?

State-based models:

Differential equations

Open (contin. time & state) dynamical system



- Time is continuous: $\mathbb{R}_{\geq 0}$,
- internal state is a bunch of real-valued (or complex-valued) functions of time: $\vec{x}(.) : \mathit{Time} \longrightarrow \mathbb{R}^n$,
- observable state is a time-invariant function (usually projection) thereof,
- environment influence is a bunch of real-valued (or complex-valued) functions of time: $\vec{u}(.) : \mathit{Time} \longrightarrow \mathbb{R}^m$.

Continuous modeling with DEs

1. Add further, derived state components: the *derivatives* $\dot{\vec{x}}(.), \ddot{\vec{x}}(.), \ldots$ of the state components.
2. Formulate dynamics as equations between $\dot{\vec{x}}(.), \ddot{\vec{x}}(.), \vec{u}(.), \ldots$

N.B. Higher-order derivatives $x^{(n)}$, $n > 1$, can always be removed by

1. adding a fresh state variable $y(.)$,
2. adding the equation $y(t) = x^{(n)}(t)$,
3. replacing every occurrence of $x^{(n+1)}$ by $\dot{y}$.

Differential equation w/o input / disturbance

The DE describes dynamics of the system by

- providing a state space $\mathbb{R}^n$,
- providing a (piecewise) continuous vector field $f : \mathbb{R}^n \to \mathbb{R}^n$ constraining the possible evolutions through the equation

$$\frac{dx}{dt} = f(x)$$

The initial value $x_0 \in \mathbb{R}^n$ defines the start state of the dynamic evolution.

A solution in the sense of Carathéodory is a time-dependent signal $x : [0, a) \to \mathbb{R}^n$ such that

- x is piecewise differentiable,
- $\forall t \in [0, a) \bullet x(t) = x_0 + \int_0^t f(x(s))ds$.

Then $\frac{dx}{dt}(t) = f(x(t))$ for *almost* all $t \in [0, a)$.

Differential equation with input

The DE describes dynamics of the system by

- providing a state space $\mathbb{R}^n$,
- providing an input space $\mathbb{R}^m$,
- providing a (piecewise) contin. vector field $f : \mathbb{R}^{n+m} \to \mathbb{R}^n$ constraining the possible evolutions through the equation

$$\frac{dx}{dt} = f(x, u)$$

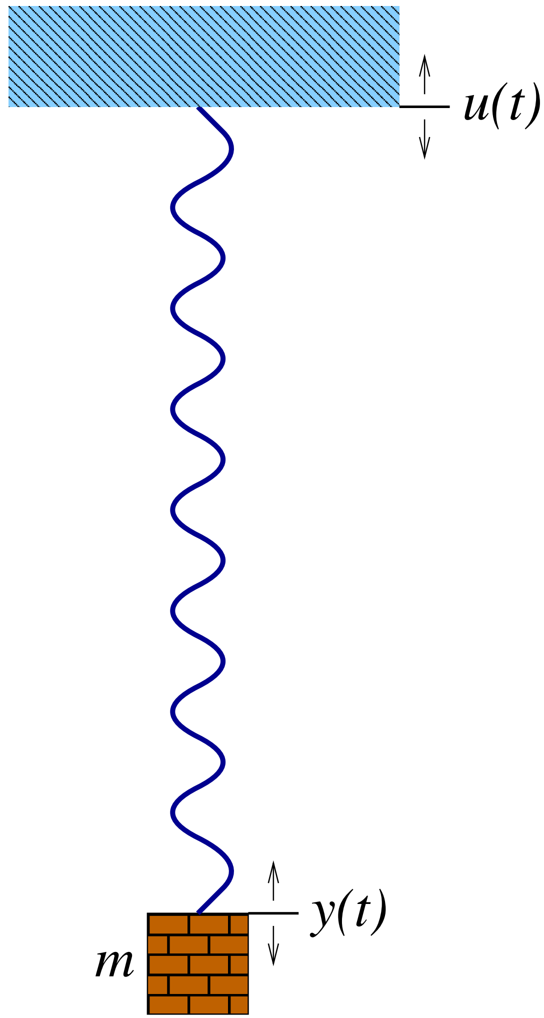
The initial value $x_0 \in \mathbb{R}^n$ defines the start state of the dynamic evolution.

A solution wrt. a (piecewise) continuous input $u : [0, a) \to \mathbb{R}^m$ is a time-dependent signal $x : [0, a) \to \mathbb{R}^n$ such that

- x is piecewise differentiable,
- $\forall t \in [0, a) \bullet x(t) = x_0 + \int_0^t f(x(s), u(s)) ds$.

Then $\frac{dx}{dt}(t) = f(x(t), u(t))$ for almost all $t \in [0, a)$.

Example: spring-mass system w. disturbance



- **Basic model:**

$$\begin{aligned} \ddot{y}(t) &= \frac{F(t)}{m} \\ F(t) &= k\,(l(t) - l_0) \\ l(t) &= u(t) - y(t) \end{aligned}$$

- **Replace higher-order derivatives:**

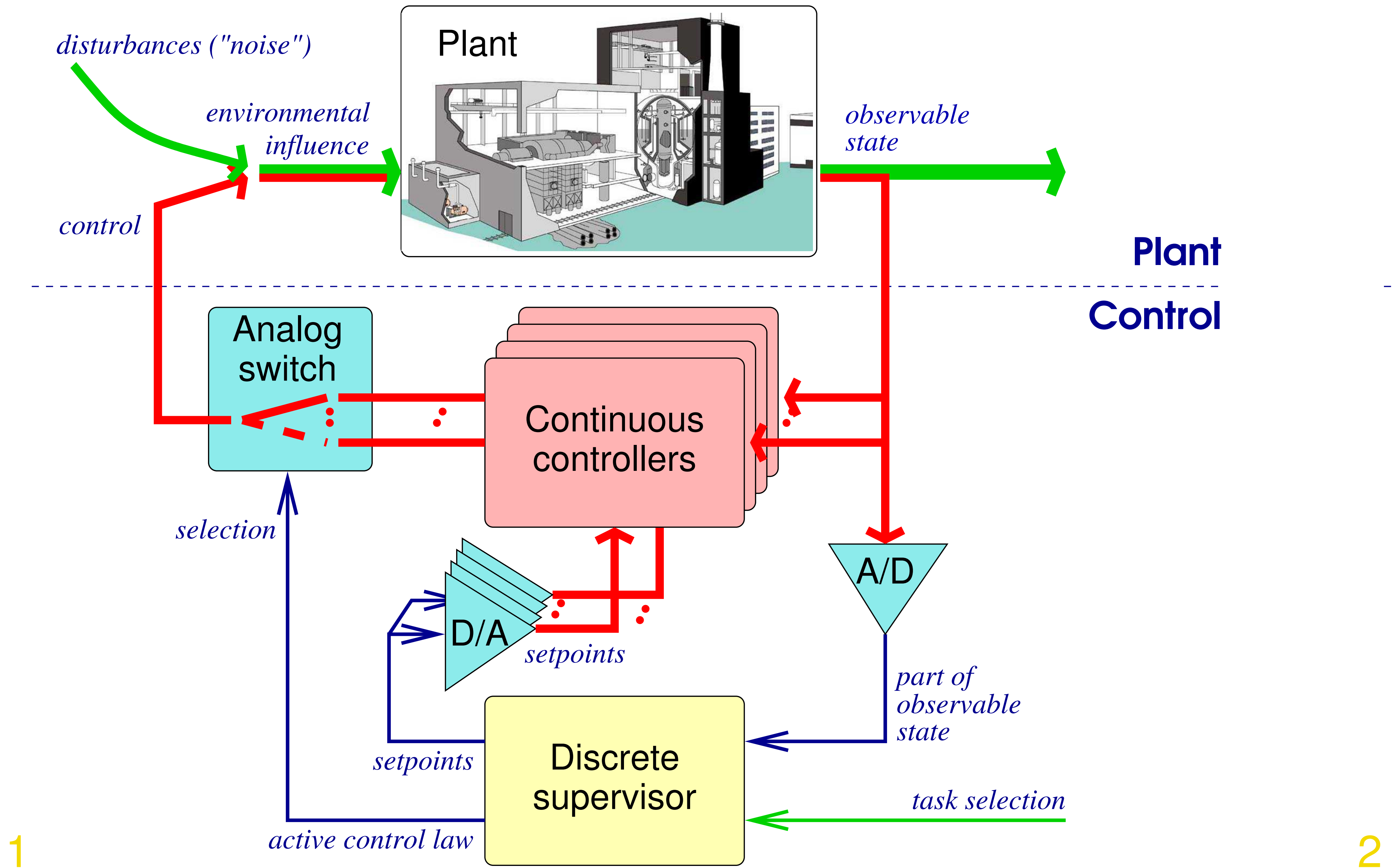
  Add $v(t) = \dot{y}(t)$.

  Gives $\begin{aligned} \dot{y}(t) &= v(t) \\ \dot{v}(t) &= \frac{k}{m}\,(u(t) - y(t) - l_0) \end{aligned}$

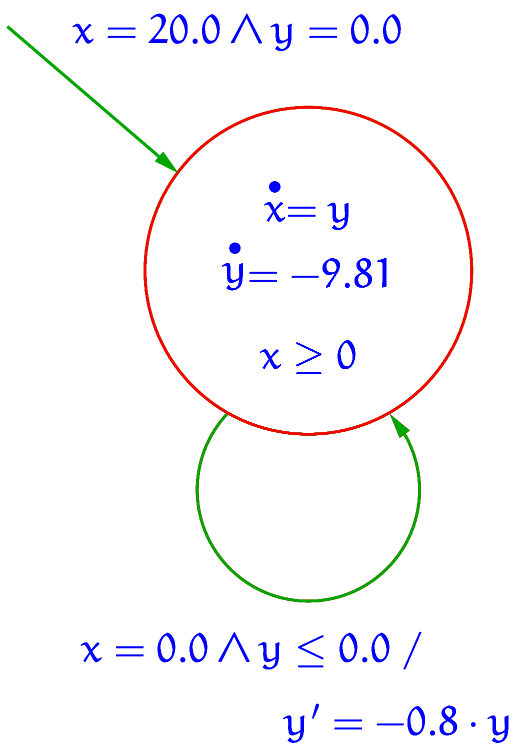
State-based modeling:

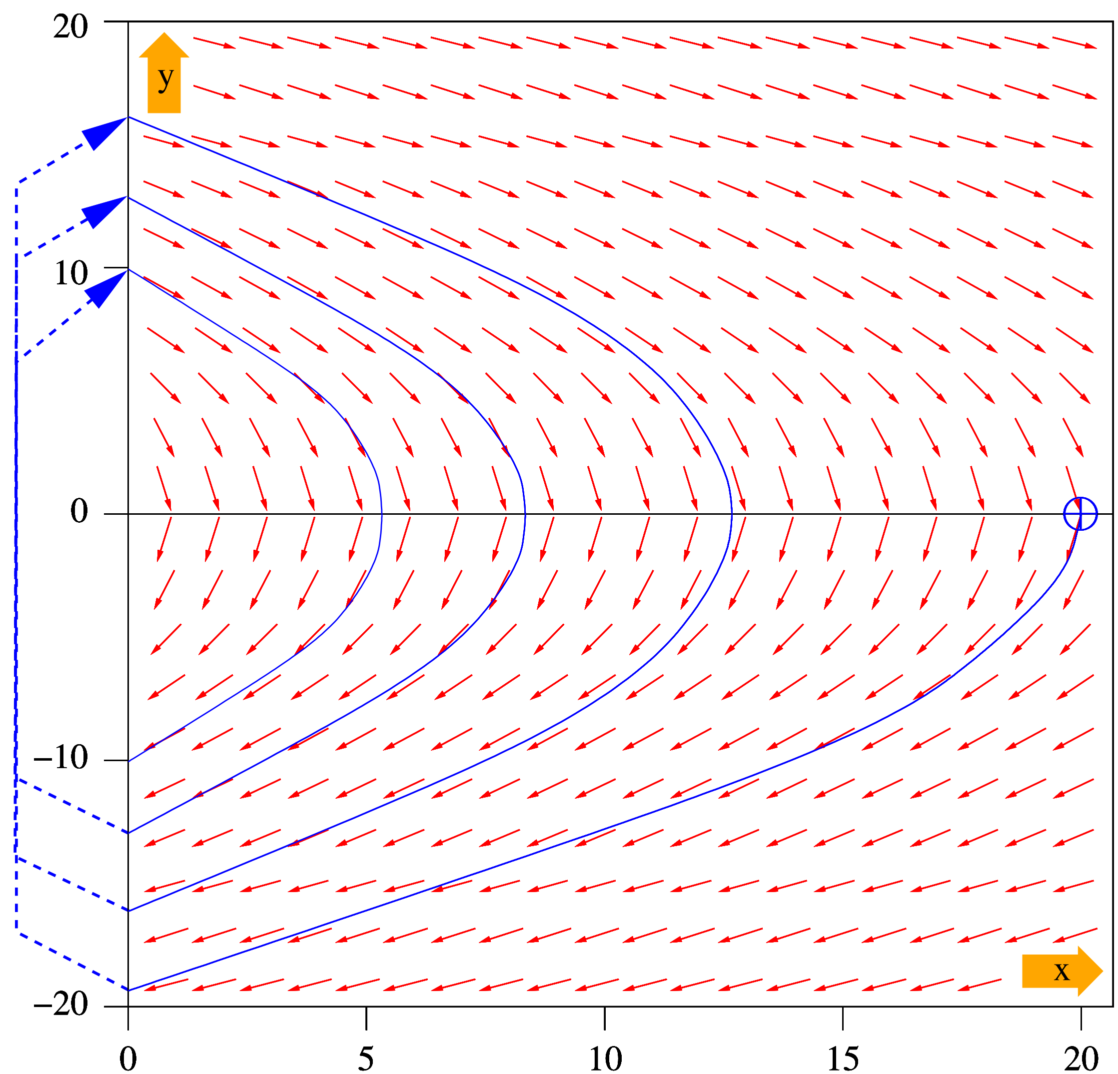
Hybrid systems

Hybrid Systems



Hybrid Automata





x : vertical position of the ball

y : velocity

- $y > 0$ ball is moving up
- $y < 0$ ball is moving down

State and Dimension Explosion

Number of continuous variables linear in number of cars

- Positions, speeds, accelerations,
- torque, slip, ...

Number of discrete states exponential in number of cars

- Operational modes, control modes,
- state of communication subsystem, ...

Symmetry reduction often impossible

- Latency in ctrl. loop depends on number of cars due to communication subsystem.
- Hidden channels due to coupled dynamics.

- Need a scalable approach
- Trying to achieve this through strictly symbolic methods.

Semantic Modeling of ES

“Embedded Systems are Predicates”
(©E. Hehner)

Symbolic transition system

Given a predicate language L, a symbolic transition system STS over L comprises

- a set V of variable names belonging to L,
  plus a *sort-preserving* renaming operation $.'$ assigning to each variable in V a "copy" $v' \notin V$,
- an initialization predicate $I \in L$ with $\mathsf{free}(I) \subseteq V$,
- a (symbolic or predicative) transition relation $T \in L$ with $\mathsf{free}(T) \subseteq V \cup V'$.

A run of symb. trans. sys. STS is a (finite or infinite) sequence $r = \langle \sigma_1, \sigma_2, \sigma_3, \ldots \rangle$ of L-interpretations such that

Initiation: $\sigma_1 \models I$ and

Consecution: $\sigma_{i,i+1} \models T$ for each $i < \mathsf{len}(r)$, where

$$\sigma_{i,i+1}(x) = \begin{cases} \sigma_i(x) & \text{iff } x \notin V' \\ \sigma_{i+1}(x) & \text{iff } x \in V' \end{cases}$$

Parallel composition

Assume $STS_1 = (V_1, I_1, T_1)$ and $STS_2 = (V_2, I_2, T_2)$

- control (i.e., constrain v') a subset $\mathrm{Out}_i \subset V_i$ of variables,
- leave the remaining variables unconstrained.

Synchronous execution: If $\mathrm{Out}_1 \cap \mathrm{Out}_2 = \emptyset$ then

$$\mathrm{STS}_{\|} = (V_1 \cup V_2, I_1 \wedge I_2, T_1 \wedge T_2)$$

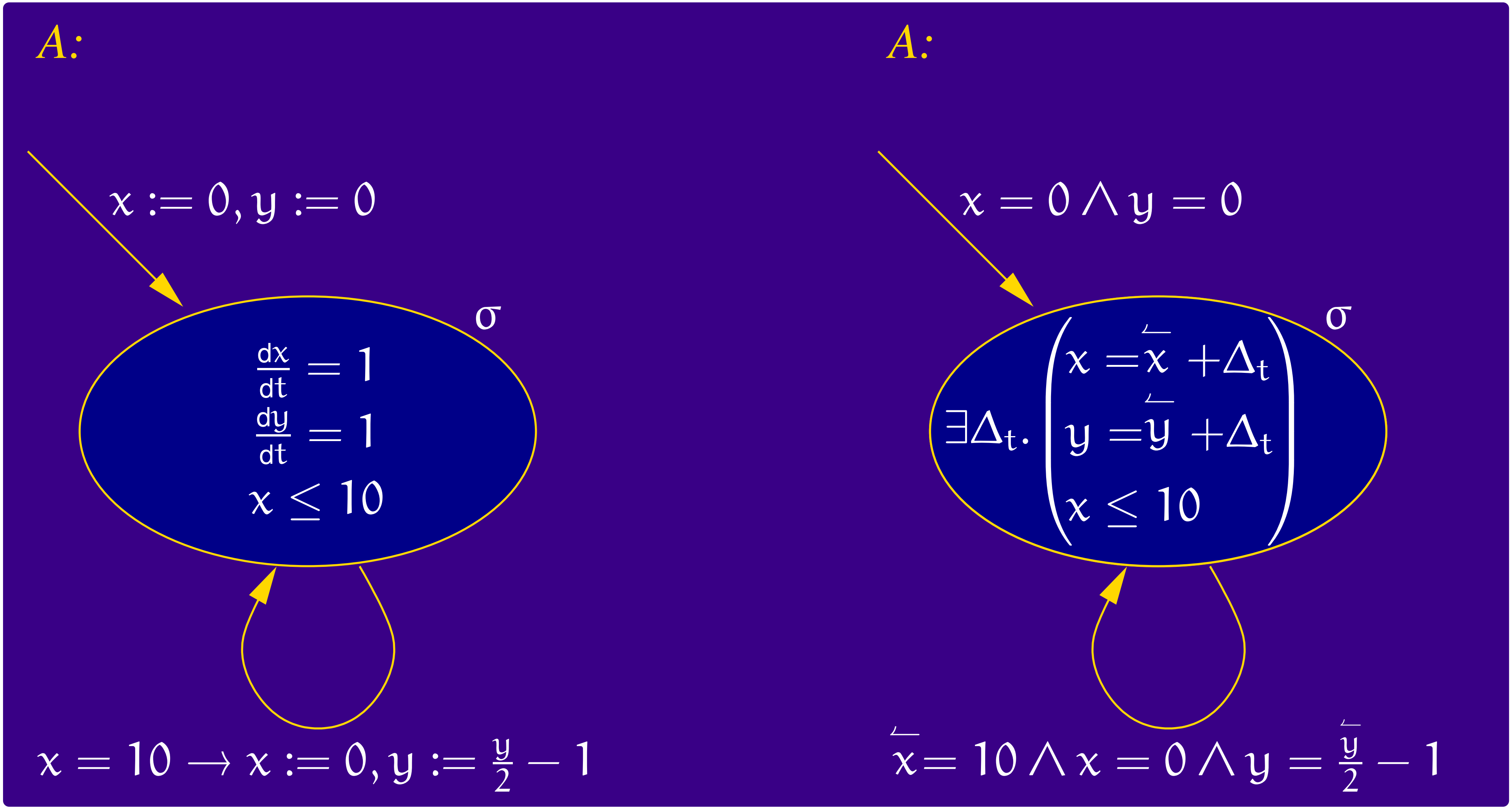
yields step-synchronous parallel execution.

Asynchronous execution:

$$\mathrm{STS}_{\|} = (V_1 \cup V_2, I_1 \wedge I_2, (T_1 \wedge \underbrace{\bigwedge_{v \in O_2 \setminus O_1} v' = v}_{\text{framing}}) \vee (T_2 \wedge \underbrace{\bigwedge_{v \in O_1 \setminus O_2} v' = v}_{\text{framing}})$$

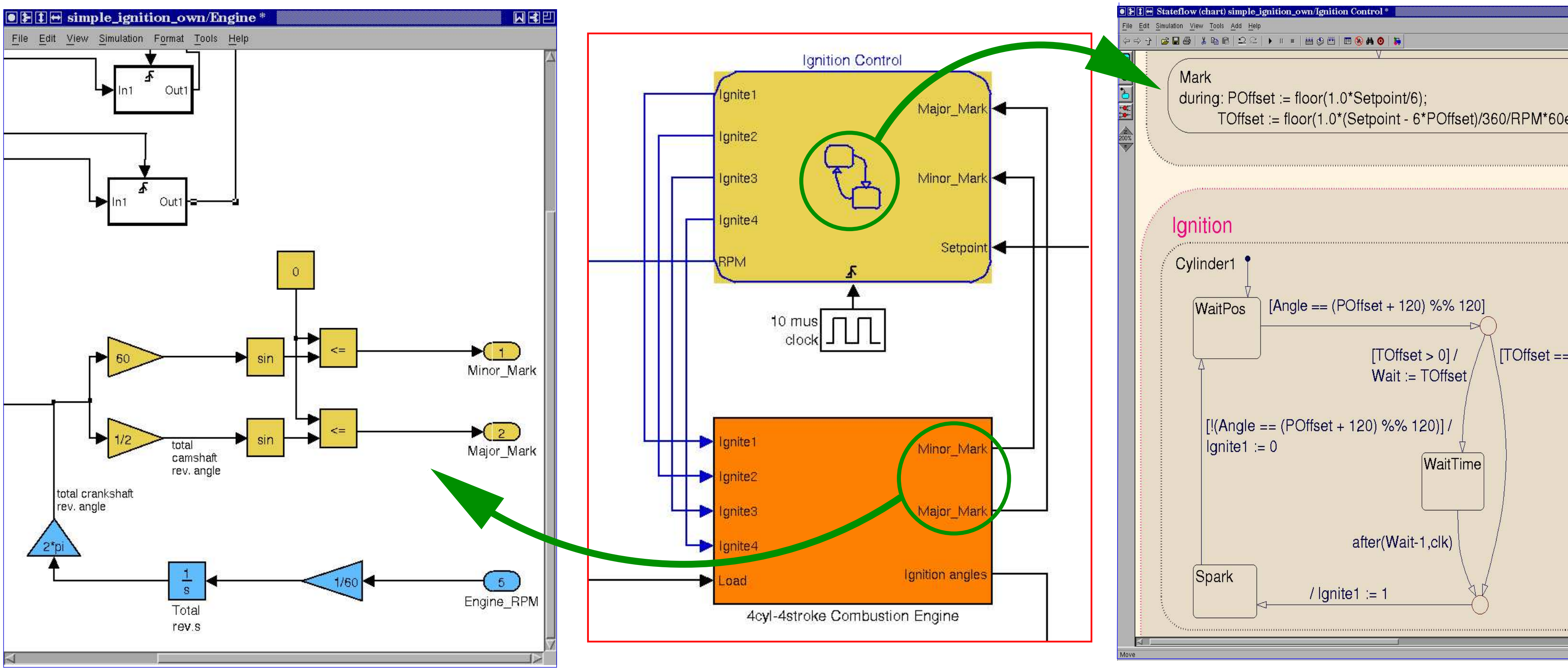
yields (non-fair) interleaving.

Symbolic Representation: Principle

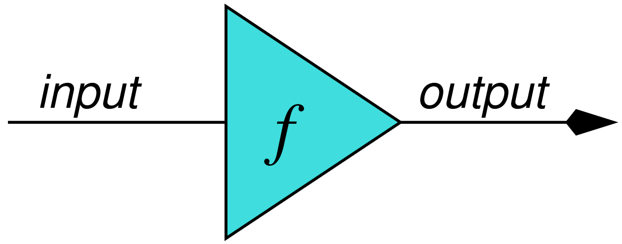


- symbolic representation of linear size
- provided ODEs are of appropriate kind.

Generalizing the concept: Simulink+Stateflow



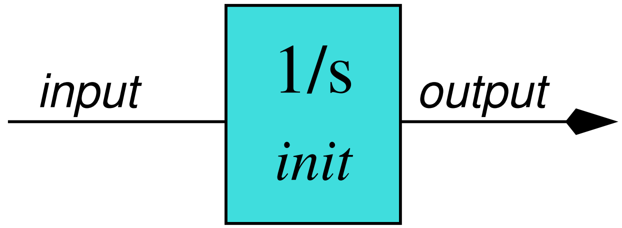
‘Algebraic’ blocks



- time-invariant transfer function $\mathit{output}(t) = f\,(\mathit{input}(t))$
- made 1st-order by making time implicit: $\mathrm{Flow} \equiv \mathit{output} = f\,(\mathit{input})$
- no constraints on initial value: $\mathrm{Init} \equiv \mathsf{true}$,
- discontinuous jumps always admissible $\mathrm{Jump} \equiv \mathsf{true}$,

All the formulae are elements of a suitably rich 1st-order logics over $\mathbb{R}$.

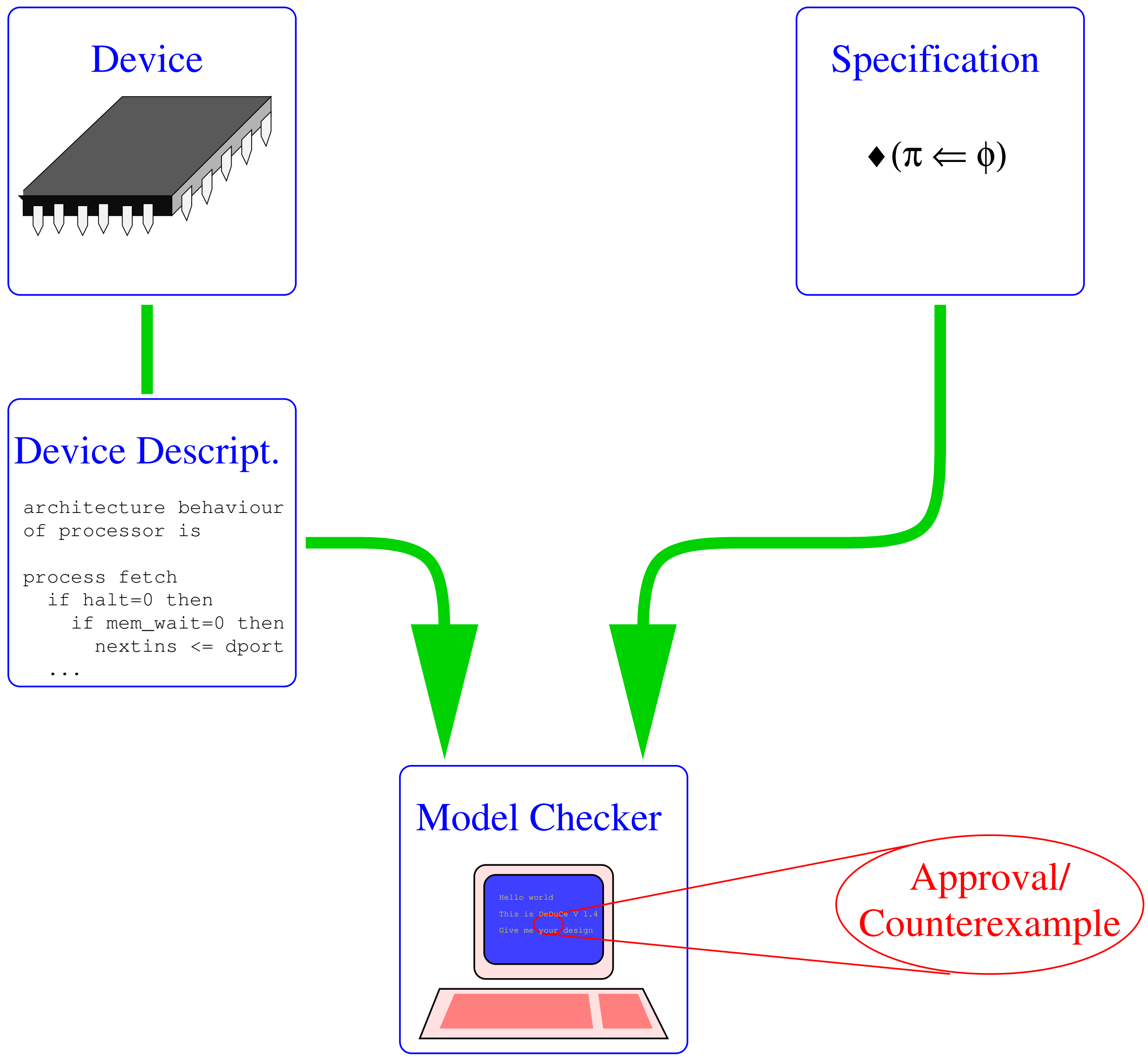
Integrators



- integrates its input over time: $\mathit{output}(t) = \mathit{init} + \int_0^t \mathit{input}(u)\,du$.
- made semi-1st-order by using derivatives: $\mathrm{Flow} \equiv \frac{d\mathit{output}}{dt} = \mathit{input}$
- initial value is rest value: $\mathrm{Init} \equiv \mathit{output} = \mathit{init}$,
- discontinuous jumps don't affect output $\mathrm{Jump} \equiv \mathit{output} = \overleftharpoon{\mathit{output}}$,

Getting started with STS: NuSMV

Model checking



NuSMV

- NuSMV (A. Cimmati, E. Clarke, F. Giunchiglia, A. Morichetti, M. Roveri et al.) is an optimized reengineering of the symbolic model checker SMV (K. McMillan, 1993)
- It is dedicated to finite state systems, providing
  - Booleans, bounded integers, enumerations as data types.
- It has a structured, symbolic language for describing initial state sets and transitions:
  - either *declarative*

    ```
    TRANS
      next(output) = !input;
    ```

    where totality of the transition relation has to be guaranteed by the user, or
  - *"imperative"* (single assignment!) providing such guarantee

    ```
    ASSIGN
      next(output) := !input;
    ```

The imperative syntax

An inverter taking one step delay:

```
MODULE main
VAR
  input : boolean;
  output : boolean;
ASSIGN
  init(output) := 0;
  next(output) := !input;
```

Note that `input` is unconstrained!

The imperative syntax

An inverter taking arbitrary delay (and missing transient inputs):

```
MODULE main
VAR
  input : boolean;
  output : boolean;
ASSIGN
  init(output) := 0;
  next(output) := (!input) union output;
```

Note that `output` has a non-deterministic assignment!

The imperative syntax

```
MODULE inverter(input)
VAR
  output : boolean;
ASSIGN
  init(output) := 0;
  next(output) := !input;

MODULE main
VAR
  gate1 : inverter(gate3.output);
  gate2 : inverter(gate1.output);
  gate3 : inverter(gate2.output);
```

This is synchronous execution, while...

The imperative syntax

```
MODULE inverter(input)
VAR
  output : boolean;
ASSIGN
  init(output) := 0;
  next(output) := !input;

MODULE main
VAR
  gate1 : process inverter(gate3.output);
  gate2 : process inverter(gate1.output);
  gate3 : process inverter(gate2.output);
```

this is asynchronous execution, and...

The imperative syntax

```
MODULE inverter(input)
VAR
  output : boolean;
ASSIGN
  init(output) := 0;
  next(output) := !input;
FAIRNESS
  running;

MODULE main
VAR
  gate1 : process inverter(gate3.output);
  gate2 : process inverter(gate1.output);
  gate3 : process inverter(gate2.output);
```

is *fair* asynchronous execution.

Rules governing the “imperative” fragment

In order to ensure totality of the transition relation, there is a

single assignment rule: For each variable, there is *at most one* assignment (which may contain case distictions).

```
next(abr) :=
    case forget = 0 : ab;
         forget = 1 : abr;
    esac;
```

Note that assignments to both v and next(v) do also constitute multiple assignments to v!

non-circular dependencies rule: Circular dependencies are to be broken by “delays”, i.e. a variable may only depend on older values of itself.

Specification patterns for the exercises

You'll need the following types of CTL (computation tree logic formulae):

ϕ holds invariably: `AG` ϕ

- On all computation paths, ϕ holds generally.

ϕ leads to ψ: `AG` $(\phi$`->AF` $\psi)$

- On all computation paths, it holds generally that if ϕ holds then necessarily ($=$ on all computation paths), ψ holds eventually.

> Tutorial available at
>
> `http://nusmv.irst.itc.it/NuSMV/tutorial/v24/tutorial.pdf`

Course Schedule

(this week)

Schedule

4 Slots a day, 2 in the morning, 2 in the afternoon.

Monday:

1. Introductory lecture

2.+3. Exercise class: State-exploratory verification using NuSMV

4. Lecture: CTL and CTL model checking

Tuesday:

1. Lecture: Checking Circuit Equivalence

2.+3. Exercise class: dito

4. Lecture: Satisfiability solving of large propositional formulae

Schedule (cntd.)

Wednesday:

1. Lecture: Symbolic methods for finite-state model checking

2.+3. Exercise class: Circuit equivalence (cntd.)

4. Lecture: Symbolic methods for real-time

Thursday:

1. Lecture: Arithmetic satisfiability solving

2.+3. Exercise class: Using arith. SAT solving for scheduling

4. Lecture: Hybrid state-space exploration I

Friday:

1. Lecture: Hybrid state-space exploration II
2. Exercise class: Introduction to the projects,
   time to continue with previous exercises
3. Lecture: Game-theoretic synthesis