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A bilevel approach to the traveling salesman problem

In a recent paper, Labbé, Marcotte and Savard (A bilevel model of taxation and its application to optimal highway pricing, Management Science 44, 1998) proposed a general taxation model involving two levels of decision-making. The upper level (leader) imposes taxes on a specified set of goods or servcies while the lower level (follower) optimizes its own objective taking into account the taxation scheme devised by the leader. This problem belongs to the class of bilevel optimization problems where both objective functions are bilinear. They focused on an application of this model called the Toll Setting Problem (TSP). In this problem, the government or a private company (leader) maximizes revenues raised from tolls applied on a specified subset of arcs of a transportation network and the users (the follower) minimize the total costs of their paths using both tolled and toll-free arcs between their respective origins and destinations.

Considering notations of Table 1, the Toll Setting Problem is as follows:

$$
\begin{array}{lll}
\text{TSP:} & \max_{T,x} & \displaystyle\sum_{a\in\mathcal{A}_1} T_a \sum_{k\in\mathcal{K}} x_a^k \\
& \text{s.t.} & T_a \geq l_a \qquad \forall a \in \mathcal{A}_1, \\
& & \\
& \min_{x,y} & \displaystyle\sum_{k\in\mathcal{K}} \Big(\sum_{a\in\mathcal{A}_1} (c_a + T_a) x_a^k + \sum_{a\in\mathcal{A}_2} d_a y_a^k \Big) \\
& \text{s.t.} & \displaystyle\sum_{a\in i^+} (x_a^k + y_a^k) - \sum_{a\in i^-} (x_a^k + y_a^k) = b_i^k \qquad \forall k \in \mathcal{K}, \\
& & x_a^k \geq 0 \qquad \forall k \in \mathcal{K}, \quad \forall a \in \mathcal{A}_1, \\
& & y_a^k \geq 0 \qquad \forall k \in \mathcal{K}, \quad \forall a \in \mathcal{A}_2.
\end{array}
$$

In this talk, we show how to reduce the Traveling Salesman Problem (TSP as well) to the Toll Setting Problem. This new formulation of the Traveling Salesman Problem can be transformed in a mixed integer programming formulation. By deriving the MIP formulation, the classical Miller-Tucker-Zemlin subtour elimination constraints are obtained in a novel way. These results suggest new algorithmic approaches for solving the Traveling Salesman Problem. These will be discussed.

$\mathcal{N}$	node set
$\mathcal{A}$	arc set
$\mathcal{A}_1$	set of tariff arcs
$\mathcal{A}_2$	set of free arcs
$k \in \mathcal{K}$	set of users with the same origin-destination pair
i^-	$\{(j,i) \in \mathcal{A} \ : \ j \in \mathcal{N}\}$
i^+	$\{(i,j) \in \mathcal{A} \ : \ j \in \mathcal{N}\}$
b_i^k	demand at node i for commodity k
c_a	cost per unit of $a \in \mathcal{A}_1$
d_a	cost per unit of $a \in \mathcal{A}_2$
T_a	toll on $a \in \mathcal{A}_1$
l_a	lower bound on T_a
x_a^k	number of users of commodity k on $a \in \mathcal{A}_1$
y_a^k	number of users of commodity k on $a \in \mathcal{A}_2$

Table 1: Notations.