COORDINATED VEHICLE ROUTING

WITH UNCERTAIN DEMAND

Alan L. Erera\textsuperscript{2} and Carlos F. Daganzo,
Institute of Transportation Studies
University of California
Berkeley, CA 94720, USA.

E-mail: alerera@ieor.berkeley.edu
daganzo@ce.berkeley.edu

Web: \url{http://www.ce.berkeley.edu/~daganzo}

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Alan L. Erera and Carlos F. Daganzo
Room 416 McLaughlin Hall, Institute of Transportation Studies,
University of California, Berkeley, CA 94720, USA.

e-mail: alerera@ieor.berkeley.edu and daganzo@ce.berkeley.edu

ABSTRACT

Numerical optimization methods have been developed and applied successfully to many deterministic variants of the so-called vehicle routing problem (VRP). Unfortunately, existing numerical methodologies are not as effective for planning and design problems when uncertainty is a significant issue. In view of this, this presentation will show how approximation models for large-scale uncertain VRP’s can complement conventional optimization methods and allow for the exploration of a broader set of design and operating strategies than is currently possible. The presentation will consider vehicle routing problems where vehicles have a finite capacity and demand is uncertain, focusing on strategies that coordinate the actions of all vehicles in the fleet in real time as information becomes available.

When uncertainty exists, systems should be designed with degrees of flexibility that allow for efficient control in real time. In the case of "single-period" vehicle routing problems, we should determine two things: (i) the system configuration, including the fleet size and composition and an initial set of vehicle routes, and (ii) a dynamic control plan (algorithm) which specifies how vehicle routes are modified in real time as information becomes available. Uncertainty should be considered when designing both the system configuration and its control algorithm. Furthermore, configuration decisions should be made with both the flow of information and the control method in mind. For the capacitated
VRP with uncertain demand, the desirability and feasibility of specific designs will depend on how and when lot size information becomes available and the degree of control that a dispatcher can exert over en-route vehicles.

Researchers have attempted to obtain optimal designs minimizing expected operating costs for problems in which customer lot size information becomes known only after the arrival of a vehicle. Unfortunately, all the solutions proposed to date are based either on configurations that are unlikely to be feasible in practice, such as single-vehicle fleets, or on feasible operating plans that are too restrictive to be appealing in practice. A possible alternative system design that may be more practical and efficient would allow tour failures to be consolidated into secondary “sweeper” routes. The approach here would be to plan initial routes as if the vehicle capacity were smaller (q- < q) to ensure that few primary tours would fail, and then to serve the overflow customers with a set of secondary tours where vehicles are allowed to cooperate. Unfortunately, although this configuration is simple to describe, it is already too difficult to optimize exactly. More promising designs where vehicles would be allowed to cooperate during the primary tours are even more difficult to treat exactly.

The presentation will show how a system in which vehicles are allowed to cooperate during the primary phase can be designed and operated by minimizing and approximate "logistic cost function" of key design parameters. The effectiveness of the proposed strategies is compared against (a) current strategies in which there is little or no coordination, and (b) against deterministic strategies for equivalent problems without uncertainty. It is shown that the introduction of coordination in proper ways lowers the operation cost from the best levels that can be achieved without coordination (a) to levels close to (b).
BASIS OF THE PRESENTATION

EXAMPLES OF SPACE-CONSTRAINED VRP’s WITH UNCERTAINTY

EXAMPLES OF CONTINUUM APPROXIMATION VRP’s
REVIEWS AND OTHER BACKGROUND


OUTLINE

♦ DEFINITIONS AND BACKGROUND

♦ MODELING APPROACH: COORDINATED STRATEGIES
   Static
   Dynamic

♦ DYNAMIC CAPACITY SHARING BY A VEHICLE FLEET
   Formulation
   Modeling
   Optimization
   Proof of concept

♦ CONCLUSIONS
Single-period vehicle routing

Decisions

– depot location, fleet composition, operating strategy

Possible system characteristics

– space-constraints: vehicle size
– time-constraints: time-windows, deadlines
– uncertainty
  • demand-side (locations, lot sizes, service times)
  • supply-side (travel times)
Vehicle routing and uncertainty

Deterministic vehicle routing problem

- $NP$-hard problem for minimum total distance (cost)
  - solution: set of vehicle tours
- Extensive literature: bounds, asymptotic behavior, heuristics, exact (IP) methods

Complications from demand uncertainty

- more difficult: planned tours may fail!
Large-scale approximations

Exploit scale under uncertainty

- continuous approx. of discrete locations, demands
- large-number laws, central limit theorem

Deterministic vehicle routing problem

- A large-number approximation for total distance
- Daganzo and Newell (1984, 1986)

\[
D_T \approx 2\bar{\rho} + k_f \delta^{1/2} A
\]
Space-constrained vehicle routing with uncertain demand

\( \text{VRP}_{\text{SC}}(\text{UD}) \)

**Given:** Depot, fleet of vehicles with space capacities, customer demands random variables with known distributions, point-to-point travel costs

**Find:** Minimum expected cost operating strategy:
- all customer demand satisfied
- no vehicle exceeds capacity

### Operating strategies

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<th>Strategy</th>
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<td><strong>Uncoordinated</strong></td>
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<td>Single vehicle-tour</td>
<td>Dror et al (1989); Bastian and Rinooy Kan (1992); Bertsimas (1992)</td>
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<td>Multiple vehicle-tours; single-zone sweeper tours</td>
<td>Gendreau, Laporte, Seguin (1995,1996)</td>
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<td><strong>Static coordination</strong></td>
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<td><strong>Dynamic coordination</strong></td>
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<tr>
<td>Multiple vehicle-tours; vehicle reassignments</td>
<td>Today</td>
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VRP_{SC}(UD): uncoordinated strategies

Single vehicle-tour

Multiple vehicle-tours
Importance of operating strategy

Single-period deterministic VRP

Single-period stochastic VRP
**Uncoordinated operations**

**Expected tour cost calculation tractable**

- Example: Recursion in Bertsimas (1992)
- $O(K^2n^2)$ per tour; $K$ discrete demand levels

$$E[L] = \sum_{i=0}^{n} d(i, i+1) + \sum_{i=1}^{n} [\delta_i s(i, i) + \gamma_i s(i, i+1)]$$

$$s(i, j) = s(i, 0) + s(0, j) - s(i, j)$$
**VRP_{SC}(UD): approximation model**

**Daganzo and Erera (1999)**
- static coordination; multiple-zone sweep strategy
- consolidation of overflows

**Modeling approach**
- large-scale problem focus
- obtain approximate *logistics cost function* (LCF)
- optimization and testing
Proposed modeling approach

(1) Formulation

(2) Cost modeling
   – approximate logistics cost function (LCF)

(3) Optimization

(4) Implementation w/ details and testing
Capacity-sharing strategies

Partially-planned

(1) Operate tours to predetermined customers

(2) Non-full vehicles dynamically assigned to unserved customers

Local sharing strategies

2-vehicle capacity sharing  4-vehicle capacity sharing
$N$-vehicle capacity-sharing strategy

Region 1: Preplanned tours

Region 2: Dynamically-assigned tours
Formulation

Idealized service region

Design decisions

– number of vehicles, \( N \)
– region 2 radius, \( r \)
– shape of region 1 zones (\( w, L \))
– strategies for:
  • Sweeping region 1 excess demand
  • Allocating vehicles to region 2 customers
Formulation: operating strategy

(1) Line-haul travel to region 1 zone
(2) Local travel between region 1 customers
(3) Line-haul return to region 2 perimeter
(4) Reposition along region 2 perimeter
   (4b) Serve set of unserved region 1 customers
(5) Serve pie-shaped region 2 zone enroute to depot

Region 2 overflow customers: depot-based sweeper tours
Region 2 dynamic assignment

(1) Capacity proportional
(2) Minimal repositioning distance
Modeling

Total line-haul and region 1 local distance

Assumptions

- Equal-sized zones ⇒ \( A = \pi (R^2 - r^2)/N \)
- Near-optimal dimensions (Daganzo (1984))

Expected distance

\[
2N \left( \frac{2(R^3 - r^3)}{3(R^2 - r^2)} \right) + k_f \delta^{1/2} \pi (R^2 - r^2)
\]
Modeling

Repositioning distance

Assumptions

- region 1 tour demand ~ Normal R.V.
- redistribute remaining capacity uniformly

Expected distance

\[
\Phi \left( \frac{\lambda A - C}{(\gamma \lambda A)^{1/2}} \right) N(2\pi r) \sqrt{\frac{\pi \gamma \lambda A}{32 N (C - \lambda A)^2}}
\]
Modeling: repositioning distance

Cumulative excess capacity: diffusion process
- \( X(n) = (C - D)n \)
- \( X(n) \sim \eta((C - \lambda A)n, \gamma \lambda An) \) by CLT

Target curve, given \( X(N) \)
- \( T(n) = (n/N) X(N) \)

Expected reposition distance per vehicle
- \( E[\text{area between curves}] / E[X(N)] \)
Modeling

Region 2 local distance

Assumptions
- width of vehicle pie zone proportional to capacity
- upper bound on remaining capacity variance used

Expected distance

\[
N \left( \frac{\delta r^3}{9} \right) \left( \frac{2\pi}{N} \right)^2
\]
Modeling

**Region 1 overflow service distance**

– assignment distance: included in repositioning
– lateral distance: included in region 1 local distance
⇒ model expected radial distance

\[ 2N \left( \frac{2(R^3 - r^3)}{3(R^2 - r^2)} - r \right) \Phi \left( \frac{C - \lambda A}{(\gamma \lambda A)^{1/2}} \right) G \left( \frac{C - \lambda A}{(\gamma \lambda A)^{1/2}} \right) \]

– \( G \): expected number of vehicles with remaining capacity needed to serve an overflow zone
Expected Distance Analysis

Demand rate: 20 items/area
Index of Dispersion: 5.4
Vehicle Capacity: 75

Minimum internal radius: 5.4
Distance: 1980

Minimum line haul distance: 1227
Simulation

Inputs
\[ R, \delta, F_D, r, N \]

Generate instance
- locations :: 2-D Poisson process
- demands :: \( F_D^{-1} \)

Create region 1 tours
- modified “sweep” heuristic
- cluster-first, route-second

Operate region 1 tours
- full vehicles :: to depot
- non-full :: to perimeter

Assign region 1 overflows
- greedy “local-matching” method

Operate overflow tours

Create region 2 tours
- pie size (\( \theta \)) ~ vehicle capacity

Operate region 2 tours

Create sweep tours
- modified “sweep” heuristic

Operate sweep tours
Simulation Validation of Expected Cost Model

Distance vs. Internal Radius

Minimum line haul distance: 1227

6.5% and 4.3% changes in distance as compared to the model.
Comparisons: an example

N-vehicle coordinated strategy (radius 5.4)
- predicted OC: 1980; simulated OC: 1953
- # vehicles: 92
- # customers missed on first tour: 1.3%

Uncoordinated single-zone strategy lower bound
- predicted OC: 2240
- # vehicles: 120
- # customers missed on first tour: 1.3%

Savings
~ 25% fewer vehicles, 7% less distance

Comparison with deterministic bound
- TSP-tour partitioning bound OC: 1704
- Cost of uncertainty
  - N-vehicle strategy: 16%
  - uncoordinated strategy: 31%
Expected Distance Analysis

Minimum line haul distance 1227

Uncoordinated single-zone strategy 2240

Deterministic TSP-partitioning bound 1704

Distance vs. Internal Radius
Conclusions

Assessment

– New coordinated strategies for VRP_{sc}(UD)
– Large-scale approximations
– Preliminary proof-of-concept and validation
– Results
  • strategies improve status quo
  • large-scale approximation methods promising

Extensions

– Coordinated strategies for time-constrained vehicle routing
  • deadline problem
  • application: overnight package delivery collections
– Improved control