Exercise 4

Write a discrete event simulation program for a blocking system, i.e. a system with m service units and no waiting room. The offered traffic A is the product of the mean arrival rate and the mean service time.

1. The arrival process is modelled as a Poisson process. Report the fraction of blocked customers, and a confidence interval for this fraction. Choose the service time distribution as exponential. Parameters: m $=$ 10, mean service time $=$ 8 time units, mean time between customers $=$ 1 time unit (corresponding to an offered traffic of 8 Erlang), 10 x 10.000 customers.

   This system is sufficiently simple such that the analytical solution is known. See the last slide for the solution. Verify your simulation program using this knowledge.

2. The arrival process is modelled as a renewal process using the same parameters as in Part 1 when possible. Report the fraction of blocked customers, and a confidence interval for this fraction for at least the following two cases
   (a) Experiment with Erlang distributed inter arrival times The Erlang distribution should have a mean of 1
   (b) hyper exponential inter arrival times. The parameters for the hyper exponential distribution should be $p_1 = 0.8, \lambda_1 = 0.8333, p_2 = 0.2, \lambda_2 = 5.0$.
3. The arrival process is again a Poisson process like in Part 1. Experiment with different service time distributions with the same mean service time and m as in Part 1 and Part 2.
   (a) Constant service time
   (b) Pareto distributed service times with at least $k = 1.05$ and $k = 2.05$.

(c) Choose one or two other distributions.

4. Compare confidence intervals for Parts 1, 2, and 3 then interpret and explain differences if any.

Exercise 4 - exact solution

- With arrival intensity λ and mean service time s
- Define $A = \lambda s$
- Erlang's B-formula

$$B = P(m) = \frac{\frac{A^m}{m!}}{\sum_{i=0}^{m} \frac{A^i}{i!}}$$

- Valid for all service time distributions
- But arrival process has to be a Poisson process