02427 Advanced Time Series Analysis

Computer exercise 2

The main topic of this exercise is parameter estimation.

Part 1

Estimate the parameters in at least one of the systems generating the time series that you simulated in part 1 of computer exercise 1. Use the prediction error method (also refered to as the conditional least squares method) for the estimation.

Hints

See section 5.5 on page 115. Implement and minimize the loss function $Q_n(\theta)$ to estimate the parameters. As to make Part 2 feasible it is advicable to estimate two parameters and keep any remaining parameters fixed. *R:* Use for example the function `optim()` to do the minimization, see ?optim. You have to give the function that is to be optimized as an argument. So make a function, which takes the parameters and returns the value of the loss function, like:

```
RSSSetar <- function(theta)
{
  p1 <- theta[1]
  p2 <- theta[2]
  ## Calculate the objective function value
}
```

then give the initial values of `theta` and the function to `optim()`.
Matlab: You may use `fminsearch`.

Part 2

Use the model chosen in Part 1 and plot contour curves for the loss function for different values of the number of observations, N. Discuss your findings in the report.

Hints

Chose two parameters p_1 and p_2 and calculate $Q_N(p_1, p_2)$ on a regular grid. Plot the contours of $Q_N(p_1, p_2)$ and repeat for subsets of different sizes, for example if 3000 steps were simulated plot contours based on:

```
1:3000
1:300
1:30
1001:1300
1001:1030
```

R: Use for example `filled.contour()`, which makes a contour plot with a key next to the plot. To change the colors, use `color.palette` as an argument, e.g. like:

```
color.palette=colorRampPalette(c("#00007F","blue","#007FFF",
"cyan","#7FFF7F","yellow","#FF7F00","red","#7F0000"))
```

Another tip is to set the plot labels with `title()` and `expression()`, see `?plotmath`, e.g.:

```
title(xlab=expression(a[1]), ylab=expression(a[2]), line=1.5)
```

Matlab: Use `imagesc()` for contour plots.

Part 3

Chose a model for a doubly stochastic system. Write the model on state space form and simulate this model.

Hints

Write the model on state-space form with math in the report. This is done by reparameterizating the model the same way as in the book on page 149. Implement and simulate, and comment on the results.

Part 4

The last part of this exercise concerns the Extended Kalman Filter (EKF).

Introduction

First, consider the following simple state space model:

$$\begin{aligned} x_{t+1} &= a x_t + v_t \\ y_t &= x_t + e_t \end{aligned} \tag{1}$$

where a is an unknown parameter and v_t and e_t are mutually uncorrelated white noise processes with variences σ_v^2 and σ_e^2 respectively.

Your task is to use an EKF to estimate a by including it in the state vector, and study the implications of (erroneous) specifications of the variance σ_v^2 and initial values in the filter.

Part 4a

Do a number of simulations (for example 20) of the model (Eq. 1) having $a = 0.4$ and the variances of the noise processes $\sigma_v^2 = \sigma_e^2 = 1$.

Rewrite the model by including the parameter in the state vector, such that it can be used for parameter estimation with the EKF, as described on page 152 (Section 7.4) in the book.

Part 4b

Do the following to draw some conclusion about the convergence properties for the EKF used:

Estimate the parameter a in all of the simulations, with the initial value of a set to 0.5 and afterwards to -0.5. Plot the estimates and do this for the four combinations:

- In the filter set $\sigma_v^2 = 10$ and initial value of the variance for the parameter a to 1.
- In the filter set $\sigma_v^2 = 1$ and initial value of the variance for the parameter a to 1.
- In the filter set $\sigma_v^2 = 10$ and initial value of the variance for the parameter a to 10.
- In the filter set $\sigma_v^2 = 1$ and initial value of the variance for the parameter a to 10.

Hints

R: The EKF is implemented in `EKF-example.R`.
Matlab: The EKF is implemented in `ekf_example.m`.