02424 Week 2

Exercise 1

The following are heart rate measurements (beats/minute) of one person measured throughout the day.

```
71  74  82  76  91  82  82  75  79  82  72  90
```

Assume that the data are an iid sample from $\mathrm{N}(\theta, \sigma^2)$, where σ^2 is assumed to be known at the observed sample variance. Sketch the likelihood function for θ if

a) the whole data are reported.

b) only the sample mean $\bar{y}$ is reported.

Find the MLE, $\widehat{\theta}$, and the Hessian (for case a)) using the `optim` function in R.

Exercise 2

The measurements $y_1, y_2, ..., y_n$ are an iid sample from the Poisson distribution with density

$$f(y) = \frac{\lambda^y \exp(-\lambda)}{y!}.$$

a) Write down the combined likelihood function, the log-likelihood function, $l'_\lambda(\lambda; \mathbf{y})$ and $j(\lambda; \mathbf{y})$.
b) Derive the MLE, $\widehat{\lambda}$, and calculate the observed information.

Exercise 3

The following data are number of customers arriving at a cafe per 10 minutes:

```
4  6  3  7  2  4
```

Assume that the data are an iid sample from the Poisson distribution.
Plot the log-likelihood function and the quadratic approximation. Set the maximum of the log-likelihood to zero and check a range of λ such that the log-likelihood is approximately between between -4 and 0. Do the same plot again but this time not on log-scale.