# Compression

- Compression
- Lempel-Ziv
- Re-Pair and Grammars

Philip Bille

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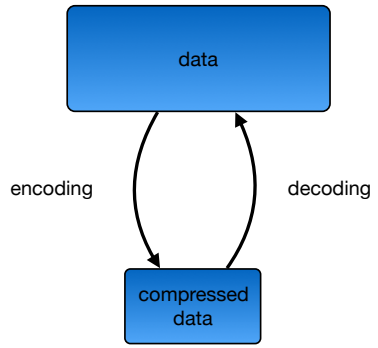
# Compression

- Encoding and decoding.
- Lossless and lossy
- Compressed computation.




# Compression

- Statistical compression.
  - Huffman, arithmetic encoding, Burrows-Wheeler, PPM, ...
- Dictionary compression.
  - Lempel-Ziv 77, Lempel-Ziv 78, Lempel-Ziv-Welch, …
- Grammar based schemes.
  - Re-Pair, sequitur, greedy, bisection, …
- Kolmogorov compression.
  - Ultimate compression scheme.
- Transformation techniques.
  - Differencing, Burrows-Wheeler, run-length encoding, Fourier transform, …

# Compression

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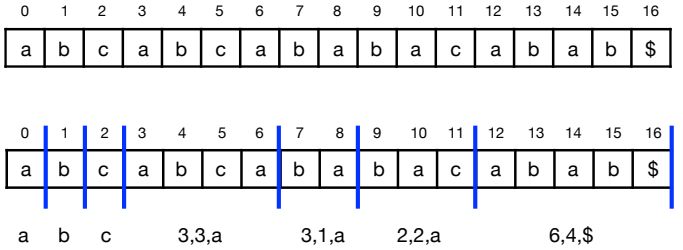
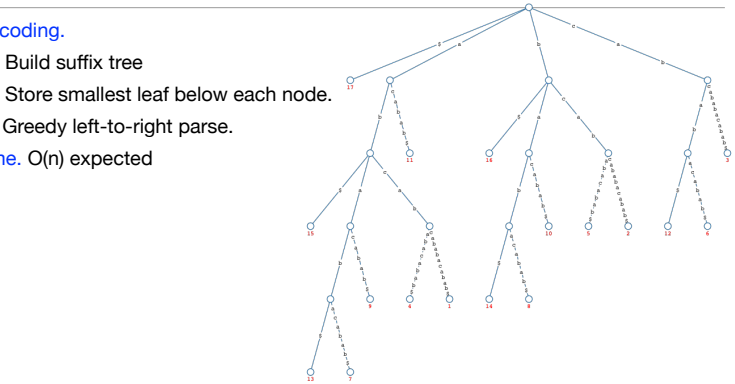
## Lempel-Ziv 77

- Encoding.
  - Parse from left-to-right into phrases.
  - Select longest matching substring starting before current position + 1 character.
  - Encode phrases by (previous occ dist, length, extra character) or single character.



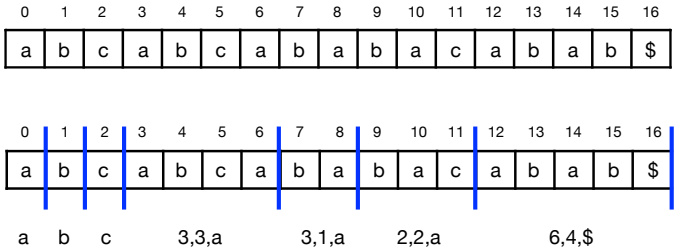

## Lempel-Ziv 77

- Encoding.
  - Build suffix tree
  - Store smallest leaf below each node.
  - Greedy left-to-right parse.
- Time. O(n) expected



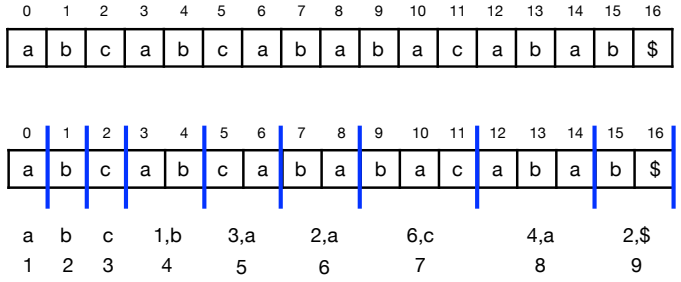

## Lempel-Ziv 77

- Decoding. Read and decode left-to-right.
- Time. O(n)



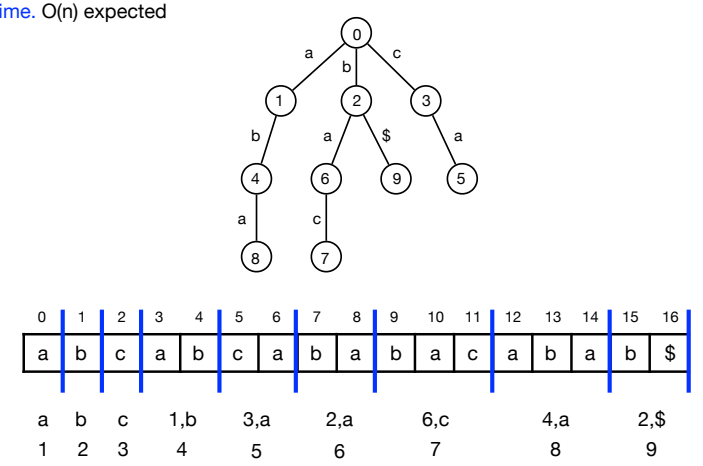
## Lempel-Ziv 78

- Encoding.
  - Parse from left-to-right into phrases.
  - Select longest phrase seen before + a single character.
  - Encode phrases (previous phrase, character) or single phrase



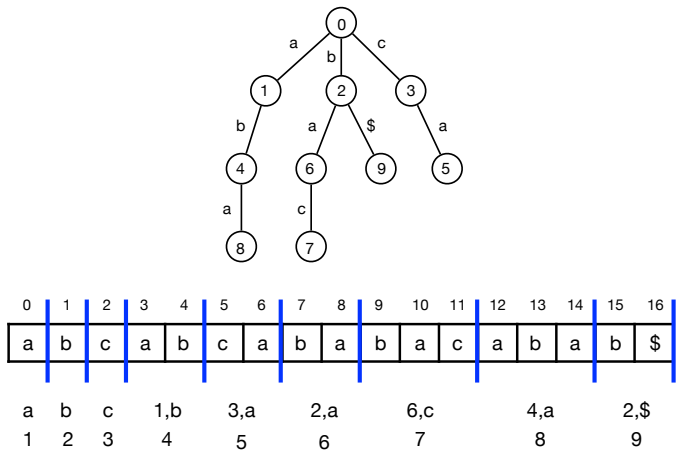

## Lempel-Ziv 78

- Encoding.
  - Dynamically build and traverse the LZ78 trie.
- Time. O(n) expected




## Lempel-Ziv 78

- Decoding. Read and decode left-to-right.
- Time. O(n)

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## Re-Pair Compression

- Recursive-pairing compression [Larsson and Moffat 2000].
  - Start with string S.
  - Replace a most frequent pair ab by new character $X_i$. Output rule $X_i \rightarrow ab$.
  - Repeat until we have a single pair.
- Decoding. Unfold rules top-down.

| | |
|---|---|
| $X_9$ | |
| $X_8X_6$ | $X_9 \rightarrow X_8X_6$ |
| $X_3X_7X_6$ | $X_8 \rightarrow X_3X_7$ |
| $X_3X_4X_5X_6$ | $X_7 \rightarrow X_4X_5$ |
| $X_3X_4X_5X_1X_2$ | $X_6 \rightarrow X_1X_2$ |
| $X_3X_4acX_1X_2$ | $X_5 \rightarrow ac$ |
| $X_3X_1X_1acX_1X_2$ | $X_4 \rightarrow X_1X_1$ |
| $X_2X_2X_1X_1acX_1X_2$ | $X_3 \rightarrow X_2X_2$ |
| $X_1cX_1cX_1X_1acX_1X_1c$ | $X_2 \rightarrow X_1c$ |
| abcabcababacababc | $X_1 \rightarrow ab$ |

## Grammar Compression

- Grammar compression. Encode string S as an grammar G that generates S.
- Parse tree. Unfolded set of rules.

$X_{12} \rightarrow X_{11}X_9$
$X_{11} \rightarrow X_6X_{10}$
$X_{10} \rightarrow X_7X_8$
$X_9 \rightarrow X_4X_5$
$X_8 \rightarrow X_1X_3$
$X_6 \rightarrow X_5X_5$
$X_5 \rightarrow X_4X_3$
$X_4 \rightarrow X_1X_2$
$X_3 \rightarrow c$
$X_2 \rightarrow b$
$X_1 \rightarrow a$