Jacques Desrosiers

École des Hautes Études Commerciales & GERAD

Airline Fleet Assignment with Homogeneity

In the usual sequential process of planning the operations for an airline, the fleet assignment problem arises between the problem of determining a flight schedule given the passenger demand curves for each pair of origin and destination, and the problem of routing individual aircraft to cover all scheduled flight legs while satisfying maintenance requirements. It consists of determining the aircraft type to assign to each flight leg of a given flight schedule. These decisions must be made in order to maximize the sum of the expected profits for each leg (which depend on the chosen aircraft type), while satisfying a certain number of constraints described later on. The sequential planning process is then completed by computing crew rotations and monthly schedules for individual crew members that minimize crew costs while satisfying government regulations and collective agreement working rules.

This research addresses the fleet assignment problem for a weekly flight schedule where it is desirable to assign the same type of aircraft to the legs operating on different days of the week but with the same flight number. Even though it reduces schedule profitability, aircraft type homogeneity is sought in order to improve customer service and the planning of operations. Indeed, when the same aircraft type is assigned to legs with the same flight number, the same gate can be used for these legs, which is considered as more convenient for regular passengers. Also, the ground equipment needed to service and resupply the aircraft can remain near that gate. Finally, when crews are assigned to one-day rotations, as it is often the case for regional carriers, homogeneous assignments allow building the same crew rotations day after day. Such rotations are appreciated by the crew members, which usually prefer regular working days.

To our knowledge, no papers have been published on this extension of the airline fleet assignment problem. However, several papers addressing the classical version can be found in the literature. The problem is often formulated as a mixed integer, linear, multicommodity flow problem with side constraints defined on a time-space network (see Abara (1989), Subramanian *et al.* (1994) and Hane *et al.* (1995)). In Desaulniers *et al.* (1997) and Rexing *et al.* (2000), a variant of the problem is tackled where some flexibility on the flight departure times is allowed. These departure times must fall within given time intervals called time windows. Such flexibility opens up new feasible flight connection opportunities and, thus, can yield a more profitable fleet assignment. Formulating the problem as a special case of the unified formulation for time constrained vehicle and crew scheduling problems subsequently presented in Desaulniers *et al.* (1998), Desaulniers *et al.* (1997) solve it using a column generation approach embedded in a branch-and-bound search tree. Based on a discretization of the time windows, Rexing *et al.* (2000) formulate the problem as an integer linear program similar to the one proposed for the case with fixed departure times and solve it using a preprocessor, an LP solver and a branch-and-bound scheme.

Sometimes, solving the fleet assignment problem for a single day of the week can be a good starting point to obtain a fleet assignment for the whole week. Indeed, if the daily schedules are sufficiently similar from one day to the other, one can obtain an initial (perhaps infeasible) solution by duplicating the one-day solution over the week. This initial weekly solution often needs to be adapted to take into account the minor differences existing between the days (for instance, when a flight is not flown every day of the week or when the passenger demand of a flight varies substantially over the week). In the latter case, it might be advantageous to assign different aircraft types to flight legs having quite different demands.

A weekly assignment derived from a daily solution tends to be homogeneous. However, in most cases, it is not optimal since the aircraft type assignment is determined from the schedule of a given day rather than from the whole week schedule. This is all the more true when the differences between the days are important. In particular, significant differences can often be observed between weekdays and weekend days.

On the other hand, when the problem size allows it, one may solve the problem (without homogeneity) considering the whole week schedule by using a model similar to that of Hane *et al.* (1995). Such an approach can produce optimal solutions in terms of profits but does not favor homogeneity since it is not taken into account.

The first contribution of this research is to propose a model for the fleet assignment problem with homogeneity that can be used to yield high profitable homogeneous solutions. Its second contribution consists of developing different heuristic solution approaches for this model that provide solutions of various qualities in various solution times. Third, these methodologies can be used to evaluate the impact of trading-off profits for homogeneity. Finally, this research shows that optimization tools can be used to produce solutions directly usable in practice by introducing elements that facilitate the operations.

Computational results obtained on Air Canada instances involving up to 4400 flight legs are reported. The system produces realistic solutions arising from a trade-off between profits and homogeneity, and solves large-scale instances in short times.

References

ABARA, J., "Applying Integer Linear Programming to the Fleet Assignment Problem," *Interfaces 19*, 20–28, 1989.

DESAULNIERS G., J. DESROSIERS, Y. DUMAS, M.M. SOLOMON, AND F. SOUMIS, "Daily Aircraft Routing and Scheduling," *Management Science 43(6)*, 841–855, 1997.

DESAULNIERS G., J. DESROSIERS, I. IOACHIM, F. SOUMIS, M.M. SOLOMON, AND D. VILLENEUVE, "A Unified Framework for Time Constrained Vehicle Routing and Crew Scheduling Problems," *Fleet Management and Logistics*, T. Crainic and G. Laporte (eds), Kluwer, Norwell, MA, 57–93, 1998.

HANE C., C. BARNHART, E.L. JOHNSON, R. MARSTEN, G.L. NEMHAUSER, AND G. SIGISMONDI, "The Fleet Assignment Problem: Solving a Large-Scale Integer Program," *Mathematical Programming 70*, 211-232, 1995.

Subramanian R., R.P. Scheff, J.D. Quillinan, D.S. Wiper, and R.E. Marsten, "Coldstart: Fleet Assignment at Delta Air Lines," *Interfaces 24(1)*, 104–120, 1994.

Rexing B., C. Barnhart, T. Kniker, A. Jarrah, and N. Krishnamurthy, "Airline Fleet Assignment with Time Windows," *Transportation Science 34(1)*, 1–20, 2000.