Stochastic Simulation Discrete simulation/event-by-event

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Discrete event simulation

- Continuous but asynchronous time
- Systems with discrete state-variables
  - Inventory systems
  - Communication systems
  - Traffic systems - (simple models)
- even-by-event principle

Elements of a discrete simulation language/program

- Real time clock
- State variables
- Event list(s)
- Statistics

The event-by-event principle

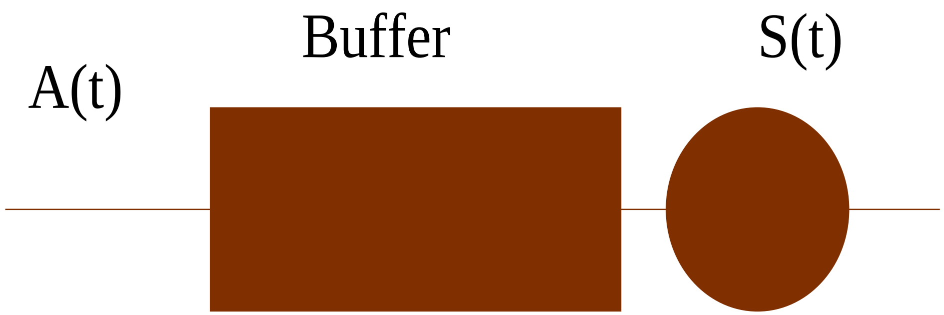
- Advance clock to next event to occur
- Invoke relevant event handling routine
  - collect statistics
  - Update system variables
- Generate and schedule future events - insert in event list(s)
- return to top

Analysing steady-state behaviour

- Burn-in/initialisation period
  - Typically this has to be determined experimentally
- Confidence intervals/variance estimated from sub-samples

Queueing systems

- Arrival process
- Service time distribution(s)
- Service unit(s)
- Priorities
- Queueing discipline

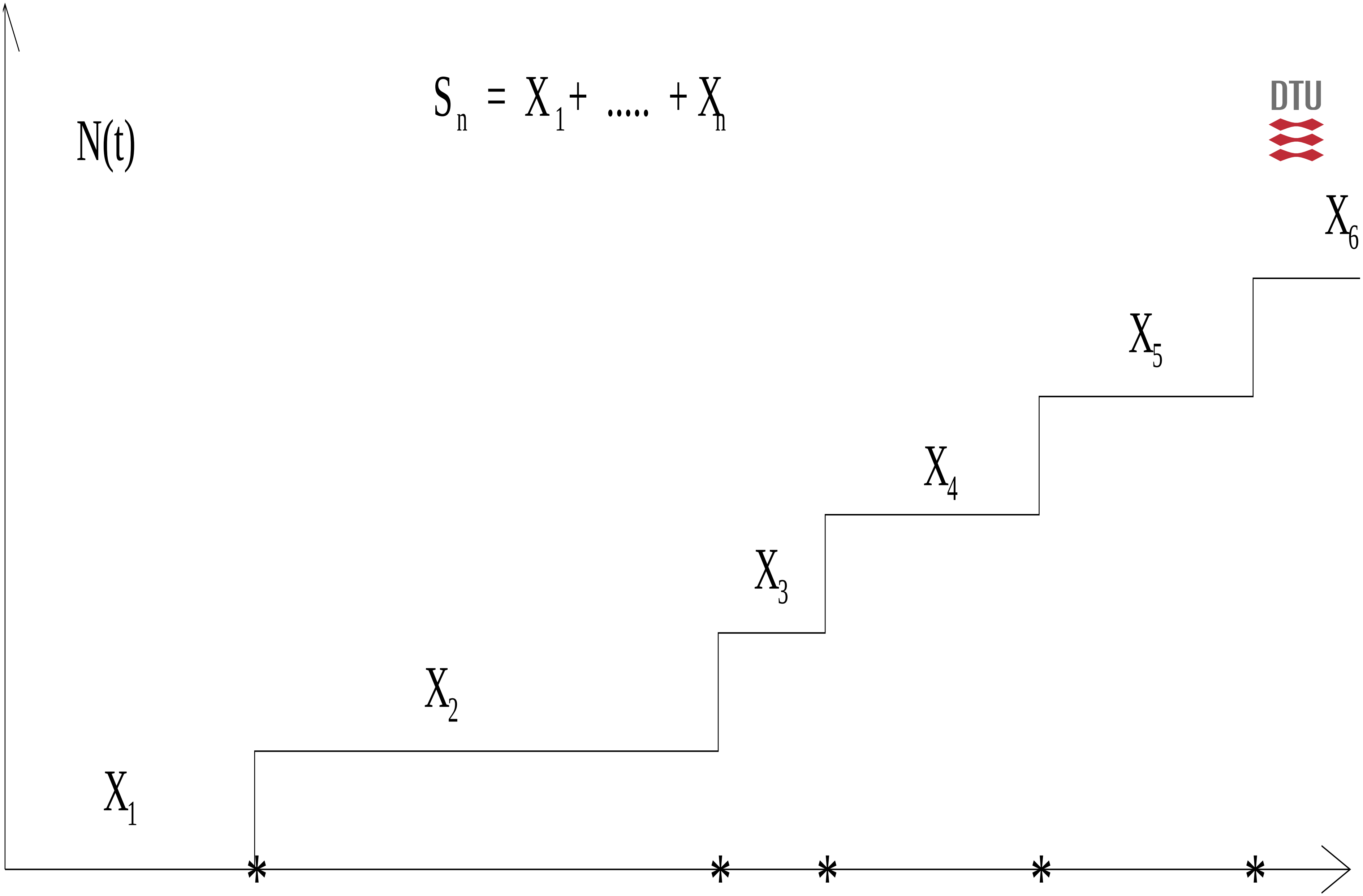


- $A(t)$ - Arrival process
- $S(t)$ - Service process (service time distribution)
- Finite or infinite waiting room
- One or many servers
- Kendall notation: $A(t)/S(t)/N/K$
  - N - number of servers
  - K - room in system (sometime K only relates to waiting room)

Performance measures

- Waiting time distribution
  - Mean
  - Variance
  - Quantiles
- Blocking probabilities
- Utilisation of equipment (servers)
- Queue length distribution

$$S_n = X_1 + \ldots\ldots + X_n$$



Poisson process

- Independently exponentially distributed intervals

$$P(X_i \leq t) = 1 - e^{-\lambda t}$$

- Poisson distributed number of events in an interval. Number of events in non-overlapping intervals independent

$$N(t) \sim P(\lambda t) \Leftrightarrow P(N(t) = n) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}$$

- If the intervals X_i are independently but generally distributed we call the process a renewal process

Sub-samples - precision of estimate

- We need sub-samples in order to investigate the precision of the estimate.
- The sub-samples should be independent if possible
- For independent subsamples the standard deviation of our estimate will be proportional to $\sqrt{n}^{-1}$

Confidence limits based on sub-samples

- We want to estimate some quantity θ
- We obtain n different (independent) estimates $\hat{\theta}_i$.
- The central limit theorem motivates us to construct the following confidence interval:

$$\bar{\theta} = \frac{\sum_{i=1}^{n} \hat{\theta}_i}{n} \qquad S_{\theta}^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} \hat{\theta}_i^2 - n\bar{\theta}^2\right)$$

$$\left[\bar{\theta} + \frac{S_{\theta}}{\sqrt{n}} t_{\frac{\alpha}{2}}(n-1); \bar{\theta} + \frac{S_{\theta}}{\sqrt{n}} t_{1-\frac{\alpha}{2}}(n-1)\right]$$

Confidence limits based on sub-samples - based on normal distribution

$$\left[\bar{\theta} + \frac{S_\theta}{\sqrt{n}} u_{\frac{\alpha}{2}} ; \bar{\theta} + \frac{S_\theta}{\sqrt{n}} u_{1-\frac{\alpha}{2}}\right]$$

General statistical analysis

- More generally we can apply any statistical technique
- In the planning phase - experimental design
- In the analysis phase
  - Analysis of variance
  - Time-series analysis
  - $\vdots$